Minimum K-Broadcast Graphs In Communication Networks

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Abstract

Multi-broadcasting is the process of disseminating information in a network by placing a series of calls over the nodes in the network. Each call requires one unit of time and a node is allowed to call at most k of its adjacent nodes simultaneously. Following these constraints, at most $(k+1)^t$ vertices can be informed in t time steps. Alternatively, the time taken to complete a k-broadcast on n vertices is $\lceil \log_{(k+1)} n \rceil$. Multi-broadcasting has been extensively studied and some lower bounds for $B_k(n)$, the number of edges in a minimum k-broadcast graph (k-mbg) with n vertices, for k = 2, 3 & 4 have been presented for small n. We present some more values for $B_2(n)$ for $13 \le n \le 16$ with proofs of optimality and 2-broadcast schemes with different vertices as originators.

Keywords – Broadcasting, Multi-broadcasting, Multi-broadcast Graphs, Communication Networks

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1. Introduction

A graph G = (V, E) can be used to represent a communication network where the vertices represent communicating stations and edges represent connections between them. Here we consider G as a finite undirected simple connected graph. The Broadcasting problem, as introduced in one of the earliest papers by Farley [1], is a process of delivering information from an originator vertex to all the nodes in the network. This is done by placing a series of calls by the vertices in G. The calling process follows the constraints that each call requires one time unit, and, each vertex is allowed to call only one of its connected neighbors in one call. A broadcast graph is one in which broadcasting can be completed in $\lceil log_2 n \rceil$ time steps in the graph, regardless of the originator vertex. Construction of broadcast graphs having minimum number of edges, Minimum broadcast graphs (mbgs), and sparse broadcast networks has been the interest of many researchers [1-16]. The problem has been extensively studied where mbgs for some small values of n, the number of nodes in a network, have been determined and construction methods for larger n have been proposed. The broadcasting problem was extended by Konig and Lazard [17,18], wherein, each vertex is enabled to call more than one, say k, of its neighbors simultaneously. This model was defined as k-broadcasting or multi-broadcasting. k-broadcasting is defined as information dispersal in a graph where a message originated at one vertex is to be distributed to the entire network. This message transmission follows the constraints that each call requires one discrete time step and a vertex is allowed to call at most k of its neighbors simultaneously in each call. All informed vertices aid the originator in spreading the message. Following this scheme, we can see that at any time step, the number of vertices receiving the message increases by a factor of k+1. Thus, the number of vertices informed in t time steps is at most $(k+1)^t$. Or, in other words, the time required to inform n vertices is $\lceil log_{(k+1)} n \rceil$ time units.

A k-broadcast graph is one in which a message originating at any vertex can be distributed to the entire network in $\lceil log_{(k+1)} n \rceil$ time steps. A Minimum k-broadcast Graph (k-mbg) is a k-broadcast graph having the minimum number of edges. Construction of k-mbgs for arbitrary n has been extensively researched. Farley [1] speculated that determination of B(n), number of edges in a minimum broadcast graph, is NP-Complete. It is natural to infer that determining $B_k(n)$, number of edges in a minimum k-broadcast graph, is NP-Complete as well.

From an application perspective, k-mbgs represent the most economical multi-broadcast networks in which information can be transmitted in the fastest possible way. k-broad casting finds applications in parallel and distributed computing. Certain organizations such as military, government, business, can be modeled as a distributed network in which a general order or an alert message is to be communicated from an originator to the rest of the network in minimum time using minimum number of connections. The process of k-broadcasting is also likened to the spread of a contagion or influence spreading in a social network.

It can be understood that there is no known method for constructing k-mbgs and determining exact values of $B_k(n)$ and is extremely difficult. Lazard [18] presented 2-mbgs for $n \leq 12$, 3-mbs and 4-mbgs for $n \leq 8$. In [24], Lee *et.al* presented 3-mbgs for n = 9, 10 and 4-mbgs for n = 9, 10, 11. Sparse broadcast graphs for larger n can be constructed by compounding known k-mbgs for small n. For this reason, designing of k-mbgs for small n becomes important.

In this paper, we present values of $B_2(n)$ for $13 \leq n \leq 16$, and the corresponding k-mbgs for each case. We support our results with extensive proofs of optimality and illustrate the k-broadcast protocols with figures.

