



# Investigation of symmetrical two-cylinder lens.

Ali A. Al-Azawy

Ph.D. in Physics / Department of Medical Physics / College of Science / Al-Karkh University of Science

Address: Baghdad / Iraq

E-mail: [aliphysics74@gmail.com](mailto:aliphysics74@gmail.com)

## Abstract

In the current work, the immersion lens was investigated as a two-cylinder symmetrical lens with different radii, and this lens is studied in the accelerated mode and in the deceleration mode. Their main points have been studied in these cases. Where firstly the lens has a negligible gap width and secondly with a dysfunctional gap width, and all the cardinal points can be obtained as a function of the voltage ratio.

**Keywords:** Electro-Optics, Symmetric lens, immersion lens, cardinal points, electrostatic lenses.

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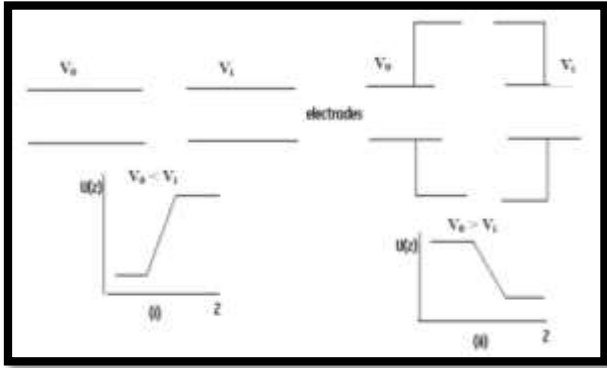
## Introduction

Development in technology and science journals. The electrostatic lenses have many applications and uses in modern electronic devices, we mention the electrostatic devices used in the assembly or formation of ion beams adopted in the study of ion mass spectrum analysis. Today, with the help and characteristics of advanced electrostatic lenses, ionic sensors are being used to implant ions inside semiconducting materials, for example, to change some of the properties of these materials, and thus the use of the electron probe appeared in a wide range of industries in the manufacture of miniature semiconductor devices and others. In general, electrostatic lenses consist of an axially symmetric electrostatic field, and asymmetry may occur completely as the axial voltage distribution and the derivative of this voltage are not equal to zero [ $\ddot{U}_0(z) \neq 0$ ], depending on the shape of the electrode and the electrostatic field distribution.

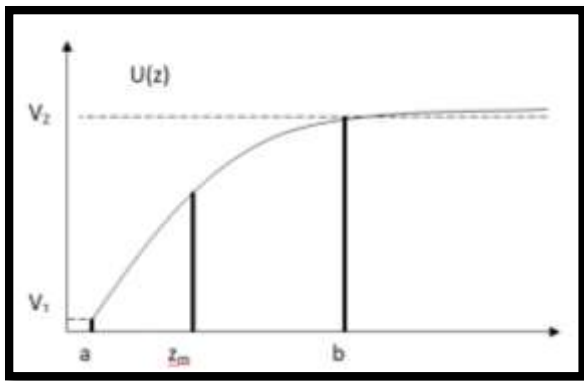
## THEORETICAL ASPECTS

The immersion lens was investigated as a two-cylinder symmetrical lens with different radii, and this lens is studied in the accelerated mode and in the deceleration mode. Their main points have been studied in these cases. Where firstly the lens has a negligible gap width and secondly with a dysfunctional gap width, and electrostatic lenses can be classified into several types as in The figures (1, 2, 3 and 4) below (Szilagy, 1988), (Baranova, 1997) & (Al-Azawy, 2016).

