Some Types of Closed Functions

Nassir Ali Zubain[1*], Ali Khalaf Hussain[2*]

Corresponding author: Nassir Ali Zubain

[1*] College of Education for Pure Sciences, Wasit University, Iraq

[1*]E-mail: nasseerali480@gmail.com , alhachamia@uowasit.edu.iq

Abstract

The goal of this search is to get the definition of the closed function. through the concepts that, we study to clarify and benefit from the properties that, we obtain with these definitions, proofs and applied example necessary to solve mathematical problems. The class also sits strictly in between the class of semi-generalized closed sets. We also introduce and study a new class of spaces, we will present the concepts that like both of what we have previously taken to obtain a new study, that the generalized set through its definition as well as the generalized closed set, prepares for us the scientific material to be the starting base between those concepts. We start working again, each concept with the function , for example, the α-closed function, and so on with all the concepts that we passed through the study.

Key Words: α-closed function, generalized closed function, α-generalized closed function, generalized α-Closed function.

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Introduction

To study new concepts he took, N. Levine,[1] "Generalized Closed Sets in Topology Rend." And provided us with content that if useful information that he presented, P. Bhattacharya and B. K. Lahiri. [2] generalized the concept of closed sets to semi-generalized closed sets via semi-open sets. The complement of a semi-open (resp. semi-generalized closed) set is called a semi-closed [2] (resp. semi-generalized open [7]) set. It was taken to clarify some requirements, S. P Arya and T Nour,[3] Characterizations of S-normal space, and then added all of H. Maki, R. Deri and K. Balachandran.[4], Generalized α-closed Sets. In a recent study, the concepts became more powerful, M. CALDAS, S. JAFARI AND R. K. SARAF, [5] "On Some Maps Concerning g α-open Sets". The class of g- closed sets, the class of gα-closed sets and the class of αg-closed sets. Moreover this lesson sits properly in between the class of semi-closed sets and the course of semi-generalized closed sets. And between semi-continuity[8] also αg-continuity [9]

2 On *semi* α-open [*semi* α-closed] sets

In this part ,we shall introduce the concepts of *semi* α-open set (*semi* α-closed), And the basic definition with some examples to explain these sets, also, we shall introduce important definition, remarks, theorem and properties about these sets .

Definition 2.1. [7]

i- A subset A of a topological space X is named to be $\boldsymbol{semi\ \alpha}$-open set,

ii- If there exists an $\boldsymbol{\alpha}$-open set M in X, such that $\boldsymbol{M \subset A \subset Cl\ M}$.The family of all *semi* α-open sets of X is denoted by S $\boldsymbol{\alpha\ O}$ $(\boldsymbol{X})$.

iii- A subset A of a topological space(X, τ)is said to be $\boldsymbol{semi\ \alpha}$-open set. If there exists an open set M in X,such that $\boldsymbol{M \subset A \subset Cl\ Int\ Cl\ M}$.

iv- A subset M of a topological space (X, τ) is called to be $\boldsymbol{semi\ \alpha}$-open Set. If $\boldsymbol{M \subset Cl\ Int\ Cl\ Int\ M}$.

v- A subset A of a topological space X is termed to be semi $\boldsymbol{\alpha}$-open set.

vi- If there exists a semi $\boldsymbol{\alpha}$-open set M in

X, such that $\boldsymbol{M \subset A \subset Cl\, M}$.

Definition 2.2. [7]

The complement of *semi* α-open set is named ***semi* $\boldsymbol{\alpha}$**-closed set.

the family of all *semi* α-closed sets. A subset A of a topological space (X,T)is said to be semi $\boldsymbol{\alpha}$–closed Set. If There exists a closed set M in X such that, $\boldsymbol{Int\, Cl\, Int\, M \subset A \subset M}$.

A subset M of a topological space (X , T) is called to be ***semi* $\boldsymbol{\alpha}$**-closed. If $\boldsymbol{Int\, Cl\, Int\, Cl\, M \subset M}$.

Proposition 2. 3. [7]

Every $\boldsymbol{\alpha}$-open set is ***semi*** -open set.