The paper is organized into four sections. In Section 2, we review some of the work presented in extant literature. Section 3 contains our main results for values of $B_2(n)$ for $13 \leq n \leq 16$ with proofs and the corresponding k-mbgs. In Section 4, we conclude our study and suggest directions for future work in this area.

2. Related Work

Determination of exact values of B(n), the number of edges in a minimum broadcast graph, has caused much intrigue and a number of researchers have worked towards construction of broadcast graphs with minimum number of edges. The first mbgs appeared in [2] for $n \leq 15$. Mbgs for n = 17, 18, 19 were presented in [3, 4]. Maheo [5] gave the values of B(n) for n = 20, 21, 22. For n = 26, 27, 28, 58, 61, the mbgs were presented in [6], and for n = 30, 31 in [5]. Labahn [7] constructed the mbg on 63 vertices, and Harutyunyan [8] for n = 127. Bin Shao [9] presented the mbgs for n = 1023, 4095. Chau and Liestman [13] proposed methods for constructing broadcast graphs with very few edges using split construction and compounding methods. These methods use fewest possible edges to connect smaller mbgs to obtain sparse broadcast graphs for large n. In [14], Averbuch *et al.* proposed a new construction method using binomial trees and hypercubes, giving improved upper bound for B(n) for some values of n. In another study, Harutyunyan *et. al* [15], presented a construction method that combines knödel graphs, instead of hypercubes, with binomial trees. In [16] the authors discussed construction methods involving vertex addition to knödel graphs.

The multi-broadcasting problem was introduced by Grigni and Peleg [11] as a model for placing conference calls of a certain limited size, where each communicating station in the network is enabled to inform at most k of its adjacent nodes in one call. A broadcast scheme, or protocol, is a sequence of vertices informed by the originator at each time step. This determines a spanning tree of the vertices of the graph. In [19, 20], k-broadcasting in trees was explored to compute the size of a tree in which k-broadcasting can be accomplished in t time steps. Bin Shao [9, 21], researched k-broadcasting in graphs and in trees. He presented algorithms for finding the k-broadcast center of a tree, where k-broadcast center is the set of vertices with the smallest k-broadcast time.

In a recent study [22], the authors delve on the problem of communicating information to a group of robots in the fastest and most economical manner. The problem is a direct application of the broadcasting model where the robots are represented by vertices edges represent the connections between them. They suggest different heuristics to construct an optimal route from a given originator to the rest of the network by exploring the shortest path between vertices. Harutyunyan *et.al* [23] presented a heuristic algorithm for finding the minimum broadcast time for a broadcast originating at a central vertex in k-cycle graphs.

Some 2-mbgs for $n \leq 12$, 3-mbs for $n \leq 8$, and 4-mbgs for $n \leq 8$, were presented by Lazard in [18]. This research was extended by Lee *et.al* [24], where 3-mbgs for $n = 9, 10$ and 4-mbgs for $n = 9, 10, 11$ were presented. They also proposed a network compounding algorithm for constructing sparse k-mbgs of larger order by connecting copies of known k-mbgs for smaller n. In this study, we present some more 2-mbgs for $n = 13, 14, 15$ and 16.

3. Main Results

Our main results are summarized in Theorem 1. In this section, we present values of $B_2(n)$ for $13 \leq n \leq 16$, and 2-mbgs for each case. To get an upper bound, say l, on $B_2(n)$, the number of edges in a minimum 2-broadcast graph on n vertices, we construct a 2-broadcast graph having n vertices and l edges. For obtaining a lower bound we make use of some graph properties, such as minimum possible degree of a vertex, number of vertices having the minimum degree, and arrive at lower bound value for $B_2(n)$. When the upper and lower bounds match, we have an optimal result.

Theorem 1. The values of $B_2(n)$ for $13 \leq n \leq 16$, are as given in Table 1.

Table 1. Number of edges in 2-mbg for $13 \leq n \leq 16$.

N	13	14	15	16
$B_2(n)$	18	20	22	25

3.1. Basic Methodology Used for Proof of Optimality

Optimality of each result is proven in two parts. Part I explores graph properties and topology, establishing a lower bound on $B_2(n)$, whereas, whereas, Part II shows the 2-mbg having the lower-bound number of edges. We make the following observations:

Proposition 1. If G is a k-mbg on n vertices, then G can have a vertex of degree 1 only when $n = 1 + (k+1)^{t-1}$, where $t = \lceil log_{(k+1)} n \rceil$.