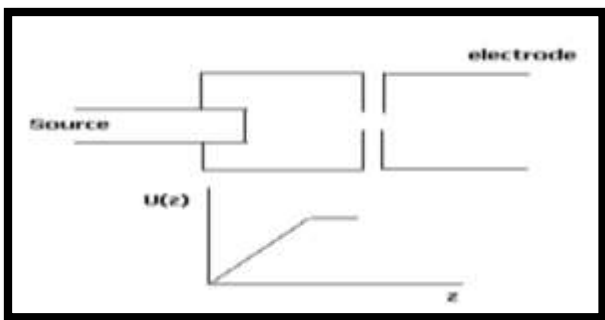




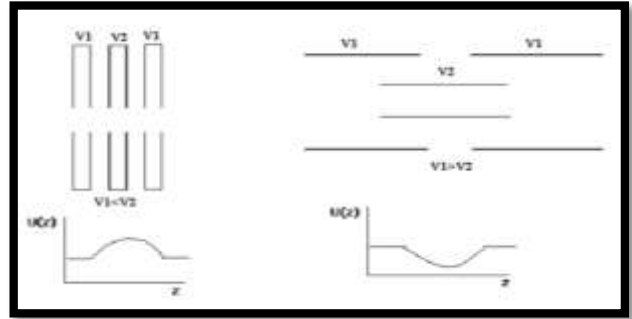
**Figure 1A. the electrostatic immersion lens (Al-Azawy, 2016).**



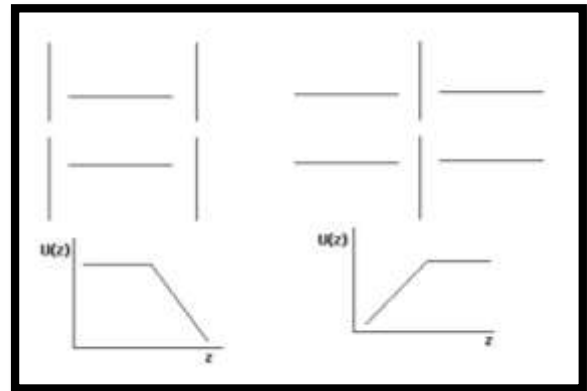
**Figure 1B. Potential distribution of a two-electrode immersion lenses (Al-Azawy, 2016).**



**Figure 2. the cathode lens (Al-Azawy, 2016).**

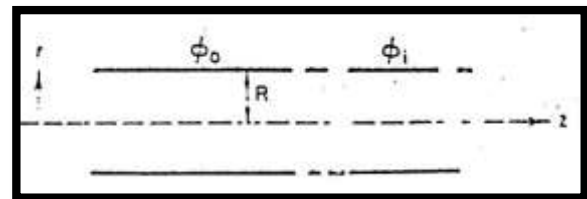


**Figure 3. the Einzel lens (Al-Azawy, 2016).**



**Figure 4. the Diaphragm lens (Al-Azawy, 2016).**

Figure-5 shows the submerged lens, and all the cardinal points can be obtained as a function of the voltage ratio (El-Kareh, 1970).



**Figure 5. Equidiameter two-cylinder electrostatic lens.**

For the image and object side, the focal length are related to the potentials. A similar relationship holds for the principal planes as shown by eq. (El-Kareh, 1970):

$$\frac{P_i G}{P_0 G} = \frac{\phi_i^{1/4} - \phi_0^{1/4}}{\phi_i^{1/4} - \phi_0^{1/4}} = \left(\frac{\phi_i}{\phi_0}\right)^{1/4} = \left(\frac{n_i}{n_0}\right)^{1/4} \dots (1)$$

Here  $0 > (P_i G/P_0 G) > 1$ , and  $P_i$  must be on the left of  $P_0$ . The focal lengths obtained from the reduced equation are equal in magnitude for all values of  $\phi_0$  and  $\phi_i$ .

From apply Eq.(1), can get (El-Kareh, 1970):

$$\phi(0, z) = \frac{V_1 + V_2}{2} + \left(\frac{V_1 + V_2}{2}\right) \tanh wz \dots (2)$$

From eq.(2), to determine analytically the cardinal points of this lens for the axial potential distribution. The focal length of weak lens is given by:

$$\frac{1}{f_i} = \frac{3}{16} \left(\frac{\phi_i}{\phi_0}\right)^{1/4} \int_{z_0}^{z_i} \left|\frac{\dot{\phi}(z)}{\phi(z)}\right|^2 dz \dots (3)$$

From eq. (3) and (2) we obtain:

$$\frac{\dot{\phi}(z)}{\phi(z)} = wz \frac{\text{sech}^2 wz}{1 + z \tanh wz} \dots (4)$$

Where  $w=1.318$ . the focal length is then:

$$\frac{1}{f_i} = \frac{3}{16} \sigma^{-1/4} z^2 w^2 \int_{-z}^{+z} \frac{\text{sech}^4 wz}{(1 + z \tanh wz)^2} dz \dots (5)$$

