New Wavelet Collocation Method for Solving Nonlinear Korteweg-de Vries Equation

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ABSTRACT

In this paper, To solve nonlinear PDE, we employed a new wavelet formula derived from the notion of convolution. Analyzing the operational matrix of integration, we developed the general formulas for n of integrals to new wavelets. To solve the nonlinear Korteweg-de Vries problem, the proposed approach with collocation points is applied (KDV). Even though the step size utilized was huge, the results were good, and the accuracy of the derived answers is pretty high despite the tiny number of calculation points. The procedure is more precise, better, and gets closer to the correct solution.

Key words: Haar wavelets, Cosine and Sine (CAS) wavelets, Nonlinear partial differential equations, Operational Matrix.

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We get if we limit the values of a and b to discrete values

$$a = a_0^{-k}\ ,\ b = nb_0\ a_0^{-k}\ ,\ a_0 > 1\ , b_0 > 0$$

When n and k are positive integers, the following family of discrete wavelets exists.

$$\Psi_{k,n}(t) = |a_0|^{\frac{k}{2}}\ \Psi\left(a_0^k\ t - n\ b_0\right) \quad \ldots\ldots (2)$$

In which case the wavelet basis for $L^2(R)$. Is $\Psi_{k,n}(t)$. When, specifically,

$a_0 = 2\ , b_0 = 1\ , then\ \Psi_{k,n}(t)$ Creates an orthonormal structure [2]

The wavelet method has gained prominence in the field of numerical methods in recent years. Many different forms of wavelets and approximation functions have been used for this. The stochastic integral equations were solved using the Haar wavelet and CAS wavelet methods, and the operational matrix of integration of these wavelets was created by Siddu C. S. and Lata [19, 17] Siddu C. S. and R. A. M. describe the wavelet full-approximation approach in [18]The Voltera-Frodholm integral equations were numerically computed with great precision to tackle nonlinear problems. In [6,] To solve nonlinear fractional order Volterra integral equations, the CAS wavelet function approach is utilized; in [8,] To solve three-

INTRODUCTION

Wavelet analysis is a new numerical idea that allows a function to be expressed in terms of spatially confined basic functions called wavelets. Wavelets are a form of data structure that debuted in the early 1980s. They grabbed the interest of the mathematics community as well as members from a variety of disciplines where wavelets could be useful. As a result of this interest, numerous books and research publications on the subject have been released [11]

The wavelet decomposition analysis is the most often used in wavelet signal processing. It is used in signal compression as well as signal identification. The Fourier transform of a function, often known as the Wavelet transform, is a strong approach for analyzing the components of stationary occurrences. When the Fourier transform fails, the wavelet has the advantage of being able to study non-stationary data events [14, 15, and 3].

When the dilation and translation parameters a and b fluctuate continuously, we have the following family of continuous wavelets:

$$\Psi_{a,b}(t) = |a|^{\frac{-1}{2}}\ \Psi\left(\frac{t-b}{a}\right).\quad a, b \in R\ , a \neq 0 \quad \ldots\ldots (1)$$

Hermite [22], as well as finite difference approaches [24].

The following is the paper's structure. Section 2 contains the integrals of Haar wavelets. Section 3 discusses CAS wavelets and integrals. In section 4, we derive the generic integration formulas for the new wavelet. Section 5 walks through the phases of the suggested approach for solving nonlinear PDEs. The numerical solution of the Kdv problem is then solved in section 6. Finally, the numerical results are reported, together with suggestions for future research.

dimensional and time-dependent PDEs, the Haar wavelet approach is used; in [10], The nonlinear two-dimensional BBM-BBM system is solved, and the numerical solutions have a high degree of accuracy even when the number of calculation points is limited; and in [7], The nonlinear two-dimensional BBM-BBM system is solved, and numerical solutions are provided, In [9], A numerical solution for a non-linear third-order bossiness system was obtained using an operational integration matrix based on the Haar wavelet technique. The numerical solution to Kdv problem has been discovered in a variety of approaches, including Haar wavelet analysis [21, 23], Finite element approach based on cubic

HAAR WAVELETS

We considered the interval $t \in [0,1)$, The Haar wavelets family is designated as [4, 5, 12].

$$h_i(t) = \begin{cases} 1 & for\ t\ \in [\gamma_1(i)\ ,\gamma_2(i)\ ) \\ -1 & for\ t\ \in [\gamma_2(i)\ ,\gamma_3(i)\ ) \\ 0 & elsewhere \end{cases} \quad \ldots\ldots\ldots (3)$$

