Quantum Interpretation to Decision Making Under Risk: The Observer Effect In Allais Paradox

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ABSTRACT

As it has served as a basement for all sciences; Newtonian Physics has served for economics as well. Consequently Newtonian causality appeared as economic determinism in economic theory. Recently quantum physics called into question the philosophy of determinism and focused on possibility rather than certainty. Decision making under risk and uncertainty has been reviewed by scientists from different disciplines. Since it is revealed that individual behavior contradicts with the basic tenets of the theory; economists, psychologists, neurologists and physicists have devoted much to better understand the basic motives underlying irrational behavior. This study adapts quantum approach to decision making under risk considering Allais Paradox and reinterprets it not from the side of the subject but from the side of the observer. In this way it is aimed to reconsider Allais Paradox referring to quantum measurement problem and the observer effect.

Key Words: quantum economics, decision making under risk, Allais paradox, Schrödinger's cat, the observer effect

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Introduction

Having a deterministic approach, physics has been an attractive model to economics as to other social sciences. Thus, the existing economic paradigm is strongly influenced by Newtonian physics and its mechanistic approach. The mechanistic paradigm considers atoms as rigid, solid, non-divisible particles so does economic theory individual preferences or individual behavior. The Newton's "law of universal gravitation" and "laws of motion" allow determining the complete history of a system precisely. The whole universe can be explained by exact values by Newtonian approach.

As a matter of Newtonianism in economic theory, economic behavior is viewed as a mechanical phenomenon. Consistent with this claim an individual is considered as a mechanical robot whose acts are programmable, controllable, predictable and finally modelable reminding "Newtonian Clockwork Universe". The next step was to label this economic human as "homoeconomicus" who is:

- •selfish
- •rational
- •unemotional
- •fully-informed
- •a perfect utility-maximizer.

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This study examines the evolution of the research of decision making processes under risk, from the spectrum of the theory of expected utility to quantum physics. It is aimed to discuss Allais Paradox from the observer's side with reference to Schrödinger's view. The rest of the paper is organized as follows. In section I, decision making under risk is summarized within the concept of expected utility theory. Allais Paradox presented in Section II. Section III harmonizes quantum physics and progress in behavioral economics. Section IV considers the observer in Allais Paradox from Schrödinger's point of view. Section V concludes.

I. Decision Making Under Risk Within The Concept of Expected Utility Theory

Individual decision making under risk is explained by the theory of expected utility in economics. Actually the theory was the final link of the chain:

Following their correspondence during 1654; Blaise Pascal and Pierre de Fermat suggested that expected value of a gamble is the sum of the multiplication of offered payoffs (x_1, . . .,x_n) and their probabilities (p_1,..., p_n):

Expected Value = ∑xipi.

However, in 1728 by "St. Petersburg Paradox" it is understood that individual consideration is beyond expected value. The paradox grilled the fair price that would be paid for a game offering an infinite expected value in a casino. People are expected to be willing to pay higher amounts to enter such a game. But the reality was totally different! Daniel Bernoulli claimed that individuals consider situations not according to their expected values but to their utilities in 1738. Finally in 1944 John Von Neumann and Oscar Morgenstern suggested "the theory of expected utility" which was accepted and adopted with a great respect because of its solid mathematical and axiomatic structure. The theory is still predominant and is the basic reference in consumer behavior theory in economics. Von Neumann and Morgenstern described expected utility for a two-outcome lottery L=(P, A, B) as E[U(L)]=PU(A)+(1-P)U(B) where L, P, A, B, U(A), U(B) refer to lottery, probability, outcome A, outcome B, the utility of outcome A and the utility of outcome B respectively. According to the theory; expected utility is the basic motivation to make a choice between risky prospects. However it didn't take long for the assumptions of the theory of expected utility to be questioned. French Economist Maurice Allais (1953) firstly revealed that individual behavior under risk violates the axioms of the theory of expected utility.