Proof: Let u be a vertex of degree 1 in a k-broadcast graph. In the first time step, u informs vertex v. Now, in the remaining (t-1) time steps, v can pass the message to at most $(k+1)^{t-1}$ vertices (including v), as shown in Figure 1. This makes a total of $(k+1)^{t-1}$ +1 informed vertices in t time steps (including u).

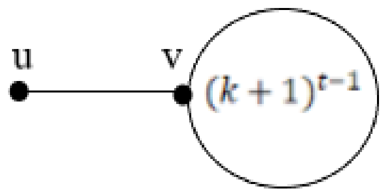
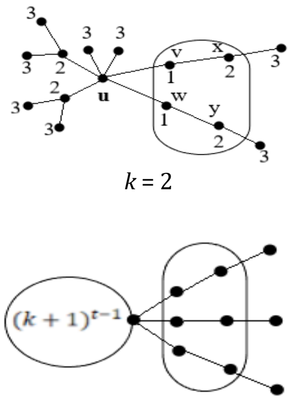


Figure 1. G having a vertex of degree 1.

Proposition 2. For $n > 1 + (k+1)^{t-1}$, $t = \lceil log_{(k+1)} n \rceil$, the minimum degree of a vertex can be 2, however, G can have at most 2*k vertices of degree 2.

Proof: Let $u \in$ G be a vertex of minimum degree 2, $(k+1)^{t-1} < n \leq (k+1)^t$. If u is the originator of the message, in the first time step u can call k vertices, which will inform 1 more vertex each, in the second time step. As the degree has to kept 2, each of the k vertices informed in the second time step will relay the message to 1 more each in the final step. This way at most k*t vertices will be informed and the number of vertices of degree 2 will be at most 2*k. This situation is illustrated in Figure 2 for $k = 2$, $k = 3$ and $(k+1)^2 < n \leq (k+1)^3$.



$k = 3$

Figure 2. G having vertices with minimum degree 2, for k = 2, 3.

Proposition 3. When G has 2*k vertices of degree 2, the maximum number of informed vertices can be $(k+1)^{t-1}$ + k*t.

Proof: As explained above, the originator, u, informs k vertices in the first time step and continues to call $(k+1)^{t-1}$ vertices in the remaining time steps. In order to maintain minimum degree 2, these k vertices call only 1 vertex each, in the next time step. These k, newly informed vertices, further call 1 vertex each in the final step. This way we have $(k+1)^{t-1}$ + k*t total vertices informed.

The proof of optimality for values of $B_2(n)$ are presented in the following subsections. In Part I we establish a lower bound |E(G)| as:

|E (G)| ≥ (degree sum of all the minimum degree of each vertex in G)/2, since we are considering minimum degree that a vertex may have here. Let this value of |E(G)| be l.

A 2-broadcast graph having n vertices and l edges is presented in Part II, indicating that $B_2(n) \leq l$. Part I and Part II together establish that $B_2(n) = l$.

3.2. *n* = 13, $B_2(13)$ = 18.

The proof includes two parts, Part I and Part II. In Part I we analyze the graph to obtain $B_2(13)$ ≥ 18. In Part II, n2-mbg on 13 vertices and 18 edges is presented, implying that $B_2(13) \leq 18$. These two parts complete the proof, $B_2(13)$ = 18.

Part I: Lower bound for$B_2(13)$.

Let G be a 2-mbg on 13 vertices. According to Proposition 1, 2 and 3, we make the following observations:

1. G cannot have a vertex of degree 1.
2. G can have at most 2*2 = 4 vertices of degree 2.
3. A vertex of degree 2 must be adjacent to a vertex of degree at least 3.

We note that the 2-regular graph on 13 vertices, C_{13}, is not a 2-mbg. Therefore, a degree 2 must be connected to at least one vertex having degree at least 3. Thus, G will have at most 4 vertices of degree 2, some vertices of degree 3, and some other vertices of degree 4 or more. Let the number of vertices having degree 4 or more be p. Let U be a subset of V, containing these p vertices.

It follows that $\sum_{u \epsilon U} \deg \quad u \geq 4{*}p$. (1)

Also, each of the remaining 13-p vertices of degree 2 and 3, must be connected to a vertex of degree at least 4.

Therefore, $\sum_{u \epsilon U} \deg \quad u \geq 13 - p$. (2)

1. If 4*p ≥ 13 – p, then p > 2.

And, |E (G)| ≥ [4*p + (13 – 4 – p)*3 + 4*2] / 2 ≥ 37/2 ≥ ⌊ 18.5⌋ ≥ 18.

