Comparison Study of variousnumerical integration methods

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Abstract

Formally, the calculation of the integral is based on the area between the provided function and the integral in a specific range.In this paper, we examine thevarious numerical integration methods so we get the more appropriate values of definite integral.Paper and pencil approaches can be used to assess these integrations. However, such evaluation takes time, thus computers are required to handle such challenging issues. The paper-pencil method is extremely time-consuming and has a significant likelihood of human error. Computers save the day by streamlining busywork and efficiently completing repetitive activities. The C++ programming language makes up the standard calculation tools for interacting with computers. These tools offer a more accurate, precise, and error-free method of solving the integral.All the methods are broadly studied,after study data is outlined in the form of comparison practically,rules are performed on C^{++}.Error of all the rules is taken into consideration.

Keywords: Simpson 1/3rd, Quadrature,Trapezoidal Rule,Gaussian quadrature, Simpson3/8^{t}

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1. Introduction

As indicated by Madhumangal Pal [1], It is commonly known that even if a function f(x) is thoroughly understood, evaluating the definite integral of it using the analytic approach is not always achievable. Again, we are required to integrate a function between two provided limits in many real-world scenarios, but the function is not directly known, but it is known in a tabular form (equally or unequally spaced). Then, to address all such problems, a procedure called as numerical integration or quadrature can be utilized.

Numerical data that cannot be integrated theoretically are approximated through the technique of numerical integration. Quadrature is the process of substituting the area of a square for the space under a curve.Interpolating polynomials are used in Newton-Cotes procedures [3].Special examples of first, second, and third order polynomials utilised in Newton-Cotes methods are the Trapezium rule, Simpson 1/3, and Simpson 3/8 rules, respectively.Another option is the Gaussian Quadrature Rule, which is a quadrature rule outcome for polynomials of degree 2n-1. [5]

2. Reasons for numerical integration

The integrand f(x) may only be known at certain places. For these reasons, some installed frameworks and other PC applications may demand numerical joining.

1 Sometimes it is difficult to apply the Formula of Integration to elementary functionin spite of Known integration.

2 It might be conceivable to locate integration, yet it might be simpler to figure a numerical value estimate than to register the integration. That might be the situation if the integration is given as a limitless arrangement or item, or if its assessment requires an uncommon capacity that isn't available. There are different tenets to figure numerical mix of an integrand.

3. Types of NumericalIntegration Methods

Depending on our requirements, we can employ techniques that enable us to determine a result with a particular inaccuracy. There are a few fundamental approaches to numerical integration that vary by how they approximate:

- trapezoidal rule,
- Simpson's 1/3 rule
- Simpson's3/8 rule
- Gaussian quadrature.

3.1. Trapezoidal rule

To approximation an area under the curve, Riemann's sums are utilized

This method, trapezoidal rule takes a similar method. The area under the curve split into several trapezoids, each of whose area is calculated and combined to provide a rough estimate of the total area.

1.) Let us assume that straight line be the curve through $(x_0, y_0)\ and\ (x_1, y_1)\ by$ taking n=1 in Equation. When a polynomial is first order, the variations between higher orders can be ignored, and we get

$\int_{x_0}^{x_0+h} f(x)dx = h[y_0 + \frac{1}{2}\Delta y_0] = \frac{h}{2}(2y_0 + y_1 - y_0) = \frac{h}{2}(y_0 + y_1)$.

Similarly, $\int_{x_0+h}^{x_0+2h} f(x)dx$= h $(y_1 + \frac{1}{2}\Delta y_1) = \frac{h}{2}(y_1 + y_2)$

……

, $\int_{x_0+(n-1)h}^{x_0+nh} f(x)dx = = \frac{h}{2}(y_{n-1} + y_n)$.

$$= h\int_0^n [y_0 + p\Delta y_0 + \frac{p(p-1)}{2}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{6}\Delta^3 y_0 + \cdots\ldots\ldots\ldots]dp,$$

On simplification;

$$I = nh[y_0 + \frac{n}{2}\Delta y_0 + \frac{n(2n-1)}{12}\Delta^2 y_0 + \frac{n(n-2)^2}{24}\Delta^3 y_0 + \cdots\ldots\ldots\ldots] \quad (1)$$

The Newton–Cotes quadrature formula is the formula above.

Different integration formulas can be produced by substituting n=1, 2, 3, in the general formula.

When these n integrals are added together, we get:

$\int_{x_0}^{x_0+nh} f(x)dx = \frac{h}{2}(y_0 + 2(y_1 + y_2 + \cdots + y_{n-1}) + y_n)$ (2)

It's called the trapezoidal rule.

2) Thisrule can be applied to any number of subintervals odd or even.