Proof :

Since M is an α-open set. Then $\subset Int\, Cl\, Int\, M$.

Obviously $Int\, Cl\, Int\, M \subset Cl\, Int\, Cl\, Int\, M$.

Thus M is *semi* α-open set. By definition [2.1.(iii)]

Remark 2.4. [7]

The converse of proposition [2.3.] is not necessarily true in universal .

To see this , the following counter example is given .

Example 2.5.

Let $X = \{1,2,3\}$ and $T = \{\emptyset, \{1\}, \{3\}, \{1,3\}, X\}$.

{2 ,3} is *semi* α-open set, but it is not α-open set .

Example 2.6.

If $X = \{a, b, c, d\}$, $T_X = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, X\}$,

The α-open set; $T^{\alpha}_{(X)} = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}, X\}$, and the *semi* α-open; $S\alpha O(X) = T^{\alpha}_{(X)} \cup \{b, c, d\}, \{a, c, d\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, c\}\}$,

Let $M = \{b, c\}$, thus A is *semi* α-open set,

But M it is not α-open set. Such that α-open $\notin T^{\alpha}_{(X)}$.

On generalized closed

Definition : 3.1. [1]

Let a topological space (X, τ), and $M \subseteq X$, M is called generalized closed set.

If $\boldsymbol{Cl\, M \subseteq N}$, for each N is open set containing M. we denoted by $\boldsymbol{g}$-closed.

Definition 3.2. [2]

Let a topological space (X, τ), and $M \subseteq X$, M is called semi generalized

closed set. If $\boldsymbol{Cl\, semi\, M \subseteq N}$, for each N is *semi*-open set containing M. we

denoted by ***semi* $\boldsymbol{g}$**-closed.

Definition 3.3. [3]

Let a topological space (X, τ), and $M \subseteq X$, M is called generalized semi

closed set. If $\boldsymbol{Cl\, semi\, M \subseteq N}$, for each N is open set containing M. we

denoted by ***g semi*-closed.**

Proposition 3.4.

For each closed set in a topological space (X, τ), is g-closed set.

Proof :

If M is closed set in a (X, τ), and $M \subseteq N$, that is N open set in (X, τ).

As M is closed set thus $Cl\, M = M$,

therefore $Cl\, M \subseteq N$. So M is g-closed set.

Remark 3.5.

a) If M is a subset of the topological space (X,τ), then the following conditions are satisfy.

I- $Cl\,\alpha\,M = N \cup Int\,Cl\,M$.

II- $Int\,\alpha\,M = N \cap Cl\,Int\,M$.

b) The opposite case of the property [3. 4.] is not necessarily true for example.

Example 3.6.

If $X = \{a,b,c\}, \tau = \{\emptyset,\{a\},\{c\},\{a,c\},X\}$.
The closed sets are $F = \{X,\{b,c\},\{a,b\},\{b\},\emptyset\}$, *let* $M=\{a\}$,
and $N = \{a,b,c\}$ *is open set in* (X,τ)
As $Cl\,M = \{a,b\}$, *thus* $Cl\,M \subseteq N$.
then $M = \{a\}$ *is g-closed set, but it is not closed set.*

Proposition 3.7.[4]

For each α*-closed set in a topological space* (X,τ), *is* αg*-closed set.*

Proof :

If M *is* α*-closed set in a* (X,τ), *and* $M \subseteq N$, *that is* N α*-open set in* (X,τ).
As M *is* α*-closed set thus* $Cl\,Int\,Cl\,M = M$,
Since $M \subseteq N$, *then* $Cl\,Int\,Cl\,M \subseteq N$.
And $Cl\,\alpha\,M$ *is smallest* α*-closed set containing* M.
We obtain $Cl\,\alpha\,M = M \cup Cl\,Int\,Cl\,M \subseteq M \cup N \subseteq N$.
So M *is* αg*-closed set.*

Proposition 3.8.[4]

For each g-closed set in (X,τ), *it is* αg*-closed set.*
Proof :
Let M *is g-closed set in* (X,τ), *and* $M \subseteq N$.
There is N *is open set in* (X,τ).
Since M *is g-closed set. Then* $Cl\,M \subseteq N$ *we get* $Int\,Cl\,M \subseteq Int\,N$.
As N *is open set then* $Int\,N = N$. *And obtain* $IntCl\,M \subseteq N$,
Since $cl\,\alpha\,M$ *to be* α*-closed set smallest contain* M.
Therefore $Cl\,\alpha\,M = M \cup Cl\,Int\,Cl\,M \subseteq M \cup Cl\,N \subseteq N$,
Then $Cl\,\alpha\,M \subseteq N$, *thus* M *is* αg*-closed set.*
Remark 3.9.
The opposite case of the property is not necessarily true for example.