Where

$$\gamma_1(i) = \frac{k}{2^j}, \gamma_2(i) = \frac{k+0.5}{2^j}, \gamma_3(i) = \frac{k+1}{2^j} \quad \ldots\ldots\ldots (4)$$

[0, 1) is subdivided into 2M equal-length subintervals, where t=1/ (2M) is the breadth of each subinterval. Integer j = 0, 1, 2... J represents the wavelet level, and integer; k = 0, 1, 2... 2 j -1 represents the translation parameter. J is the highest degree of resolution. The formula for calculating the index I is as follows: i= 2^j +k+1.When it comes to minimal values, 2^j =1, k=0 , we have i=2 , the maximal value of i is i= 2M = 2^{J+1} . It is assumed that the value i=1 determines the scaling function in [0, 1] for which h1 (t) = 1

Let the collocations be:

$$t_l = \left(l - \frac{1}{2}\right) \Delta t\ ,\ \ l = 1,2, \ldots, 2M \quad \ldots\ldots\ldots (5)$$

It's a good idea to start with the Haar matrices.$H(i\ ,l\ ) = h_i(t_l)$ With the dimensions of 2M*2M. The integrals for the Haar wavelets in equation (3) can be determined analytically, and these integrations can then be utilized to numerically solve higher-order differential equations. These integrals will be used to numerically solve a one-dimensional nonlinear system. We get the operational matrix of integration p by integrating equation (3) from (0) to (t).

$$p_{i\ ,1}\ (t) = \int_0^t h_i\ (t')dx'$$

$$p_{i\ ,1}\ (t) = \begin{cases} t - \gamma(i)\ , & t\ \in [\gamma(i)\ ,\gamma_2(i)\ ), \\ 2\ \gamma_2(i) - t - \gamma_1(i)\ , & t\ \in [\gamma(i)\ ,\gamma_3(i)\ ) \\ 0 & elsewhere \end{cases} \quad \ldots\ldots\ldots (6)$$

$$p_{i\ ,2}\ (t) = \begin{cases} \frac{(t-\gamma_1(i))^2}{2}, & t\ \in [\gamma_1(i)\ ,\gamma_2(i)\ ) \\ \frac{1}{4m^2} - \frac{(\gamma_3(i)-t)^2}{2}\ , & t\ \in [\gamma_2(i)\ ,\gamma_3(i)\ ) \\ \frac{1}{4m^2}, & t\ \in [\gamma_3(i)\ ,1\ ) \\ 0 & elsewhere \end{cases} \quad \ldots\ldots\ldots (7)$$

Overall

$$p_{i,s+1}(t) = \int_0^t p_{i,s}(t')dt' \quad , \quad s = 1, 2, \ldots \qquad \ldots\ldots\ldots (8)$$

The typical form of s − indispensable times [13]

$$p_{i,s}(t) = \begin{cases} 0 & for\ t < \gamma_1(i) \\ \frac{1}{s!}\,[t - \gamma_1(i)]^s, & for\ t \in [\gamma_1(i), \gamma_2(i)) \\ \frac{1}{s!}\,\{[t - \gamma_1(i)]^s - 2[t - \gamma_2(i)]^s\}, & for\ t \in [\gamma_2(i), \gamma_3(i)) \\ \frac{1}{s!}\,\{[t - \gamma_1(i)]^s - 2[t - \gamma_2(i)]^s + [t - \gamma_3(i)]^s\}, & for\ t > \gamma_3(i) \end{cases} \qquad \ldots\ldots (9)$$

And In the case $i = 1$, we have $\gamma_1 = 0$, $\gamma_2 = \gamma_3 = 1$ the rules are applicable for $i > 1$;

$$p_{1,s}(t) = \frac{1}{s!}(t)^s, \quad \forall\, t \in [0,1] \qquad \ldots\ldots\ldots (10)$$

CAS WAVELETS AND WITH INTEGRALS

We offer the relevant definitions and mathematical preliminaries for Cosine and Sine (CAS) wavelets in this section. The block pulse function and function approximation via CAS wavelets are next introduced

The CAS wavelet $\Psi_{w,m}(t) = \Psi(k, w, m, t)$ has four arguments ; $w = 0,1,2, \ldots, 2^k - 1$, k Assume that n is any positive integer, m can be any integer, and t can be any integer