II. Allais Paradox

In Allais Paradox subjects were first asked to make a choice between option A and option B. After they did, they were asked to make a choice between options C and D (Allais, 1953: p.527);

Experiment I: Choose between:

A) $1 million for sure

B) 10% chance of receiving $5 million
89% chance of receiving $1 million
1% chance of receiving nothing

Experiment II: Choose between:

C) 11% chance of receiving $1 million

D) 10% chance of receiving $5 million

For the first problem the expected value of option A is $1 million and the expected value of option B is $1.39 million. According to the theory of expected utility individuals are expected to choose the option that has the highest expected utility value. This expectation is consistent with "more is better than less" also with the homoeconomicus whose preferences ensure all of the assumptions of the theory attached to rational behavior and whose final goal is to maximize utility as a rational agent.

However individual choices contrast with the predictions of the theory; most people choose A over B for the first choice set. Kahneman and Tversky named this tendency as "certainty effect" (Kahneman & Tversky, 1979: p.265). Individuals prefer certain options when they are asked to make a choice between a risky and a certain alternative. In such a scenario, certainty contributes to risk averseness.

Expected utility relationship is written below for the choice between options A and B:

$u(1) > 0.10u(5)+0.89u(1)+0.01u(0)$, $\underline{0.11u(1) > 0.10u(5)}$.

Once individuals decided so, it is expected that they would choose C over D. However D was mostly choose over C in the second decision. Here is the expected utility relationship for the second experiment:

$\underline{0.10u(5) > 0.11u(1)}$ which contradicts with the first choice in the first stage and also with the axioms of the expected utility theory.

Allais prooved that decision making under risk violates the expected utility hypothesis and the independence axiom (also known as *Savage's Sure Thing Principle*) that lies at the heart of the theory of expected utility. According to the axiom; if an individual is indifferent between simple lotteries L_1 and L_2, she/he is also indifferent between L_1 mixed with lottery L_3 with probability p and L_2 mixed with L_3 with the probability p. Violation of this principle is called as "the common consequence problem" or "the common consequence effect". The independence axiom includes substitution and a reduction of compound lotteries principle (Grant, 2013). Thus the paradox also violates the substitution axiom (also known as independence of irrelevant alternatives).

III. Quantum Physics and Progress in Behavioral Economics

Although Isaac Newton assumed that space and time were linear, absolute and separate entities; Albert Einstein overturned Newtonian physics demonstrating that space and time were actually the same entity and this entity was non-linear and relative (Ray, 2011:274). By the discoveries in the quantum physics it is understood that (Lambertini, 2013:271):

- The substance is not tied to one place, to one location; the substance is in motion,
- The cause-consequence principle, the philosophy of determinism is not applicable,
- Connections between the objects matter.

As Bohr suggested; the quantum theory is not related to certainty (determinism) but to possibilities (Lambertini, 2013: p.271). All of the basic concepts of Newtonian World; determinacy, predictability, divisibility, rationality, the notion of "either-or", order, reliability and validity, objectivity and impartiality, testability, consistency, independence, entitativity, causality, bivalency, atomism, linearity, proportionality, stability, classification / categorizing, simplicity, manageability, exclusivity, reductionism are replaced by indeterminism, improbability, non-linearity, complexity, fuzziness, interconnection and interaction, duality (wave-particle), intersubjectivity, nonlocal causes, indefinitiveness, intentionality, uncertainty, lack of objectivity, complementarity, notion of "both", disproportionalities between cause and effect chain, interrelatedness, sensitivity to initial conditions (chaos theory), holism, context, potentiality, constructivism, unknowability in quantum world (Dulupçu & Okçu, 2000: pp.32, 44).