Example 3.10.

Let
$X=\{4,5,6\}, \tau=\{\emptyset,\{4\},\{6\},\{4,6\},X\}, F=\tau^C = \{X,\{5,6\},\{4,5\},\{5\},\emptyset\}$.
If $M = \{6\}$ *and* $N=X$ *is open in* (X,τ).
Since $Cl\,M=\{5,6\}$, *and* $Int\,cl\,M=\{6\} \Longrightarrow Cl\,Int\,Cl\,M = \{5,6\}$.
Then is obtain $Cl\,\alpha\,M \subseteq N$,
And from it we find M *is* αg*-closed set, but it is not g-closed set.*
If we take $N = \{4,6\}$ *is open in* (X,τ). *On the other hand* $Cl\,M = \{5,6\}$

Proposition 3.11.

For each αg*-closed set in* (X,τ), *it is the semi g-closed set.*
Proof :
If M αg*-closed set in* (X,τ), *and* $M \subseteq N$. *There is* N *is open set in* (X,τ).
Since M *is* α *g-closed set, then* $Cl\,\alpha\,M \subseteq N$,
Since $Cl\,\alpha M = M \cup Cl\,Int\,Cl\,M$, *so* $Cl\,Int\,Cl\,M \subseteq Cl\,\alpha M \subseteq N$.
But $IntCl\,M \subseteq Cl\,Int\,Cl\,M$, *thus* $Int\,Cl\,M \subseteq N$.
Since $Cl\,\alpha M$ *is semi g-closed smallest set contain* M;
$Cl\,semi\,M = M \cup IntClM$. *Take advantage of the two relationships* $M \subseteq N$, $IntCl\,M \subseteq N$, *we get* $Cl\,semi\,M \subseteq N$, *therefore* M *is semi g-closed set.*

Definition 3.12. [10],[11],[12],[13],[14]

A subset M of a topological space (X,τ) is named ;

I- A **generalized closed** (briefly, *g-closed*) set if $\boldsymbol{Cl\,M \subseteq U}$, since $M \subseteq U$, and U is **open** set of X .

II- A ***semi*-generalized closed** (concise, *sg-closed*) set if $\boldsymbol{semi\,Cl\,M \subseteq U}$, since $M \subseteq U$ and U is ***semi*-open** set of (X,τ).

III- A **generalized *semi*-closed** (abbreviated, *gs-closed*) set if $\boldsymbol{semi\,ClM \subseteq U}$, whenever $M \subseteq U$, and U is **open** set of (X,T).

g-closed.	*g-open*
sg-closed	*sg-open*
gs-closed	*gs-open*
αg-closed	*αg-open*
gα-closed	*gα-open*

IV- A **α-generalized closed** (shortened, αg-*closed*) set if $\boldsymbol{\alpha\, Cl\, M \subseteq U}$, whenever $M \subseteq U$ and U is **open** set of (X, T).

V- A **generalized α-closed** (abridged, $g\alpha$-*closed*) *set* if $\boldsymbol{\alpha\, cl\, M \subseteq U}$, whenever $M \subseteq U$ and U is **α-open** set in (X, T).

Remark 3.13.

The opposite case of the property 1.3.20. is not necessarily true for example.

Example 3.14.

If $X = \{0,1,3\}$, $\tau = \{\emptyset, \{0\}, \{1\}, \{0,1\}, X\}$,

$\tau^C = F = \{X, \{1,3\}, \{0,3\}, \{3\}, \emptyset\}$, let $M = \{1\}$, and $N = X$ is open in (X, τ),

Now $Cl\, M = \{1,3\}$, so $Cl\, M \subseteq N$.