2. If 13 – p ≥ 4*p, then p < 3, and $\sum_{u \epsilon U} \deg \quad u$ ≥ 10|E (G)| ≥ [(13 – p) + (13 – 4 – p)*3 + 4*2] / 2 ≥ 36/2 ≥ 18

Therefore, we get, $B_2(13)$ ≥ 18.

Part II. A 2-minimum broadcast graph on 13 vertices with 18 edges and broadcasting schemes.

We present a 2-mbg on 13 vertices with 18 edges. From the 2-mbg in Figure 3, we have $B_2(13)$ ≤ 18, thus completing the proof that $B_2(13)$ = 18.

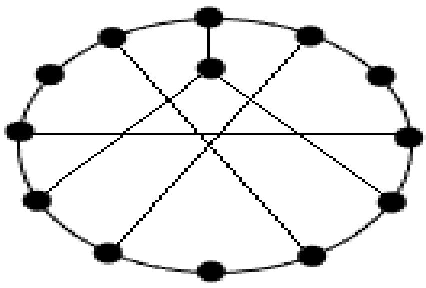


Figure 3. A 2-MBG on 13 vertices.

Figure 4 depicts some 2-broadcast schemes, with different vertices as the originator. Other schemes can be obtained by the symmetry of the graph. In a scheme, the vertex labeled as '**+**' indicates the originator vertex. The label beside other vertices signifies the time step at which they get informed.

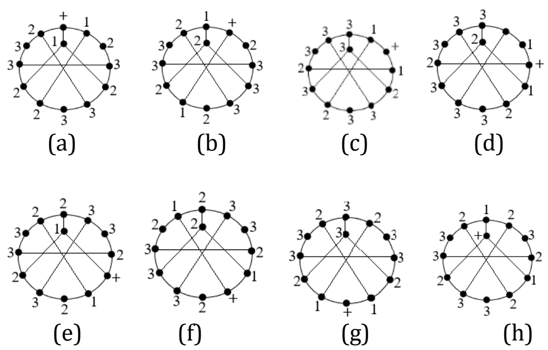


Figure 4. 2-Broadcasting schemes for 2-mbg on 13 vertices with different vertices as originator.

3.3. n = 14, $B_2(14)$ = 20.

Proof: Part I - $B_2(14) \geq 20$

Let G be a 2-mbg on 14 vertices. From proposition 1, 2 and 3, we make the following observations about G:

1. Minimum degree of a vertex can be 2.
2. There can be at most 4 vertices of degree 2.
3. As the broadcast is to be completed in $[log_3 14]$ = 3 time steps, a degree 2 vertex must be adjacent to a vertex having degree at least 3.

Let the number of vertices having degree 4 or more, be p. We collect these p vertices in a subset U ⊆ V. The remaining (14 – 4 – p) vertices will be having degree 3.

It follows that $\sum_{u\epsilon U} \deg \quad u \geq 4{*}p$ (3)

Again, each member of U must be adjacent with at least one of the other (14 – p) vertices.

Therefore, $\sum_{u\epsilon U} \deg \quad u \geq 14 - p$ (4)

1. If $4{*}p \geq 14 - p$, $p > 2$, and
|E (G)| ≥ ⌈ [4*p + (14 – 4 – p)*3 + 4*2] / 2 ⌉ ≥ 40/2 ≥ 20

2. If $14 - p \geq 4{*}p$, $p < 3$, and, $\sum_{u\epsilon U} \deg \quad u \geq 11$
|E (G)| ≥ [(14 – p) + (14 – 4 – p)*3 + 4*2] / 2 ≥ 40/2 ≥ 20

Therefore, $B_2(14) \geq 20$.

Part II. A 2-broadcast graph on 14 vertices, 20 edges, and different 2-broadcast schemes.

We present a 2-broadcast graph on 14 vertices having 20 edges. From the graph in Figure 5, we can say,
$B_2(14) \leq 20$, thus completing the proof that $B_2 14$ = 20.

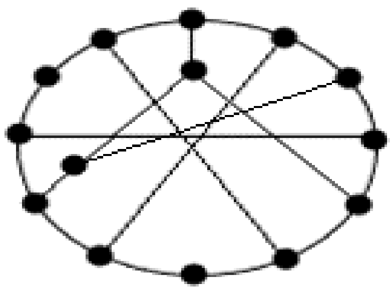


Figure 5. 2-mbg on 14 vertices.