On the interval [0, 1), CAS orthonormal wavelets are described by [16]:

$$\Psi_{w,m}(t) = \begin{cases} 2^{\frac{k}{2}}\ CAS_m\left(2^k t - w\right), & for\ \frac{w}{2^k} \le t < \frac{w+1}{2^k} \\ 0 & otherwise \end{cases} \qquad \ldots\ldots\ldots (11)$$

Where

$$CAS_m(t) = \cos(2m\pi t) + \sin(2m\pi t), \qquad \ldots\ldots\ldots (12)$$

If we solve the equation(11) from (0) to (1) we get .

$$P_{2^k(2M+1),1}(t) = \int_0^t \Psi_{w,m}^{CAS}(t')dx'$$

$$P_{2^k(2M+1),1}(t) = \begin{cases} 0 & 0 \le t < \frac{w}{2^k} \\ 2^{\frac{k}{2}}\frac{1}{2^{k+1}\pi m}\left(\sin\left(2\pi m\left(2^k t - w\right)\right)\right) - \cos\left(2\pi m\left(2^k t - w\right)\right)) & \\ -2^{\frac{k}{2}}\frac{-1}{2^{k+1}\pi m}, & \frac{w}{2^k} \le t < \frac{w+1}{2^k} \\ 2^{\frac{k}{2}}\frac{1}{2^{k+1}\pi m}(\sin(2\pi m) - \cos(2\pi m)) - 2^{\frac{k}{2}}\frac{-1}{2^{k+1}\pi m}, & \frac{w+1}{2^k} \le t < 1 \end{cases} \qquad \ldots (13)$$

$$P_{2^k(2M+1),2}(t) = \begin{cases} 0 & , \quad 0 \le t < \frac{w}{2^k} \\ 2^{\frac{k}{2}}\frac{(-1)}{(2^{k+1}\pi m)^2}\left(\cos\left(2\pi m(2^k t-w)\right)+\sin\left(2\pi m(2^k t-w)\right)\right) & \\ -\left(2^{\frac{k}{2}}\frac{(-1)}{(2^{k+1}\pi m)^2}+2^{\frac{k}{2}}\frac{(-1)}{(2^{k+1}\pi m)}\left(t-\frac{w}{2^k}\right)\right) & , \quad \frac{w}{2^k} \le t < \frac{w+1}{2^k} \\ & \ldots\ldots(14) \\ \left[1+\left(t-\frac{w+1}{2^k}\right)\right]\left[2^{\frac{k}{2}}\frac{(-1)}{(2^{k+1}\pi m)^2}\cos(2m\pi)+2^{\frac{k}{2}}\frac{(-1)}{(2^{k+1}\pi m)^2}\sin(2m\pi)\right] & \\ -\left(2^{\frac{k}{2}}\frac{(-1)}{(2^{1+k}\pi m)^2}+2^{\frac{k}{2}}\frac{(-1)}{(2^{k+1}\pi m)}\left(\frac{1}{2^k}\right)\right)] & , \quad \frac{w+1}{2^k} \le t < 1 \end{cases}$$

Reintegrating s times, we get [8]

$$P_{i,s}(t) = \begin{cases} 0 & 0 \le t < \frac{w}{2^k} \\ 2^{\frac{k}{2}}\frac{(-1)^{a_s}}{(2^{k+1}\pi m)^s}\cos\left(2\pi m(2^k t-w)\right) & \\ +2^{\frac{k}{2}}\frac{(-1)^{b_s}}{(2^{k+1}\pi m)^s}\sin\left(2\pi m(2^k t-w)\right) & \frac{w}{2^k} \le t < \frac{w+1}{2^k} \\ -\sum_{JJ=0}^{s-1} 2^{\frac{k}{2}}\frac{1}{JJ\,!}\frac{(-1)^{a_s}}{(2^{k+1}\pi m)^{s-JJ}}\left(t-\frac{w}{2^k}\right)^{JJ}, & \ldots\ldots(15) \\ \sum_{JJ=0}^{s-1}\frac{1}{j\,!}(t-\frac{w+1}{2^k})^j. & \frac{w+1}{2^k} \le t < 1 \\ .((2^{\frac{k}{2}}\frac{(-1)^{a_s}}{(2^{k+1}\pi m)^s}\cos(2\pi m)+2^{\frac{k}{2}}\frac{(-1)^{b_s}}{(2^{k+1}\pi m)^s}\sin(2\pi m)-\sum_{JJ=0}^{s-1}2^{\frac{k}{2}}\frac{1}{JJ\,!}\frac{(-1)^{a_s}}{(2^{k+1}\pi m)^{s-\mathrm{JJ}}}(\frac{1}{2^k})^{JJ}), & \end{cases}$$

