Investigation of Normal Fracture Cracks in Elastic-Layer Materials

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Abstract

In this work, based on the representation of the Papkovich –Neiber displacement and stress through 3 harmonic functions, dual integral equations are obtained, the solution of which is reduced to finding one Helder function. To find this function, a singular integral equation with a Cauchy kernel of the 1st kind is obtained. The solution of this integral equation, proposed by the method of V. D. Kuliyev, is reduced to the Fredholm integral equation of the 2nd kind with a continuous kernel. The main parameter of the mechanics of linear fracture of the stress intensity coefficient is determined and a numerical analysis is carried out. When the crack of a normal fracture is located in an infinite elastic medium, it is shown that both components of the displacement vector are nonzero. This result suggests that the crack is an oblate ellipsoid.

Keywords Papkovich-Neiber formula, harmonic functions, elastic medium, stress intensity coefficient

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INTRODUCTION

1. SETTING THE BOUNDARY VALUE PROBLEM

It is assumed that in a homogeneous isotropic elastic band $|x|<\infty$, $-h\le y\le h$, there is a crack of normal rupture $y=0,\ |x|\le \ell$, где 2ℓ – crack length (Fig. 1). This problem was considered in the works [1,2,3]. Some normal stress is applied on the banks of the crack $\sigma_y(o,x)$ (tangential stress $\tau_{xy}(x,o)=0$). Strip surfaces $y=\pm h\ \ (|x|<\infty)$ they are free from external loads. At infinity $(|y|<h,\ x\to\pm\infty)$ the voltages and displacements are equal to zero.

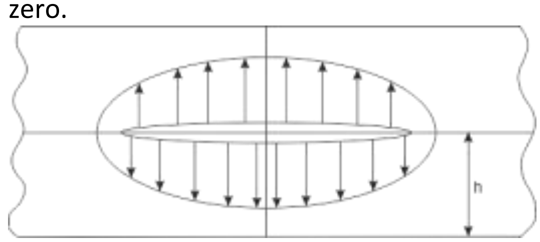

Figure 1. On the formulation of the boundary value problem

Therefore, we have come to the following mixed boundary value problem.
Boundary conditions

$|x|<\infty, \quad \sigma_y(x,\pm h)=0, \tau_{xy}(x,\pm h)=0$; (1.1)

$|x|<\infty, \quad \tau_{xy}(x,\pm 0)=0$; (1.2)

$|x|<l, \quad \sigma_y(x,\pm 0)=-\sigma(x)$; (1.3)

$|x|>\ell$, $|x|>l, \quad \upsilon(x,0)=0$. (1.4)

Conditions at the end of the crack [**Error! Reference source not found.**]:

$$\lim_{x\to l-0}\left[\sqrt{2\pi(l-x)}\frac{\partial\upsilon(x,+0)}{\partial x}\right]=-\frac{(1-\nu)}{\mu}\mathrm{K_I},$$ (1.5)

or

$$\lim_{x\to l-0}\left[\sqrt{2\pi([x-l)}\sigma_y(x,+0)\right]=\mathrm{K_I}$$ (1.6)

Conditions at infinity:

$$|y|<h, \quad |x|\to\infty\ \{\sigma_y,\sigma_x,\tau_{xy}\}\to 0,$$ (1.7)

$$(u,\upsilon)\to 0.$$ (1.8)

Since one of the three harmonic functions is arbitrary, we assume $\Phi_1(x,y)\equiv 0$. In addition, suppose that the harmonic functions $\Phi_0(x,y)$ or $\Phi_2(x,y)$ they also tend to zero when $|x|\to\infty$ и $|y|<h.$ Here $\mathrm{K_I}$ – stress intensity coefficient for normal fracture cracks to be determined, $\sigma(x)\in C[-\ell,\ell]$ – given an even function, and after that we consider, that $\sigma(x)\in H^\beta[-\ell,\ell]$, $\frac{1}{2}<\beta\le 1$, where $\sigma(x)\in H^\beta[-\ell,\ell]$ – Helder class of functions with an exponent .