Let ( $L=\tanh wz$ ), then (Zhigarev, 1975):

$$\frac{1}{f_i} = \frac{3}{16} \sigma^{-1/4} z^2 w^2 \int_{-1}^{+1} \frac{1 - L^2}{(1 + zL)^2} dL = \frac{3}{8} \sigma^{-1/4} w^2 \left(\frac{1}{z} \log \sigma^{-2}\right) \dots (6)$$

From value of ( $w$ ) we obtain (Kato, M. and Tsuno, 1990):

$$f_i = \frac{2.02 \sigma^{-1/4}}{[(\sigma + 1)(\sigma - 1)] \log \sigma^{-2}} \dots (7)$$

This formula is dependable for weak lenses i.e. for small value of  $\sigma$  (Al-Obaidi, and Al-Azawy 2014).

To define the cardinal points by sectionizing the lens. The reduced focal lengths according to the formula (El-Kareh, 1970):

$$\frac{1}{F_i} = \frac{3}{16} \int_{z_0}^{z_1} \left(\frac{\dot{\phi}(z)}{\phi(z)}\right)^2 dz \dots (8)$$

and  $\frac{1}{f_i} = -\frac{r_i}{r_0} = \left(\frac{\phi_0}{\phi_i}\right)^{1/4} \frac{1}{F_i} \dots (9)$

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The reduced paraxial-ray formula is:

$$\frac{d^2 R}{dz^2} = -\frac{3}{16} \left(\frac{\dot{\phi}}{\phi}\right)^2 R \dots (10)$$

Or we can write eq.(12) as:

$$\frac{d^2 R}{dz^2} = -M^2 R \dots (11)$$

From (El-Kareh, 1970) and (Grivet, 1972) the equations of  $\beta$ , A and B as:

$$\gamma = \int_{-1}^{+1} \frac{1 - \alpha^2}{(1 + x\alpha)^2} \log(1 + x\alpha) \log\left(\frac{1 + \alpha}{1 - \alpha}\right) d\alpha \dots (12)$$

$$\frac{1}{f} = \sigma^{1/4} (O - Q) \dots (13)$$

$$O = \frac{3\omega}{8} \left(\frac{1}{x} \log \sigma^{-2}\right) \dots (14)$$

$$Q = \left(\frac{3}{16}\right)^2 I \dots (15)$$

Where I in eq. (15) is:



$$I = \frac{\omega}{x} \left\{ 4x - 2[1 + \log(1 - x^2)] \log \sigma + \frac{1}{x} \log(1 - x^2) (\log \sigma)^2 + 2x^2 \beta \right\}$$

### Results and calculations

First we analyze from a point outside the range to ensure that the derivatives along the axis are equal to zero. Then we perform numerical analysis step-by-step until a point is reached after passing through the lens where the field is negligible. The same analysis was performed for the lens as in equation (2), where we used to determine the axial voltage and its derivatives, the results of the relationship between the voltage ratio (10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20) to the point The computed coaxiality is listed in Table 1., Table2. About the cardinal points of two equal cylinders with negligible spacing, then by using gap space of 0.1, 0.2 and 0.3 inch intervals of 0.1 which are listed in Tables-3, 4 and 5. According to table--3, 4 and 5, Figure-6 has been drawn to Show the increasing of the ( $f_i/R$ ) ratio for high voltage ratio, Figure -7 has been drawn to Show the increasing of the ( $f_o/R$ ) ratio for high voltage ratio, and Figure-8 Show the relationships of ( $Z_{mi}/R$ ), ( $Z_{pi}/R$ ), ( $Z_{mo}/R$ ) and ( $Z_{po}/R$ ) to the increasing of voltage ratio at the mentioned gap spacing

**Table1. calculated focal point according to the  $\gamma$ , O and Q values**

$V_2/V_1$	$\Gamma$	O	Q	$f_i$ mm
10	2.91	0.422	0.039	4.91
11	3.25	0.467	0.049	4.77
12	3.59	0.498	0.052	4.54
13	4.02	0.522	0.058	4.41
14	4.36	0.538	0.064	4.38
15	5.00	0.573	0.073	4.30
16	5.25	0.595	0.082	4.24
17	5.58	0.620	0.091	4.11
18	5.99	0.645	0.112	4.06
19	6.3	0.661	0.130	4.00
20	6.76	0.711	0.146	3.98