Where

$$a_s = \begin{cases} 0 & if \ \ s = 3,4,7,8,11,12, \ldots\ldots \\ 1 & if \ \ s = 1,2,5,6,9,10, \ldots\ldots \end{cases}$$

And

$$b_s = \begin{cases} 0 & if \ \ s = 1,4,5,8,9,12, \ldots\ldots \\ 1 & if \ \ s = 2,3,6,7,10,11, \ldots\ldots \end{cases}$$

The case $s = 0$ corresponds to CAS function $\Psi_{w,m}^{CAS}(t)$ in equation (11).

DERIVATION OF THE NEW WAVELET

Definition: [1]

In R, consider two functions, u and v. The convolution of u and v is thus defined by $u * v(x) = \int_R u(t)v(t-x)dt$ when that integration makes sense.

Theorem:-[1]

If Ψ is a wavelet and φ is a bounded integrate function, then $\Psi * \varphi$ is a wavelet as well. By converging two wavelets, we may now create a new wavelet formula, CAS and Haar wavelets, with CAS wavelets represented by trigonometric functions with periodic and constrained integration..

Let $H_i = Haar\ wavelet\ and\ \Psi_{w,m} = CAS\ wavelet$, since the convolution is Commutative we obtain [20]

$$W_{w,m}^{New}(t) = (\Psi_{w,m} * H_i)(t) = \int_0^x \Psi_{w,m}(t).H_i(t-x)dt \quad \dots\dots\dots(16)$$

$$W_{w,m}^{New}(t) = \begin{cases} \frac{2^{\frac{-k}{2}-1}}{\pi\, m}[\cos\left(2m\pi\left(2^k t - w\right)\right) - \sin\left(2m\pi\left(2^k t - w\right)\right) + 2\sin(m\pi) \\ \quad -2\cos(m\pi) + 1\,] \qquad , \quad \frac{w}{2^k} \le t < \frac{w+1}{2^k} \\ \quad 0 \qquad , \quad \text{otherwise} \end{cases} \quad \dots..(17)$$

Any function $u(t) \in L^2[0,1)$ might be extended involving $W_{w,m}^{New}$ wavelets as $u(t) = \sum_{w=1}^{\infty}\sum_{m\in z} C_{w,m}\ W_{w,m}^{New}(t)$, $\dots\dots\dots\dots(18)$

$C_{w,m} = < f(x), W_{w,m}^{New} >$. Where

In the event that the boundless series in condition (18) is shortened, condition (18) can be composed as

$$u(x) = \sum_{w=0}^{2^k-1}\sum_{m=-M}^{M} C_{w,m}\ W_{w,m}^{New}(t) = C^T\ W_{w,m}^{New}(\mathrm{t}), \quad \dots\dots\dots\dots(19)$$

Where C and $W_{w,m}^{New}$ are $2^k\ (2M+1)\times 1$ matrices given by

$$C = \left[c_{0,(-M)}, c_{0,(-M+1)}, \dots., c_{0,M}, c_{1,(-M)}, \dots, c_{1,(M)}, c_{2^K-1,(-M)}, \dots\dots, c_{2^K-1,(M)}\right]^T \quad \dots. \quad (20)$$

$$W_{w,m}^{New}(t) = \left[W_{0,(-M)}(t), W_{0,(-M+1)}(t), .., W_{0,M}(t), W_{1,(-M)}(t), .., W_{2^K-1,(-M)}(t), .., W_{2^K-1,M}(t)\right]^T \dots(21)$$

For accommodation, in mathematical arrangement, we revamp condition (19) as follow:

Set $i = w(2M+1) - M + m$, and then

$$u(x) = \sum_{i=1}^{2^k(2M+1)} C_i\ W_{2^k(2M+1),i}^{New}(t), \quad \dots\dots\dots..(22)$$

DERIVATION A GENERAL FORMULA FOR THE INTEGRATION OF THE NEW WAVELET

Analytically, we get the integrals for the new wavelet specified in equation (17). The new wavelet is explained in terms of periodic trigonometric functions with periodic integration. We can find a generic formula for n of these wavelet integrals. If we integrate equation, (17) from $0\ to\ t$, we get