Trapezoidal laws are ubiquitous in engineering concepts to calculate displacements in creating academic reading.Only the trapezoidal rule has been used for their practice to fill a triangulated irregular. Kinematics and the largest area of back; hydraulic analysis of ordinates are displayed only includes the trapezoidal rule has been used for their practice to fill a triangulated irregular. In the use of the trapezoidal rule, the entire distance between such codes tends to accomplish.

3.2. Simpson's 1/3 rule

1.) Let us assume that the curve through $(x_0, y_0)\ and\ (x_1, y_1)\ by$ taking n=2 in Equation is of second order polynomial. If a polynomial is second thedifferences of higherorders except second order can be neglected, then

$\int_{x_0}^{x_0+2h} f(x)dx$= 2h $(y_0+\Delta y_0 + \frac{1}{6}\Delta^2 y_0)$.

$= \frac{h}{3}(6y_0 + 6\Delta y_0 + \Delta^2 y_0)$

$= \frac{h}{3}(y_0 + 4y_1 + y_2)$.

Similarly,

$\int_{x_0+2h}^{x_0+4h} f(x)dx = \frac{h}{3}(y_2 + 4y_3 + y_4)$,

…….

$\int_{x_0+(n-2)h}^{x_0+nh} f(x)dx = \frac{h}{3}(y_{n-2} + 4y_{n-1} + y_n)$, n being even.

Add all theintegrals, we have

$\int_{x_0}^{x_0+nh} f(x)dx = \frac{h}{3}[y_0 + 4(y_1 + y_3 + \cdots + y_{n-1}) + 2(y_2 + y_4 + \cdots + y_{n-2}) + y_n]$ (3)

The above formula is the Simpson's 1/3rd Rule.

2.) This rule of integration can be applied when the given interval is divided in even number of intervals.

3.3. Simpson's 3/8 rule

Let us assume that the curve through $(x_0, y_0)\ and\ (x_1, y_1)\ by$ taking n=3 in Equation is of third order polynomial. If a polynomial is third the differences of higher orders except third order can be neglected, then

$\int_{x_0}^{x_0+3h} f(x)dx$= 3h $(y_0+\frac{3}{2}\Delta y_0 + \frac{3}{4}\Delta^2 y_0 + \frac{1}{8}\Delta^3 y_0)$

=3h $(y_0+\frac{12}{8}\Delta y_0 + \frac{6}{8}\Delta^2 y_0 + \frac{1}{8}\Delta^3 y_0)$

$= \frac{3h}{8}(8y_0 + 12\Delta y_0 + 6\Delta^2 y_0 + \Delta^3 y_0)$

$= \frac{3h}{8}(y_0 + 3y_1 + 3y_2 + y_3)$

Similarly,

$\int_{x_0+3h}^{x_0+6h} f(x)dx = \frac{3h}{8}(y_3 + 3y_4 + 3y_5 + y_6)$,

…….

$\int_{x_0+(n-3)h}^{x_0+nh} f(x)dx = \frac{3h}{8}(y_{n-3} + 3y_{n-2} + 3y_{n-1} + y_n)$,

Add all the integrals, we have

$\int_{x_0}^{x_0+nh} f(x)dx = \frac{3h}{8}[y_0 + 2(y_3 + y_6 + \cdots + y_{n-3}) + 3(y_1 + y_2 + \cdots + y_{n-1}) + y_n]$ (4)

The above formula is the Simpson's 3/8 Rule.

It's worth noting that Simpson's rule has a variety of applications in real-world situations. Simpson's rule gives the distance traversed during the period if the y - values, i.e. the ordinates, denote the velocities of the body at equal intervals of time. If the ordinate values correspond to equal-spaced cross-sectional areas, this rule calculates the solid's volume. This formula can also be used to solve problems involving determining the volume of revolution of areas as well as the moment of inertia of regions.

This rule can be applied to a variety of civil engineering issues.Calculating the amount of earth that needs to be moved to build a level highway, a dam, or the earth needed to fill a depression are just a few of the

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applications. Profiles of the ground level along different sections are created using surveys and contour maps, and the average height is determined using Simpson's 1/3rd rule up to the target level. It is simple to complete the total volume of the earth to be moved using the information obtained thus far.

3.4. Gaussian Quadrature

1) Let us assume $x_{1,}x_{2,}\ldots\ldots,x_n$ in the interval $[a,b]$ and coefficients $c_{1,}c_{2,}\ldots\ldots,c_n$ to reduce the predicted inaccuracy in the approximation

$$\int_a^b f(x)dx \approx \sum_{i=1}^{n} c_i f(x_i) \qquad (5)$$

The optimum combination of these values yields the exact solution for the biggest class of polynomials.