2. SOLVING THE BOUNDARY VALUE PROBLEM (1.1) – (1.8)

The problem under consideration is obviously symmetric in relation to the planes x =0 и y =0. Therefore, we will construct a solution to the boundary value problem (1.1)–(1.8) in the range $x\ge 0, 0\le y\le h.$

Harmonic functions $\Phi_0(x,y)$ и $\Phi_2(x,y)$ are represented

$$\Phi_0(x,y)=\sqrt{\frac{2}{\pi}}\int_0^\infty\left[\mathrm{A}_0(\lambda)\ \mathrm{ch}\lambda\mathrm{y}+\mathrm{B}_0(\lambda)\ \mathrm{sh}\lambda\mathrm{y}\right]\cos\lambda x d\lambda,$$

$$\Phi_2(x,y)=\sqrt{\frac{2}{\pi}}\int_0^\infty\left[\mathrm{A}_2(\lambda)\ \mathrm{ch}\lambda\mathrm{y}+\mathrm{B}_2(\lambda)\ \mathrm{sh}\lambda\mathrm{y}\right]\cos\lambda x d\lambda.$$

Here $\mathrm{A}_j(\lambda)$ and $\mathrm{B}_j(\lambda)(j=0,2)$ – unknown functions. Taking into account (2.1), (2.2) in the equation of the dependence of the stress tensor components [3,4,5,6] due to the conditions (1.1) and (1.2) we have

$$\mathrm{B}_2(\lambda)=\lambda\mathrm{A}_0(\lambda)\frac{\mathrm{sh}^2\lambda h}{(1-2\nu)\mathrm{sh}^2\lambda h+\lambda^2h^2},$$

$$\mathrm{A}_2(\lambda)=-\lambda\mathrm{A}_0(\lambda)\frac{\mathrm{sh}\lambda h\mathrm{ch}\lambda h+\lambda h}{(1-2\nu)\mathrm{sh}^2\lambda h+\lambda^2h^2},$$

$$\lambda\mathrm{B}_0(\lambda)=-\lambda\mathrm{A}_0(\lambda)\frac{(1-2\nu)[\mathrm{sh}\lambda h\mathrm{ch}\lambda h+\lambda h]}{(1-2\nu)\mathrm{sh}^2\lambda h+\lambda^2h^2}.$$

By means of (2.15)–(3.3.19) and (3.3.3) we get

$$2\mu\upsilon(x,0)=-2(1-\nu)\sqrt{\frac{2}{\pi}}\int_0^\infty\lambda\mathrm{A}_0(\lambda)\frac{\mathrm{sh}\lambda h\mathrm{ch}\lambda h+\lambda h}{(1-2\nu)\mathrm{sh}^2\lambda h+\lambda^2h^2}\cdot\cos\lambda x d\lambda.$$

Sought function $\lambda\mathrm{A}_0(\lambda)$ is represented as

$$\lambda\mathrm{A}_0(\lambda)=-\frac{(1-2\nu)\mathrm{sh}^2\lambda h+\lambda^2h^2}{\mathrm{sh}\lambda h\mathrm{ch}\lambda h+\lambda^2h^2}\sqrt{\frac{2}{\pi}}\int_0^\ell f(t)\frac{\sin\lambda t}{\lambda}dt.$$

Here $f(t)$ – new unknown function.