$P_{2^k(2M+1),1}(t) = \int_0^t W_{w,m}^{New}(t^*)d\,t^*\ then$

$$= \begin{cases} 0 \qquad , 0 \le t < \frac{w}{2^k} \\ \frac{2^{\frac{-3k}{2}-2}}{\pi^2 m^2}[sin\left(2m\pi\left(2^k t - w\right)\right) - 4\,\pi m\, cos(m\pi)\left(2^k t - w\right) \\ + cos\left(2m\pi\left(2^k t - w\right)\right) + \pi\, 2^{k+1} mt + \pi\, 2^{k+2} mt\, sin(m\pi) \\ -2\pi nm - 4\pi nm\, sin(m\pi) - 1] \qquad , \frac{w}{2^k} \le t < \frac{w+1}{2^k} \\ \frac{2^{\frac{-3k}{2}-2}}{\pi^2 m^2}[sin\left(2m\pi\left(2^k t - w\right)\right) - 4\pi m\, cos(m\pi)\left(2^k t - w - 1\right) \quad \dots\dots\dots(23) \\ + cos\left(2\pi m\left(2^k t - w\right)\right) + \pi 2^{k+1} mt + \pi 2^{k+2} mt\, sin(m\pi) \\ -2\pi mn - 4\pi mn\, sin(m\pi) - 4\pi m\, cos(m\pi) - 1] \qquad , \frac{w+1}{2^k} \le t < 1 \end{cases}$$

$$P_{2^k(2M+1),2}(x) =$$

$$= \begin{cases} 0 & ,0 \le t < \frac{w}{2^k} \\ \begin{cases} \frac{2^{\frac{-5k}{2}-3}}{\pi^3 m^3}[\sin\left(2m\pi\left(2^k t-w\right)\right)-\cos\left(2m\pi\left(2^k t-w\right)\right)+ \\ +(-4t^2(m\pi)^2\cos(\pi m)+4t^2(m\pi)^2\sin(m\pi)+2t^2(m\pi)^2)\ \left(2^k\right)^2+ \\ +(8tn(m\pi)^2\cos(m\pi)-8tn(m\pi)^2\sin(m\pi)-4tn(m\pi)^2-2tn\pi m)\ (\ 2^k)+ \\ +(-4n^2(\pi m)^2\cos(\pi m)+4n^2(\pi m)^2\sin(\pi m)+2n^2(\pi m)^2)\ \left(2^k\right)^2+2n\pi m\ ] \end{cases} & ,\frac{w}{2^k} \le t < \frac{w+1}{2^k} \\ \begin{cases} \frac{2^{\frac{-5k}{2}-3}}{\pi^3 m^3}[(2m-1)\cos(2\pi m)+(2m+1)\cos(2\pi m)-12m^2\cos(\pi m)+ \\ +12m^2\sin(\pi m)+6m^2-4m+1)] \end{cases} & \dots\dots(24) \\ & ,\frac{w+1}{2^k} \le t < 1 \end{cases}$$

Repeating the integration w times, we obtain $P_{i,s}(t)$ as follows:

$$= \begin{cases} 0 & ,0 \le t < \frac{w}{2^k} \\ \begin{cases} \frac{1}{(2\,2^k)^r(m\pi)^{r+1}}(\left(\frac{(-1)^r\,(2)^{r+1}}{r!}\right)\sin(m\pi)\,(m\pi)^r\,w^r+\left(\frac{(-1)^{r+1}\,(2)^{r+1}}{r!}\right)\cos(m\pi)\,(m\pi)^r\,w^r+ \\ +(-1)^{b_s}\sin\left(2m\pi\left(2^k t-w\right)\right)+(-1)^{a_s}\cos\left(2m\pi\left(2^k t-w\right)\right)+(-1)^{c_s}+ \\ +\sum_{j=1}^{r}\left(\frac{(-1)^{r+j+1}(2)^{r+1}}{(r-j)!\,j!}\cos(m\pi)\right)\left(t2^k\right)^j w^{r-j}(m\pi)^r+ \\ +\frac{(-1)^{r+j}(2)^{r+1}}{(r-j)!\,j!}\sin(m\pi)\left(x2^k\right)^j w^{r-j}(m\pi)^r+ \\ +\frac{\left(2m\pi\left(2^k t-w\right)\right)^j}{j!}) \end{cases} & ,\frac{w}{2^k} \le t < \frac{w+1}{2^k} \\ \begin{cases} \left(\frac{2^{\frac{-k}{2}-1}}{(2\,2^k)^w(m\pi)^{w+1}}\right). \\ .\sum_{j=1}^{r}[(-1)^{a_v}(2m\pi)^{r-j}\cos(2m\pi)+(-1)^{b_v}(2m\pi)^{r-j}\sin(2m\pi)-\frac{2^{r+1}}{j!}\ (m\pi)^r\cos(m\pi)+ \\ +\frac{2^{r+1}}{j!}\ (m\pi)^r\sin(m\pi)+ \\ +\sum_{u=0}^{r-1}\frac{2^{r-u}}{(j-u)!}(m\pi)^{r-u}]+(-1)^{a_s} \end{cases} & \dots\dots.(25) \\ & ,\frac{w+1}{2^k} \le t < 1 \end{cases}$$