**Table2. cardinal points of two equal cylinders with negligible spacing**

$V_2/V_1$	$f_i/R$	$Z_{mi}/R$	$Z_{pi}/R$	$f_o/R$	$Z_{mo}/R$	$Z_{po}/R$
10	4.72	2.34	-2.51	-1.63	-3.31	-1.63

11	4.54	2.23	-2.51	-1.56	-3.15	-1.60
12	4.38	2.16	-2.51	-1.48	-3.01	-1.58
13	4.30	2.00	-2.51	-1.41	-2.86	-1.56
14	4.21	1.89	-2.51	-1.35	-2.71	-1.54
15	4.15	1.67	-2.53	-1.28	-2.55	-1.52
16	4.10	1.44	-2.56	-1.22	-2.41	-1.50
17	4.07	1.40	-2.58	-1.15	-2.39	-1.48
18	4.05	1.31	-2.62	-1.08	-2.32	-1.47
19	3.88	1.27	-2.66	-1.01	-2.28	-1.45
20	3.79	1.21	-2.70	-0.93	-2.24	-1.43

**Table3. cardinal points of two equal cylinders separately by a distance s=0.1**

$V_2/V_1$	$f_i/R$	$Z_{mi}/R$	$Z_{pi}/R$	$f_o/R$	$Z_{mo}/R$	$Z_{po}/R$
10	4.71	2.33	-2.51	-1.62	-3.32	-1.64
11	4.54	2.23	-2.51	-1.56	-3.15	-1.60
12	4.38	2.16	-2.51	-1.48	-3.01	-1.58
13	4.30	2.00	-2.51	-1.41	-2.86	-1.56
14	4.21	1.89	-2.51	-1.35	-2.71	-1.54
15	4.15	1.67	-2.53	-1.28	-2.55	-1.52
16	4.10	1.44	-2.56	-1.22	-2.41	-1.50
17	4.07	1.40	-2.58	-1.15	-2.39	-1.48
18	4.05	1.31	-2.62	-1.08	-2.32	-1.47
19	3.88	1.27	-2.66	-1.01	-2.28	-1.45
20	3.78	1.22	-2.70	-0.92	-2.23	-1.42

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**Table4. cardinal points of two equal cylinders separately by a distance s=0.2**

$V_2/V_1$	$f_i/R$	$Z_{mi}/R$	$Z_{pi}/R$	$f_o/R$	$Z_{mo}/R$	$Z_{po}/R$
10	4.70	2.31	-2.50	-1.60	-3.30	-1.62
11	4.54	2.23	-2.51	-1.56	-3.15	-1.60
12	4.38	2.16	-2.51	-1.48	-3.01	-1.58
13	4.30	2.00	-2.51	-1.41	-2.86	-1.56
14	4.22	1.86	-2.51	-1.33	-2.70	-1.52
15	4.15	1.67	-2.53	-1.28	-2.55	-1.52
16	4.10	1.44	-2.56	-1.22	-2.41	-1.50
17	4.07	1.40	-2.58	-1.15	-2.39	-1.48
18	4.05	1.31	-2.62	-1.08	-2.32	-1.47
19	3.88	1.27	-2.66	-1.01	-2.28	-1.45
20	3.76	1.21	-2.70	-0.91	-2.21	-1.40

**Table 5. cardinal points of two equal cylinders separately by a distance s=0.3**

$V_2/V_1$	$f_i/R$	$Z_{mi}/R$	$Z_{pi}/R$	$f_o/R$	$Z_{mo}/R$	$Z_{po}/R$
10	4.71	2.31	-2.52	-1.61	-3.29	-1.63



11	4.55	2.22	-2.51	-1.54	-3.14	-1.61
12	4.39	2.15	-2.51	-1.47	-3.01	-1.59
13	4.31	2.00	-2.51	-1.40	-2.84	-1.57
14	4.23	1.85	-2.51	-1.30	-2.70	-1.53
15	4.16	1.66	-2.53	-1.29	-2.52	-1.53
16	4.11	1.43	-2.56	-1.21	-2.40	-1.51
17	4.08	1.40	-2.58	-1.16	-2.38	-1.49
18	4.06	1.30	-2.62	-1.09	-2.34	-1.46
19	3.89	1.25	-2.66	-1.01	-2.26	-1.44
20	3.77	1.20	-2.69	-0.91	-2.20	-1.40