If the functions $f(x)$ and $\psi(x)$ are related by formulas

$$\psi(x)=\frac{2}{\pi}\int_x^l\frac{f(t)}{\sqrt{t^2-x^2}}dt,$$

$$f(t)=\frac{f_0(t)}{\sqrt{l^2-t^2}};$$

$$f(x)=-\frac{d}{dx}\int_x^l\frac{t\psi(t)}{\sqrt{t^2-x^2}}dt,$$

then, by virtue of the method proposed in the work of V. D. Kuliyev [1,2], singular integral equations of the first kind with a Cauchy kernel are reduced to Fredholm integral equations of the second kind with a continuous kernel and vice versa. Therefore, suppose that $f(x)\in K_{1/2}]-l,l[$, it means

$$f(x)=\frac{f_0(x)}{\sqrt{l^2-x^2}},\quad f_0(-x)=-f_0(x),\quad f_0(x)\in\mathrm{H}^\beta[0,l], 1/2<\beta\le 1.$$

In fact, from (2.6) and (2.7) we'll find

$$\frac{\mu}{(1-\nu)}\upsilon(x,0)=\frac{2}{\pi}\int_0^{\ell} f(t)\int_0^{\infty}\frac{\sin\lambda t}{\lambda}\cos\lambda x\, d\lambda\, dt.$$

Here the internal integral is a multiplier (integral) Dirichlet [**Error! Reference source not found.**,7,8]:

$$\int_0^{\infty}\frac{\sin\lambda t}{\lambda}\cos\lambda x\, d\lambda=\begin{cases}\pi/2, \text{если } t>x,\\ \pi/4, \text{если } t=x,\\ 0, \text{если } t<x.\end{cases}$$

It follows from (2.10) and (2.9) and [9,10] that the condition (1.4) is satisfied automatically. Let it be now $0<x<l$. Then from (2.9) by virtue of (2.10) we have

$$\frac{\mu}{(1-\nu)}\upsilon(x,0)=\lim_{\varepsilon\to+0}\left\{\frac{2}{\pi}\int_0^{x-\varepsilon} f(t)\int_0^{\infty}\frac{\sin\lambda t}{\lambda}\cos\lambda x\, d\lambda\, dt+\right.$$

$$\left.+\frac{2}{\pi}\int_{x+\varepsilon}^{\ell}\int_0^{x-\varepsilon} f(t)\int_0^{\infty}\frac{\sin\lambda t}{\lambda}\cos\lambda x\, d\lambda\, dt\right\}=\int_x^{\ell} f(t)dt,$$

where

$$\frac{\partial\upsilon(x,0)}{\partial x}=-\frac{1-\nu}{\mu}f(x).$$

Taking into account (2.11) and (1.5), by virtue of (2.9) and [11,12,13] we get

$$\mathrm{K_I}=\sqrt{\pi\ell}\,\frac{f_0(\ell)}{\ell}.$$

Therefore, the solution of the boundary value problem under consideration is reduced to finding one function $f(x)\in K_{\frac{1}{2}}]$.

In this case, all the conditions of the boundary value problem are met, with the exception of the condition (2.3).

By means of (2.3) – (2.5), (2.7), (2.1), (2.2) we have:

$$\sigma_y(x,+0)=-\frac{2}{\pi}\frac{d}{dx}\int_0^{\ell} f(t)\int_0^{\infty}\frac{\mathrm{sh}^2 xh-\lambda^2h^2}{\mathrm{sh}\lambda h\,\mathrm{ch}\lambda h+\lambda h}\cdot\frac{\sin\lambda t}{\lambda}\sin\lambda x\, d\lambda\, dt=$$

$$=-\frac{2}{\pi}\frac{d}{dx}\left\{\int_0^{\ell} f(t)\int_0^{\infty}\frac{\sin\lambda t\sin\lambda x}{\lambda}d\lambda\, dt+\right.$$

$$\left.+\int_0^{\ell} f(t)\int_0^{\infty}\frac{\mathrm{sh}^2\lambda h-\lambda^2h^2-\mathrm{sh}xh\,\mathrm{ch}\lambda h-\lambda h}{\mathrm{sh}\lambda h\,\mathrm{ch}\lambda h+\lambda h}\cdot\frac{\sin\lambda t\sin\lambda x}{\lambda}d\lambda\, dt\right\}$$