The approximation formula's coefficients $c_{1,}c_{2,}\ldots\ldots,c_n$ are arbitrary, while the nodes x_1,x_2,..., x_n are only limited by the fact that they must reside in [a, b], the integration interval.

2) This gives us a total of 2n parameters to pick from.

3) Usually calculated for a specific interval, such as $[-1,1]$

4. Implementation of C++ Programming in solving numerical integration methods

4.1 Example

To calculate an approximate value of $\int_0^{\frac{\pi}{2}} \sin x\, dx$,

Using (1) Trapezoidal rule (2) Simpson's 1/3 Rule

Using 11 ordinates.

Dividing the range of integration $(0,\frac{\pi}{2})$ $into$ =11-1=10 equal parts each of width

$h=\frac{\frac{\pi}{2}-0}{10}=\frac{\pi}{20}$=9° 624

The values of $x\ and\ y\ =\ sin\, x$ are tabulated as

	18	27	36	45	54	63	72	81	90
	543 .30901	.45399	.58779	.70711	.80902	.89101	.95106	.98769	1

(1)**By Trapezoidal Rule**

$\int_0^{\frac{\pi}{2}} \sin x dx=\frac{h}{2}[y_0+2(y_1+y_2+y_3+\ldots\ldots+y_9)+y_{10}]$

$=\frac{\pi}{40}$[0+2(.15643+.30901+.45399+.58779+.70711+.80902+.89101+.95106+.98769) +1]

$=\frac{3.1416}{40}$[2(5.85331) +1]

=0.99798

/* TRAPEZOIDAL RULE */

```
#include<iostream.h>
#include<conio.h>
#include<math.h>
Float y (float x)
{
```

```
return (sinx);
}
void main()
{
clrscr();
float h,x0,xn,s;
intn,i;
cout<<"enter the value of x0=";
cin>>x0;
cout<<"enter the value of xn=";
cin>>xn;
cout<<"enter the value of n=";
cin>>n;
h = (xn-x0)/n;
s = y(x0) + y(xn);
For (i=1; i<=n-1; i++)
{
s+=2*y(x0+i*h);
} cout<<"the value of integral is ="<<(h/2)*s<<end;
getch();
}
```


(2)**By Simpson's One-Third Rule**

$\int_0^{\frac{\pi}{2}} \sin x \, dx = \frac{h}{3}[y_0+4(y_1 + y_3+y_5+.....+y_9) +2(y_2+y_4+y_6+y_8) +y_{10}]$

$= \frac{\pi}{60}$[0+4(.15643+.45339+.70711+.89101+.98769) +2(.30901+.58779+.80902+.95106) +1]

= (.05236) (19.09908)

=1.00003

/* SIMPSON'S 1/3rd RULE */

```
#include<Iostream.h>
#include<conio.h>
#include<math.h>
float y (float x)
{
return (sinx);
}
void main()
{
clrscr();
float h,x0,xn,s,s1, s2;
intn,i;
cout<<"enter the value of x0=";
```


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```
cin>>x0;
cout<<"enter the value of xn=";
cin>>xn;
cout<<"enter the value of n=";
cin>>n;
cout<<endl;
h = (xn-x0)/n;
s = y(x0) + y(xn) + 4*y(x0+h);
for(i=3;i<=n-1;i+=2)
{
s+=4*y(x0+i*h) +2*y(x0+ (i-1)*h);
}
cout<<"the value of integral is ="<<(h/3)*s<<endl;
getch();
```

4.2 Example

Evaluate I = $\int_0^4 te^{2t}dt = 5216.926477$

T = $\frac{b-a}{2}$x + $\frac{b+a}{2}$ = 2x + 2; dt = 2dx

I = $\int_0^4 te^{2t}dt = \int_{-1}^1 (4x+4)e^{4x+4}dx = \int_{-1}^1 f(x)dx$

Two- point formula

I = $\int_{-1}^1 f(x)dx = f(\frac{-1}{\sqrt{3}}) + f\left(\frac{1}{\sqrt{3}}\right) = \left(4 - \frac{4}{\sqrt{3}}\right)e^{4-\frac{4}{\sqrt{3}}} + (4 + \frac{4}{\sqrt{3}})e^{4+\frac{4}{\sqrt{3}}}$

= 9.167657324 + 3468.376279 = 3477.43936

5. Conclusion

In order to determine which method of numerical integration is the most accurate and efficient, we compared three of them in this paper: trapezoid, Simpson's and Gaussian quadrature rules.

To compare we used the C++ Programming language and determined values for different numbers of subintervals, then examined them to determine which was the best.The most precise approximations were produced via Gaussian quadrature.

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