Broadcast schemes are illustrated in Figure 6. The originator is depicted by '**+**'. Other vertices are labeled by the time step at which they get informed.

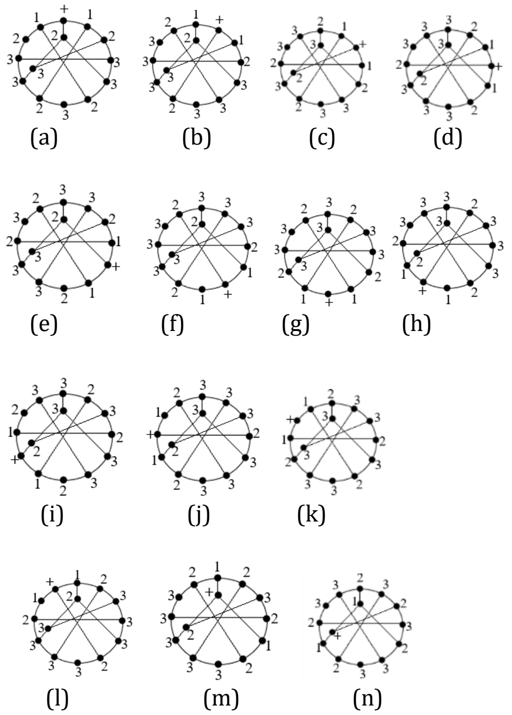


Figure 6. 2-Broadcasting schemes for 2-mbg on 14 vertices, with different vertices as originator.

3.4. n = 15, $B_2(15)$ = 22

Proof: Part I - $B_2(15) \geq 22$.

If G is a 2-mbg on 15 vertices, we conclude from proposition 1, 2 and 3:

1. Minimum degree of a vertex can be 2.
2. There can be at most 4 vertices of degree 2.
3. As the 2- broadcast is to be completed in $[log_3 15]$ = 3 time steps, a degree 2 must be connected to a node of degree t least 3.

Let the number of vertices of degree 4 or more be p. We collect these p vertices in a subset U ⊆ V. The remaining (15 – 4 – p) vertices will be of degree 3.

It follows that $\sum_{u\epsilon U} \deg \quad u \geq 4{*}p$ (5)

Also, each vertex in U must be connected to at least one of the other (15 – p) vertices.

Therefore, $\sum_{u\epsilon U} \deg \quad u \geq 15 - p$ (6)

1. If $4{*}p \geq 15 - p$, $p \geq 3$, and,
|E (G) | ≥ [4*p + (15– 4–p)*3 + 4*2] / 2 ≥ 44/2 ≥ 22
2. If $15 - p \geq 4{*}p$, $p \geq 3$, and,
|E (G)| ≥ [(15-p) + (15–4 – p)*3 + 4*2]/ 2 ≥ 44/2 ≥ 22

Therefore, we obtain, $B_2(15) \geq 22$.

Part II: A 2-Minimum Broadcast Graph having 15 Vertices and 22 edges, and different 2-broadcast schemes.

A 2-broadcast graph having 15 vertices and 22 edges is presented in Figure 7, establishing $B_2(15) \leq 22$. Thus, completing the proof that $B_2(15) = 22$.

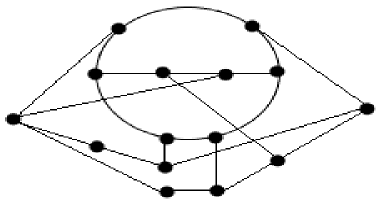
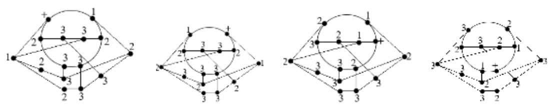


Figure 7. A 2-broadcast Graph on 15 vertices, 22 edges.

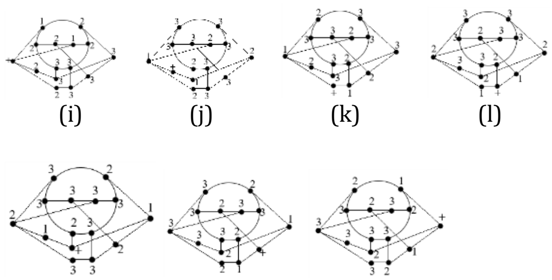
The 2-broadcast schemes, with different vertices as originator, are illustrated in Figure 8.