,

from where, having noticed that (check. [**Error! Reference source not found.**], p. 59–62) (2.9)

$$\frac{2}{\pi}\frac{d}{dx}\int_0^{\ell} f(t)\int_0^{\infty}\frac{\sin\lambda t\sin\lambda x}{\lambda}d\lambda\, dt=\frac{1}{\pi}\int_{-\ell}^{\ell}\frac{f(t)}{t-x},$$

by virtue of (1.3) and considering [14] we come to a singular integral equation with a Cauchy kernel of the first kind (2.10)

$$\sigma(x)=\frac{1}{\pi}\int_{-\ell}^{\ell}\frac{f(t)}{t-x}dt+\frac{1}{\pi}\int_{-\ell}^{\ell} f(t)\mathrm{K}(x,t)dt, \qquad (2.13)$$

$$\mathrm{K}(x,t)=-\int_0^{\infty}\frac{e^{-\lambda h}\mathrm{sh}\lambda h+\lambda^2h^2+\lambda h}{\mathrm{sh}\lambda h\,\mathrm{ch}xh+\lambda h}\sin\lambda t\cos\lambda x\, d\lambda. \qquad (2.14)$$

The singular integral equation (2.13) with a kernel (2.14) is reduced to the Fredholm integral equation of the second kind with a continuous kernel using the method developed in [1,2]

$$\frac{2}{\pi}\int_0^{x}\frac{\sigma(\tau)}{\sqrt{x^2-\tau^2}}d\tau=\Psi(x)-\int_0^{l}\Psi(t)\mathrm{K}_{\phi}(x,t)dt,$$

$$\mathrm{K}_{\phi}(x,t)=\int_0^{\infty}\frac{\lambda t\left[e^{-\lambda h}\mathrm{sh}\lambda h+\lambda^2h^2+\lambda h\right]}{\mathrm{sh}\lambda h\,\mathrm{ch}\lambda h+\lambda h}J_0(\lambda t)J_0(\lambda x)d\lambda,$$

(2.11)

$$\Psi(x)=\frac{2}{\pi}\int_x^{\ell}\frac{f(\tau)d\tau}{\sqrt{\tau^2-x^2}},$$

$\Psi(-x)=\Psi(x)$, $f(\tau)\in \mathrm{K}_{1/2}]-\ell,\ell[$, $(0\le x\le\ell)$.

Here $J_0(u)$ – zero-order Bessel function. (2.12)

From (2.17) it follows

$$\Psi(\ell)=\frac{f_0(\ell)}{\ell}.$$

From (2.18) and (2.12) we get now

$$\mathrm{K_I}=\sqrt{\pi\ell}\,\Psi(\ell).$$

Let $x=\ell\xi$, $t=\ell\eta$, $\lambda h=u$,

$$\frac{\Psi(\ell\xi)}{\sigma_0}=\Psi(\xi),\quad \frac{\Psi(\ell\eta)}{\sigma_0}=\Psi(\eta).$$

$\sigma(\ell,\upsilon)=\sigma_0\gamma_0(\upsilon)$, $\sigma_0=const>0$,

Where $\gamma_0(\upsilon)\in C[0,1]$, и $\gamma_0(\upsilon)\in \mathrm{H}^{\beta}[0,1]$.

Therefore from (2.15) and (2.16) we have

$$\frac{2}{\pi}\int_0^{\xi}\frac{\gamma_0(\upsilon)}{\sqrt{\xi^2-\upsilon^2}}d\upsilon=\Psi(\xi)-\int_0^{1}\Psi(t)\mathrm{K}_0(\xi,t)dt,$$

$$\mathrm{K}_0(\xi,t)=2\left(\frac{\ell}{h}\right)^2\int_0^{\infty}\frac{ue^{-2u}\left[1-e^{-2u}+2u(u+1)\right]}{1-e^{-4u}+2ue^{-2u}}\times J_0\left(\frac{\ell}{h}ut\right)J_0\left(\frac{\ell}{h}u\xi\right)du,$$

$\left(0 \le \xi \le 1\right)$.