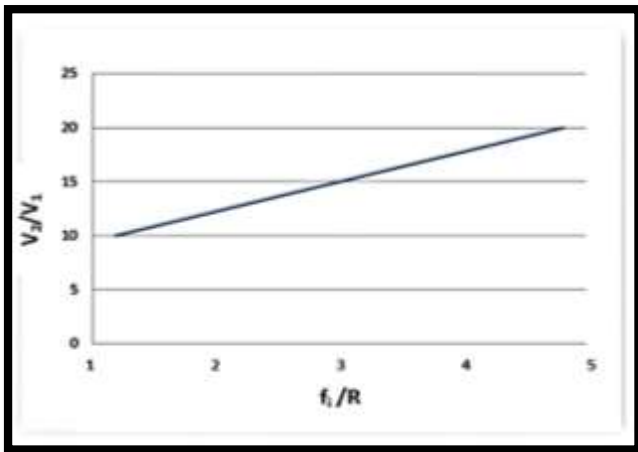


Figure 6. Show the increasing of the ( $f_i/R$ ) ratio for high voltage ratio

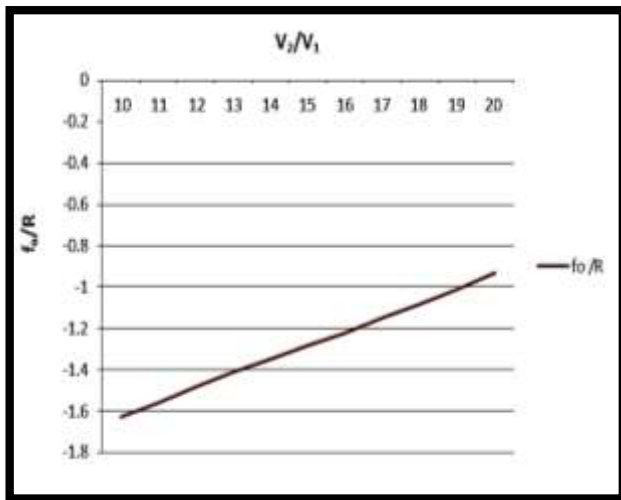


Figure 7. Show the increasing of the ( $f_o/R$ ) ratio for high voltage ratio

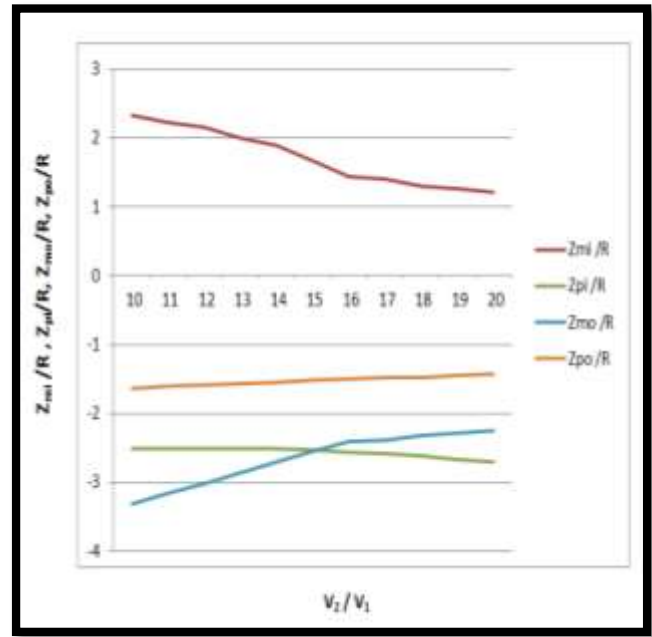


Figure 8. Show the relationships of ( $Z_{mi}/R$ ), ( $Z_{pi}/R$ ), ( $Z_{mo}/R$ ) and ( $Z_{po}/R$ ) to the increasing of voltage ratio at the mentioned gap spacing

## Conclusion

The following observations can be made:

- 1- The thickness of the specific lens increases while observing the voltage ratio, and the thickness decreases with the increase in the voltage ratio.
- 2- When increasing the width of the gap, a slight movement of the main levels occurs towards the low cylindrical voltage, but when the voltage is relatively high, this effect is small.
- 3- The focal length is always larger when the voltage is increased, while it is at a lower value when the voltage is decreasing
- 4- The focal length of the lens with two identical tubes increases with the width of the gap, in which case the lens becomes weaker with the increase in the axial spacing of the cylinders in it.



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