Similarly meanwhile it is understood that "as if" modeling which is necessary for simplicity and generality is not enough for the evaluation of real life economic behavior. At this point "behavioral economics" served to economics so did quantum physics to Newtonian Physics. In 1979 two psychologists Daniel Kahneman and Amos Tversky suggested "*Prospect Theory*" as an alternative model of decision making under risk and a critique of expected utility theory referring to Allais' example. Behavior observed in Allais Paradox was explained by rank dependent utility (Quiggin, 1981; Schmeidler, 1989), cumulative prospect theory (Tversky & Kahneman, 1992; Wakker & Tversky, 1993) and deviations from rationality assumptions are questioned by many other behavioral paradoxes (Camerer *et al.*, 2003). Individual decision making under risk is analyzed by many authors (Lindgren, 1971; Tversky & Kahneman, 1980; 1983a, Morier & Borgida, 1984; Berger, 1985; Shafir *et al.*, 1990; Weidlich, 1991; Weirich, 2001; Tentori *et al.*, 2004; Zeckhauser, 2006). It is revealed that individual decision making violates classical Bayesian probability laws in several studies (Tversky & Kahneman, 1983b; Shafir & Tversky, 1992; Tversky & Shafir, 1992). Violations of the expected utility hypothesis is revealed by Tversky (1975), Machina (1982) and Schmeidler (1989). Herbert Simon (1955, 1957) took cognitive limitations on decision making into account and suggested bounded rationality. Savage (1954) suggested subjective probabilities. While von Neumann-Morgenstern's expected utility theory refers to objective probabilities where the decision makers assess a lottery depending on the frequencies of the outcomes (La Mura, 2009), Savage's subjective expected utility theory suggests choice based subjective probabilities expressing decision maker's beliefs (Karni, 2005). The systematic departures of individual preferences from the assumptions of the expected utility theory with reference to objective or subjective probabilities reviewed by Machina (2008) and nonlinear models developed by Quiggin (1982) and Montesano (2008).

Recently it is seen that the formalism of quantum mechanics is applied to economics (Schaden, 2002; Baaquie, 2004; Haven, 2005; Bagarello, 2006). Quantum-like model of decision making process is proposed to explain irrational behavior (Khrennikov, 2009; Asano *et al.,* 2010, 2011, 2012). What is noteworthy is the deviations of individual behavior from the rationality assumptions of the theory of expected utility, individual decisions violating the expected utility hypothesis and the sure thing principle have been discussed widely in the literature of quantum physics. The violation of the expected utility hypothesis and the sure thing principle related to quantum probability and quantum interference is presented by Busemeyer et al. (2006), Franco (2014), Khrennikov & Haven (2009) and Pothos& Busemeyer (2009). Franco (2009) determined the two major causes of bounded rationality as the limitations of the human mind and the structure within which the mind operates. Aerts and Sozzo (2011) introduced "context" playing a compelling role in individual decision making under risk. They suggested "contextual risk" which is present in a situation where probability can be non-classical *i.e.,* non-Kolmogorovian. Aerts and Sozzo (2012a, 2012b) claimed that individual choices violate the sure thing principle because of a different type of reasoning not only guided by logic but also conceptual thinking which is context dependent. According to Knight (1921), risk refers to a situation where probability of the outcomes are known. Such definition is adhere to the classical probability satisfying Kolmogorov's axioms and thus called "Kolmogorov's Probability" (Kolmogorov, 1933). As it is shown by Accardi (1982) and Beltrametti (2001); quantum probability definition is not Kolmogorovian and seemingly contextual risk is consistent with the nonclassical probability of quantum mechanics.

IV. The Observer In Allais Paradox: Ultimate Perception of the Researcher From Schrödinger's Point of View

Quantum measurement problem is examined by Von Neumann (1955), Heisenberg (1958), Wigner (1963, 1967), Mermin (1985), Ballentine (1989), Schrödinger (Bitbol & Darrigol, 1993), Peres (1994) and d'Espagnat (1999). This section reviews Allais Paradox considering the observer from Schrödinger's point of view.