In this case, the stress intensity coefficient $\mathrm{K_I}$ is determined by the formula

$$\mathrm{K_I} = \sigma_0 \sqrt{\pi \ell}\, \Psi\left(1, \frac{\ell}{h}\right).$$

Here $\Psi\left(1, \frac{\ell}{h}\right) = \Psi(\xi, \ell/h)\Big|_{\xi=1}$.

Let $\gamma_o(v) = 1$. Then from (2.20), (2.21) и (2.22) we have

$$\Psi(\xi) - \int_0^1 \Psi(t)\mathrm{K}_0(\xi,t)\,dt = 1,$$

$$\mathrm{K}_0(\xi,t) = 2\left(\frac{\ell}{h}\right)^2 \int_0^\infty \frac{ue^{-2u}\left[1-e^{-2u}+2u(u+1)\right]}{1-e^{-4u}+2ue^{-2u}} \times J_0\left(\frac{\ell}{h}ut\right) J_0\left(\frac{\ell}{h}u\xi\right) du$$

$\left(0 \le \xi \le 1\right)$.

$$\mathrm{K_I} = \sigma_0 \sqrt{\pi \ell}\, \Psi\left(1, \frac{\ell}{h}\right).$$

Numerical analysis of the Fredholm integral equation [15,16] of the second kind (2.23) with a continuous core (2.24) shows (see Fig. 2): if $\frac{l}{h} \ll 1$, then $\psi(1;\frac{l}{h}) \approx 1$; with increasing $\frac{l}{h}$ correction function $\psi(1;\frac{l}{h})$ also increases. It follows that there is a steady growth of the crack under cyclic loading up to some values $\frac{l}{h}$, where $\frac{l}{h}$ is defined as the minimum root of the equation $K_{I\max}(\sigma_{\max}, \frac{l}{h}) = K_{*f}$. In this case, the number of cycles before destruction is determined by the formulas [2,3]

$$N_f = \int_{l_0}^{l_*} \frac{dl}{f(K_{Imax}, K_{Imin})},$$

$$\frac{dl}{dN} = -\beta\left(\frac{K_{Imax}^2 - K_{Imin}^2}{K_{*f}^2} + ln\frac{K_{*f}^2 - K_{Imax}^2}{K_{*f}^2 - K_{Imin}^2}\right) + \frac{v_0}{2^n\omega}\sum_{k=0}^{n}\left\{\frac{(-1)^k n!}{k!\,(n-k)!}\exp\left[\frac{\lambda}{2}(n-2k)(K_{Imax}+K_{Imin})\right] I_0\left[\frac{\lambda}{2}(n-2k)(K_{Imax}-K_{Imin})\right]\right\} \equiv f(K_{Imax}, K_{Imin}),$$

(2.22)

(2.27)

(2.23)

if the kinetic diagram of fatigue failure for a specific material is known, including if the crack resistance of the material under cyclic loading is known [17,18,19].

(2.24)

(2.25)

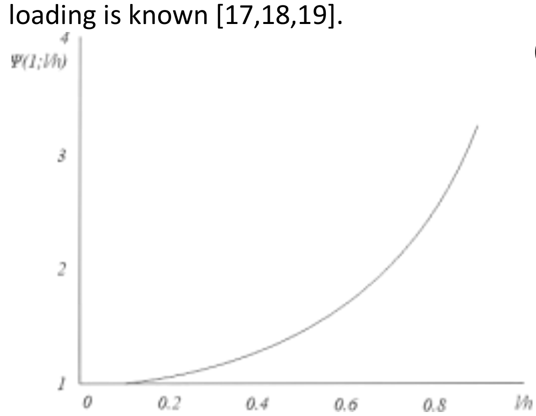

Figure 2. Dependence of the dimensionless stress intensity coefficient (correction function) on the dimensionless crack length