Where

$$a_s = \begin{cases} 0 & if \ \ s = 1,4,5,8, \dots \\ 1 & if \ \ s = 2,3,6,7, \dots \end{cases}$$

And

$$b_s = \begin{cases} 0 & if \ \ s = 1,2,5,6, \dots \\ 1 & if \ \ s = 3,4,7,8 \dots \end{cases}$$

And

$$c_s = \begin{cases} 0 & if \ \ s = 2,3,6,7, \dots \\ 1 & if \ \ s = 1,4,5,8, \dots \end{cases}$$

If u > j then $(j-u)!=0$, r is order integral

THE SUGGESTION METHOD OF SOLUTION FOR PARTIAL DIFFERENTIAL EQUATIONS

In this section, we use the novel wavelet method to solve a nonlinear partial differential equation.
The nonlinear PDE general form

$$F\left(\hat{t}, x, u, \mathcal{D}u, \mathcal{D}^2 u \dots, \mathcal{D}^{\alpha+\beta} u\right) = f\ (\hat{t}, x) \qquad \dots\dots\dots (26)$$

$$\mathcal{D}^{\alpha+\beta} u = \frac{\partial^{\alpha+\beta} u(\hat{t}, x)}{\partial t^{\alpha} \partial \hat{x}^{\beta}}$$

Where $x \in [a, b]$ and $t \in \ (0, T]$ because the variables $\hat{t}$and$x$ belong to the Ω interval
We aim to do J resolution levels, so we let $2M = 2^{J+1}$ The range [a,b] will be split into 2M sub - bands.
Hence $\Delta x = \frac{b-a}{2M}$ and the size of the matrices $2M \ \times 2M$.
This new technique is broken down into seven steps

Step 1. We transform the period using the relationship$[a, b]$ containing $\hat{x}$ to the period [0,1] containing x.

$$x = \frac{1}{L}\ (\hat{x} - a) \qquad L = b - a \qquad \dots\dots\dots (27)$$

Step 2. $(0, T]$ A is separated into N equal length parts. $\Delta t = \frac{T}{N}$ and time $t_s = \ (s-1)\ \Delta t \quad , s = 1,2 \dots., N$.
The [0,1] interval containing x is then subdivided into M_1 assembly collocation points of comparable length..

$$x_l = \frac{l-0.5}{M_1} \quad l = 12 \dots, M_1 \qquad \dots\dots\dots (28)$$

Step 3. Assume that

$$\frac{\partial^{\alpha+\beta} u(t,x)}{\partial t^{\alpha} \partial x^{\beta}} = \sum_{i=0}^{M_1-1} c_i W_i(t,x)$$

Where c_i are the coefficients of the wavelet.

Step 4. We obtain by integrating with regard to t from t_s to t and with respect to x from 0 to x.

$$u(x,t) = \frac{(\ s - s_s\ )^{\alpha}}{\alpha!} \sum_{i=0}^{M_1-1} c_i P_{\beta,i}(x) + \delta(t,x)$$

Where $\varsigma(t,x)$ is determined by the beginning and boundary conditions, and $P_{\beta,i}(t)$ is determined by the wavelet type employed in the solution (Haar, CAS, New)

Step 5. We use the problem to replace the answer $u(t,x)$ and its derivatives with regard to x and t.

Step 6. The wavelet coefficients c_i are calculated by solving a linear set of algebraic equations..

Step 7. We get the numerical solution $u(t,x)$ according to the equation.

Numerical Experiments

To demonstrate the efficacy of the suggested method, we will use the Haar, CAS, and novel wavelet methods to derive an approximate solution to the KdV equation. MATLAB R2013a was used to do all of the computations.
A nonlinear partial differential equation of the type is the expanded modified KdV equation.

$$u_t = \alpha u u_x + \beta u_{xxx} \ , \qquad 0 \le x \le 1, t \ge 0$$

Where $\beta \ and \ \alpha \ are \ parameters$.