Schrödinger's Cat is a famous thought experiment proposed by Erwin Schrödinger in 1935; there is a closed box including a cat, a radioactive source, a flask filled with poison gas. The mechanism in the box is sensitive to radioactivity. When a decay is detected the mechanism is designed to break the flask and let the poison gas release into the box. According to this scenario; if radioactive decay occurs the cat is dead, otherwise alive. Apparently the cat is both dead and alive until the observer opens the box! The Copenhagen interpretation of the two possibility occurring simultaneously is called superposition which ensues interference. At the time of the observer opens the box and sees the cat is alive (or dead); the superposition is lost, the system collapses and then we can talk about reality (which is the cat is alive or dead), not possibility.

The Schrödinger's Cat experiment sets out to explain "the measurement problem" and according to Copenhagen interpretation it is impossible to separate what is measuring and what is measured leading to the blending of both (Tarlacı, 2012a, 2012b). In quantum world the observer/experimenter and the experiment cannot be discussed separately. The quantum theory has established the unity between the subject and the object, the experiment and the scientist, people and nature for the first time in the history of science (Lambertini, 2013: p.272). According to quantum theory; objects, systems or paradigms become meaningful only in the context of their interaction with the observer (Karsten, 1990: p.387). Thus physical reality which is created by the act of the observation relating to the Copenhagen interpretation leads to "an individual quantum system" (Khrennikov, 2003: p.211). In such a concept, the result of each experiment depends on the relation between the scientist and the experiment. Because final decision about the experiment is related with the observer; the result of the experiment is considered as subjective. This deliberation of subjectivity leads to "every single observation" is accompanied with "a different reality" which is opposite to the causality principle or in other words determinism. According to Bohr's interpretation, the economist conducting Allais' experiment or Allais himself can no longer be evaluated separately from the subjects and the experiment itself leading us to the subjectivity of the results.

Allais proved that individual choices under risk violate the expected utility maximization hypothesis and the independence axiom. In economics such an individual whose choices violate theoretical assumptions is called "irrational". Consequently, those whose preferences violate the theory called irrationals and those whose preferences are consistent with the theory called rationals. Therefore the observer knows that there are two possibilities ex-ante ("*before the event*") for the subject;

- The subject behaves rational (individual choices are consistent with the axioms of the theory),
- The subject behaves irrational (individual choices violate the axioms of the theory).

Similar to Schrödinger's cat of being dead or alive; being rational or irrational for the subject coexist in the observer's mind before the experiment. Rationality and irrationality of decisions create a superposition in the observer's brain. Thus the subject is both rational and irrational until the observer sees the choices! As soon as the observer sees the subject's choice and determines that the subject is rational or irrational; the superposition is lost, the system collapses and then we can talk about reality. The Copenhagen interpretation suggests that when the observer sees the choices, the experiment is completed and rationality reality or irrationality reality is decided. The table summarize all movements and correspondent interpretation in Allais Paradox (Table 1):

Table 1. Superposition of the Possibilities and Rationality Inferences of Allais Paradox.

Experiment I	Experiment II	Violation of the Independence Axiom (Violation of the Sure Thing Principle)	The Observer's Decision
A>B	D>C	Yes	Irrational
A>B	C>D	No	Rational
B>A	C>D	Yes	Irrational
B>A	D>C	No	Rational

> refers to strong preference relation; A>B is read as A is preferred to B.

V. Conclusion

Since Allais, the assumptions of rationality have been discussed. Economists have used economic tools, psychologists have used theories in psychology and psychological methods. Recently neuroscientists and neuroeconomists have used neuroscientific methods such as fMRI, TMS, EEG, PET or injections to test individual behavior under risk. Interdisciplinary research provides researchers to assess individual behavior from different perspectives. Now it is possible to better understand irrational individual behavior by quantum physics.

According to many-worlds interpretation of quantum physics, all of the scenarios in the table above is valid. All possible futures are real as are histories. Each possibility represents a different reality and every possible outcome of each state exists in its own reality. In such a scenario, the observer cannot signify the subject as rational or irrational for sure since there is no enough information to make a decision. If such a decision could be made it would mean that the entire system is known perfectly. However the assumptions suggested for such a system is sentenced to be insufficient to explain real life situations which has been a serious critique to economic theory.

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