(a) (b) (c) (d)

(e) (f) (g) (h)

Figure 8. Contd.



(i) (j) (k) (l)

(m) (n) (o)

Figure 8. 2-broadcast Schemes for 2-mbg on 15 vertices with different originators.

3.5. $n = 16$, $B_2(16) = 25$.

Proof: Part I - $B_2(16) \geq 25$.

Let G be a 2-mbg on 16 vertices. From proposition 1, 2 and 3, we infer that:

1. Minimum degree of a vertex in G can be 2.
2. G can have maximum 3 vertices having degree 2. This follows from Proposition 3, as $16 > (2+1)^{t-1} + 2*t$, $t = \lceil log_{(3)}16 \rceil$.
3. As the 2-broadcast is to be completed in 3 time steps, a degree 2 vertex must have a neighbor having degree at least 3.

Therefore, G will have at most 3 vertices of degree 2, and other vertices of degree 3 or more. Let p be the number of vertices of degree at least 4. As before, we collect these p vertices in a subset U ⊆ V. The other $(16 - 3 - p)$ vertices will be of degree 3.

It follows that $\sum_{u \epsilon U} \deg \quad u \geq 4*p$ (7)

Again, each member of U must be adjacent with at least one of the other $(16 - p)$ vertices.

Therefore, $\sum_{u \epsilon U} \deg \quad u \geq 16 - p$. (8)

1. If $4*p \geq 16 - p$, then $p \geq 3.2 \geq 3$, and,
$|E(G)| \geq [(16 - p) + (16 - 3 - p)*3 + 3*2] / 2 \geq 49/2 \geq \lceil 24.5 \rceil \geq 25$
2. If $16 - p \geq 4*p$, $p \leq 3.2 < 4$, and,
$|E(G)| \geq [4*p + (16 - 3 - p)*3 + 3*2] / 2 \geq 49/2 \geq \lceil 24.5 \rceil \geq 25$

Therefore, $B_2(16) \geq 25$.

Part II: A 2-Minimum Broadcast Graph on 16 Vertices, 25 edges, and different 2-broadcast schemes.

Figure 9 presents a 2-broadcast graph having 25 edges, implying $B_2(16) \leq 25$.

Part I and Part II complete the proof that $B_2(16) = 25$.

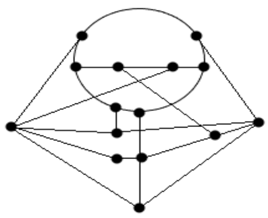
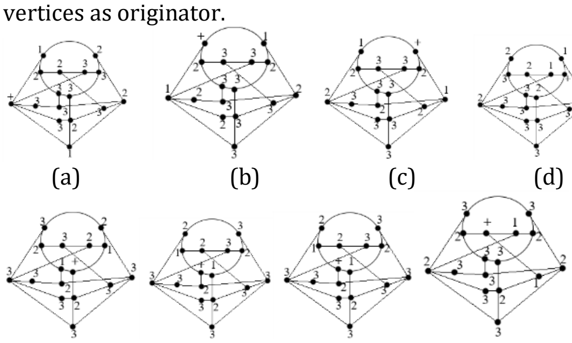


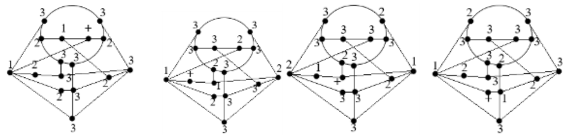
Figure 9. A 2-mbg on 16 vertices having 25 edges.

Figure 10 shows the 2-broadcast schemes, with different vertices as originator.



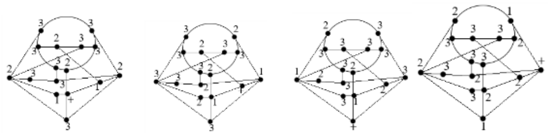
(a) (b) (c) (d)

(e) (f) (g) (h)



(i) (j) (k) (l)

Figure 10. Contd.



(m) (n) (o) (p)

Figure 10. 2-broadcast Schemes for 2-mbg on 16 vertices with different originators.