Example: let the value of $\alpha = 6$ and $\beta = -1$. Then the sample KdV equation is

$$u_t = 6uu_x - u_{xxx}$$

With elementary initial conditions $u(x,0) = 3sech^2\left(\frac{x}{2}\right)$,

The boundary conditions $u(0,t) = 3sech^2\left(\frac{t}{2}\right)$

$u(1,t) = 3sech^2\left(\frac{1}{2} - \frac{t}{2}\right), t > 0$

And the exact solution is

$u(x,t) = 3sech^2\left(\frac{x}{2} - \frac{t}{2}\right)$

We use the error norm [13]: $\delta_e = \frac{1}{N}\|u_{exact} - u_{num}\|_2$

The table compares the results achieved using the Haar, CAS wavelets, and suggestion wavelet methods for the amplitude of the matrix m=16.

Table (1): compares the approximate and precise solutions for m = 16 of an example using Haar, CAS wavelets, and a new wavelet at t = 0.0001.

(x/32)	Haar wavelet	CAS wavelet	New wavelet	Exact solution
	3.0022579	2.9994786	2.9992623	2.9992724
	2.9964168	2.9940406	2.9934015	2.9934319
1		2.9827951	2.9817363	2.9817869
3	2.9847701	2.9658306	2.9643571	2.9644279
5	2.9674074	2.9432788	2.9413977	2.9414884
7		2.9153129	2.9130327	2.9131431
9	2.9444617	2.8821443	2.8794752	2.8796052
11		2.8440196	2.8409734	2.8411228
13	2.9161074	2.8012169	2.7978067	2.7979752
14	2.8825572	2.7540414	2.7502814	2.7504688
17		2.7028209	2.6987264	2.6989323
19	2.8440589	2.6479014	2.6434886	2.6437125
21	2.8002372	2.5896427	2.5849279	2.5851697
23		2.528413	2.5234133	2.5236727
25	2.7527069	2.4645852	2.4593179	2.4595944
27	2.7011425	2.3985324	2.3930146	2.3933079
29				
31	2.6458904			
	2.5873108			
	2.5257723			
	2.4616482			
	2.3953114			

Table (2): compares the approximate and accurate solutions for m = 8 using Haar, CAS, and novel wavelets. t = 0.0001

(x/16)	Haar wavelet	CAS wavelet	New wavelet	Exact solution
1	2.9980807	2.9971755	2.9970717	2.9970816
3	2.9748114	2.9740749	2.9737842	2.9738143
5	2.9289687	2.9283964	2.9279289	2.9279 79
7	2.8619397	2.8615239	2.8608963	2.8609653
9	2.7754688	2.7754263	2.7746601	2.7747468
11	2.6724463	2.6725365	2.6716573	2.6717605
13	2.5554049	2.5556125	2.5546486	2.5547672
15	2.4272814	2.4275898	2.4265713	2.426704

The table shows the error scale for each of the Haar, CAS, and new wavelets, as well as various examples of m.

Table (3): The error norm δ_e for the approximation solutions at $t = 0.0001$

m	Haar Wavelet	CAS Wavelet	NEW Wavelet
8	$2.9719607\,23 - 04$	$2.2101e - 04$	$3.0136e - 05$
16	$6.481364070 - 04$	$8.3097e - 04$	$1.0077e - 05$
32	$1.245377461 - 03$	$6.5321e - 05$	$6.5329e - 06$
64	$2.224802387 - 03$	9.4078811e-05	9.408981e- 06