Since $Int\, Cl\, M = \{1\}$, so $Cl\, semi\, M = \{1\} \subseteq N$,

then M is *semi* g-closed set, but is not αg-closed set. Because we

take $N = \{0\}$ is open set in(X, τ),

As $Cl\, Int\, ClM = \{1,3\}$, therefore $Cl\, \alpha\, M = \{1,3\}$.

Definition 3.15.[5]

A topological space, $f: (X, T_x) \to (Y, T_y)$ *is called*

I- **α-closed function, there exists** F *closed of* X, *now* $\boldsymbol{f(F)}$ *is* **α-closed set of** Y.
II- generalized closed function, exists $\boldsymbol{F}$ *is closed set in* (X, T_x) *space, thus* $\boldsymbol{f(F)}$ *it is generalized closed set of* (Y, T_y) *space.*
III- **α-generalized closed function, there exists** $\boldsymbol{F}$ *is closed set of* (X, T_x) *space, and* $\boldsymbol{f(F)}$ *it is* **α-generalized closed set in** (Y, T_y) *space.*
IV- generalized **α-Closed function, there exists** F *is closed set of* (X, T_x) *space, so* $\boldsymbol{f(F)}$ *it* **α-generalized closed set in** (Y, T_y) *space.*

Remark 3.16.

There are complements for some of the concepts ;

Remark. 3.17.

Each closed function is α*-closed. But a reverse case is not necessarily true as*

shown by the following example.

Example : 3.18.

If $X = \{4,5,6\}$, $T_x = \{\emptyset, \{4\}, \{4,5\}, X\}$. *There be* α*-open of* X *;*

$T_x^\alpha = T_x \cup \{\{4,6\}\}$. *And if* $Y = \{1,2,3\}$, $T_y = \{\emptyset, \{1\}, Y\}$,

The α*-open of ;* $T_y^\alpha = T_y \cup \{\{1,2\}, \{1,3\}\}$. *So* $f: (X, T_x) \to (Y, T_y)$,

Define by $f(x_1) = f(x_2) = 2, f(x_3) = 3$.

Then f *is* α*-closed, however is not closed function.*

As {6} is closed of (X, T_X), *and* $f(\{6\}) = 3$. *on the other hand {3} is not*

closed of (Y, T_Y).

Remark 3.19.

Every α*-closed function, is generalized* α*-closed function. But the reverse case is not necessarily true as shown by the following example.*

Example 3.20.

If $X = Y = \{2,5,8\}$, $T_x = \{\emptyset, \{2\}, \{2,5\}, X\}$, $T_y = \{\emptyset, \{2\}, \{5,8\}, Y\}$.

$T_x^\alpha = T_x \cup \{2,8\}$, *and* $T_y^\alpha = T_y \cup \{2,5\}$.*Let* $f: (X, T_x) \to (Y, T_y)$,

is define by ; $f(x_1) = 5$, $f(x_2) = 8$, $f(x_3) = 2$.

Then f *is generalized* α*-closed function,*

But is not α*-closed function. since* $\{5,8\}$ *it is closed set of* (X, T_x) *space,*

also $f(\{5,8\}) = \{8,2\}$. *Then* $\{8,2\}$ *no* α*-closed in* (Y, T_y) *space.*

Remark 3.21.

For each generalized closed function, is α-generalized closed. However the

convers is not necessarily true as shown by the following example.

Example 3.22.

If $X = Y = \{1,2,3\}$, $T_x = T_y = \{\emptyset, \{1\}, \{1,2\}. X\}$.

Then $T_x^{\alpha} = T_y^{\alpha} = T_x \cup \{1,3\}$,

And let $f: X \to X$, *define by ;* $f(x_1) = 1, f(x_2) = 3, f(x_3) = 2.$

We know that f is generalized α-closed function,

However is not generalized closed Function.

Because $\{3\}$ *is closed in* (X, T_x) *space, but* $f(\{3\}) = \{2\}$,

However $\{2\}$ *is not generalized closed of* (Y, T_y) *space.*

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Conflict of Interest

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