4. Conclusions and Future Work

This paper explores *k*-broadcasting in communication networks represented as graphs, where every vertex is enabled to inform at most *k* of its adjacent nodes during any call. We present 2-mbgs for small values *n,* $13 \leq n \leq 16$ with proofs of optimality and the respective 2-broadcast schemes. The work can be extended by exploring *k*-mbgs for higher values of *n* and *k*.

References

[1]. M. Farley, Minimum Broadcast Networks, Networks 9 (1979) 313-332.

[2]. Farley AM, Hedetniemi ST, Mitchell SM, Proskurowski A. Minimum broadcast graphs. Discret. Math.. 1979 Jan 1; 25(2):189-93.

[3]. Mitchell S, Hedetniemi S. A census of minimum broadcast graphs. J. Combin. Inform. System Sci. 1980;5:141-51.

[4]. Bermond JC, Hell P, Liestman AL, Peters JG. Sparse broadcast graphs. Discrete applied mathematics. 1992 Apr 30; 36(2):97-130.

[5]. Maheo M, Saclé JF. Some minimum broadcast graphs. Discrete Applied Mathematics. 1994 Sep 14; 53(1-3):275-85.

[6]. Saclé JF. Lower bounds for the size in four families of minimum broadcast graphs. Discrete Mathematics. 1996 Apr 6; 150(1-3):359-69.

[7]. Labahn R. A minimum broadcast graph on 63 vertices. Discrete Applied Mathematics. 1994 Sep 14;53(1-3):247-50.

[8]. Harutyunyan HA. An efficient vertex addition method for broadcast networks. Internet Mathematics. 2008 Jan 1;5(3):211-25.

[9]. Bin Shao, On k-broadcasting in graphs, 2006-spectrum.library.concordia.ca

[10]. Jian-Guo Zhou, Ke-Min Zhang, A Minimum Broadcast Graph on 26 Vertices, Applied Mathematics Letters 14(2001) 1023-1026.

[11]. Grigni M, Peleg D. Tight bounds on minimum broadcast networks. SIAM Journal on Discrete Mathematics. 1991 May;4(2):207-22.

[12]. Grigoryan H. Problems related to broadcasting in graphs (Doctoral dissertation, Concordia University) 2013.

[13]. SC Chau, A. Liestman, "Constructing minimal broadcast networks", J. Combin. Inform. Syst. Sci, 10, pp. 110-122, 1985.

[14]. Averbuch, R. H. Shabtai, and Y. Roditty, "Efficient construction of broadcast graphs", Discrete Applied Mathematics, 171, pp. 9-14, 2014.

[15]. H. Harutyunyan, Li Zhiyuan, "A new construction of broadcast graphs", Discrete Applied Mathematics, 280, pp. 144-155, 2020.

[16]. H. Harutyunyan and A. Liestman, "Upper bounds on the broadcast function using minimum dominating sets", Discrete Mathematics 312, no. 20, pp. 2992-2996, 2012.

[17]. J. C. Konig, E. Lazard, "Minimum k-broadcast graphs", Discrete Applied Mathematics, 53, pp. 199-209, 1994.

[18]. E. Lazard, "Broadcasting in DMA-bound bounded degree graphs", Discrete Applied Mathematics, 37, pp. 387-400, 1992.

[19]. Shastri, S. Gaur, "Multi-broadcasting in Communication Networks I: Trees", In: Proc. of ISCOM, Hsinchu, Taiwan, pp. 167-170, 1997.

[20]. H. Harutyunyan, A. Liestman, "k-Broadcasting in trees", Networks: An International Journal, 38, no. 3, pp. 163-168, 2001.

[21]. H. Harutyunyan, A. Liestman, and B. Shao, "A linear algorithm for finding the k-broadcast center of a tree", Networks 53, no. 3, pp. 287-292, 2009.

[22]. M. Kenzin, I. Bychkov, and N. Maksimkin. "Emergency broadcasting strategies for distributed robotic groups under limited communication", In: Journal of Physics: Conference Series, vol. 1864, no. 1, p. 012043. IOP Publishing, 2021.

[23]. N. Conlan, H. Harutyunyan, and E. Maraachlian, "Heuristic Algorithms with Near Optimal Broadcasting in Cactus Graphs", In: 2020 28th Euromicro International Conference on Parallel, Distributed and Network-Based Processing (PDP), pp. 253-257. IEEE, 2020.

[24]. S. Lee, and J. Ventura, "An algorithm for constructing minimal c-broadcast networks", Networks: An International Journal, 38, no. 1, pp. 6-21, 2001.