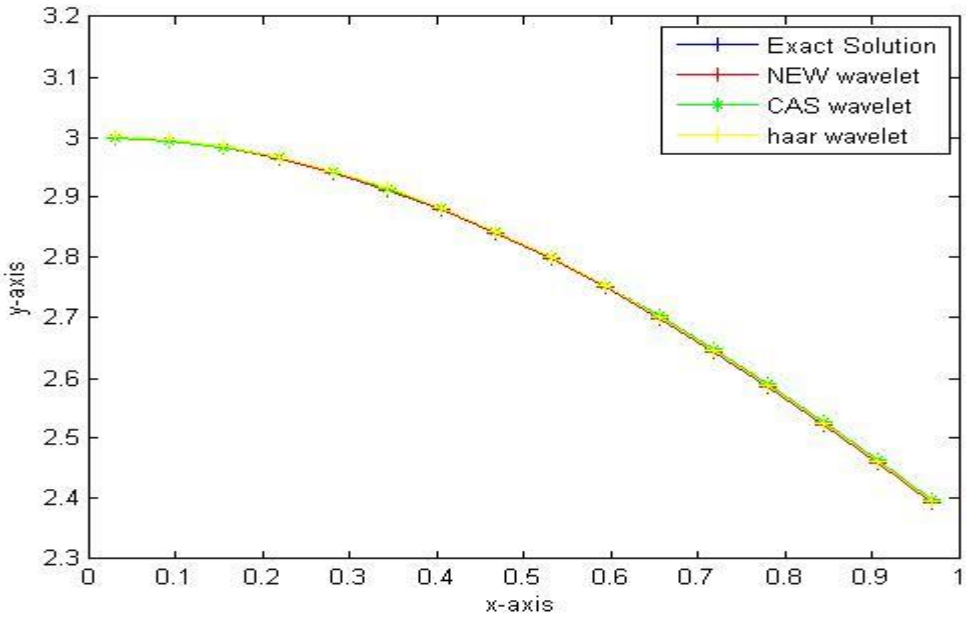

Figure (1): The numerical answers were compared to the exact solution of the KdV equation.

at t = 0.0001, m=16

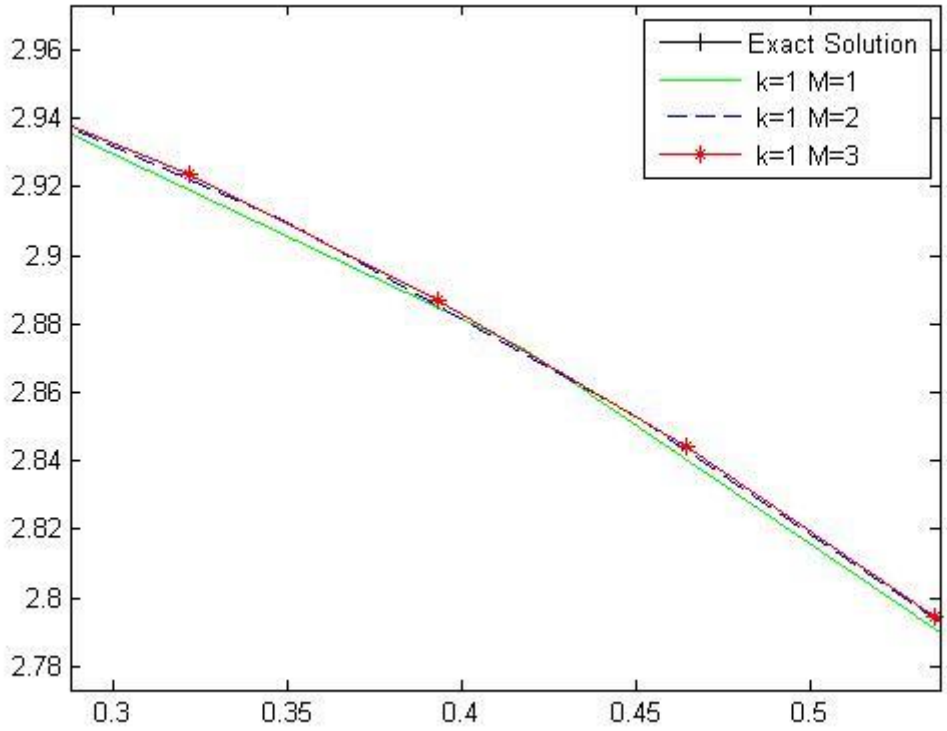

Figure (2): Illustrates the convergence of the numerical solution by new wavelet for different values M

From tables and Figures, we see that the solution of suggestion wavelet is better and nearer to the exact solution.

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Conclusions

The general integration formula for the new wavelet previously formed by the convolution of Haar and CAS wavelets is derived in this research to find the numerical solution of nonlinear PDEs, and the three wavelets approaches are employed to solve the nonlinear KdV equation. We can see from the numerical solutions in the tables (1,2,3) and figure that the solutions utilizing the new wavelet produce results that are closer to the exact answer than the Haar and CAS wavelets. It is worth noting that the new wavelet solutions produce outstanding results even for tiny values of (m and k), as illustrated in figure (2). Also when $2^k(2m+1) = 36, 2^k(2m+1) = 44, \ldots$, we can obtain the results closer to the exact values.

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