3. ANALYSIS OF SOLUTIONS AND STRESS INTENSITY COEFFICIENT $\mathrm{K_I}$

It is known that [2]

$$\frac{1}{\pi}\int_{-\ell}^{\ell}\frac{\sqrt{l^2-\tau^2}}{\tau - x}\,d\tau = \begin{cases} -x-\sqrt{x^2-l^2}, x<-\ell, \\ -x, |x|<\ell, \\ -x+\sqrt{x^2-l^2}, x>\ell. \end{cases} \quad (2.26)$$

The proof (3.1) is given in [2]. In addition, the inversion of a special integral with the Cauchy kernel is known [1, 2]

$$\frac{1}{\pi}\int_{-\ell}^{\ell}\frac{f(t)}{t-x}\,dt = \sigma(x), |x|<l$$

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Since $f(t) \in \mathrm{K}_{1/2}\,]-\ell,\ell[$ (see (2.8)), then:

1. A continuous function $\sigma(x)$ on a segment $[-\ell,\ell]$ belongs to the class of Helder functions with the exponent β ($1/2 < \beta \le 1$) (see [**Error! Reference source not found.**]).

2. The inversion formula of the special integral (3.2) by virtue of [1, 2] has the form

$$f(x) = -\frac{1}{\sqrt{\ell^2 - x^2}} \int_{-\ell}^{\ell} \sigma(\tau) \frac{\sqrt{\ell^2-\tau^2}}{\tau - x} d\tau, \quad |x| < l. \qquad (3.3)$$

In the problem under consideration, we assume that

1° $\sigma(x) = \sigma_0 \equiv const > 0$ $(\gamma_0(\upsilon) \equiv 1)$; 2° $h \to +\infty$.

Therefore from (2.13), (2.14), (3.3) и (2.15), (2.16) and considering [20] it follows

$$f(x) = -\frac{\sigma_0}{\pi\sqrt{\ell^2 - x^2}} \int_{-\ell}^{\ell} \frac{\sqrt{\ell^2-\tau^2}}{\tau - x} d\tau, \quad |x| < l, \qquad (3.4)$$

$$\Psi(x) = \sigma_0. \qquad (3.5)$$

From (3.4), taking into account (3.1), we find

$$f(x) = \frac{\sigma_0 x}{\sqrt{\ell^2 - x^2}} \quad (0 < x < l). \qquad (3.6)$$

From (3.5) by virtue of [1] we get

$$f(x) = -\frac{d}{dx}\int_x^{\ell} \frac{\tau\psi(\tau)}{\sqrt{\tau^2 - x^2}} d\tau = -\sigma_0 \frac{d}{dx}\int_x^{\ell} \frac{\tau}{\sqrt{\tau^2 - x^2}} d\tau = = -\sigma_0 \frac{d}{dx}\sqrt{\ell^2 - x^2} = \frac{\sigma_0 x}{\sqrt{\ell^2 - x^2}} \; (0 < x < l). \qquad (3.7)$$

It can be seen from (3.6) and (3.7) and in [21,22] that the value of the function obtained in two ways is the same.

From (2.12) taking into account (3.6), (2.8) or from (2.13) taking into account (3.5) we have

$$\mathrm{K}_\mathrm{I} = \sigma_0\sqrt{\pi\ell}. \qquad (3.8)$$

By means of (2.1) – (2.5), (2.7) and (3.6) we get

$$\frac{2\mu \cdot u(x,+0)}{(1-2\nu)\sigma_0} = \frac{1}{\pi}\int_{-\ell}^{\ell} \frac{\sqrt{\ell^2 - t^2}}{t - x} dt.$$

Hence, by virtue of (3.1), we have

$$u(x,+0) = \begin{cases} -\dfrac{1-2\nu}{2\mu}\sigma_0 x, & |x| < \ell, \\ \dfrac{1-2\nu}{2\mu}\sigma_0\left[\sqrt{x^2-\ell^2} - x\right], & x > \ell, \\ \dfrac{1-2\nu}{2\mu}\sigma_0\left[-\sqrt{x^2-\ell^2} - x\right], & x < -\ell, \end{cases}$$

By virtue of the formulas (3.6), (2.9) and (2.10) we get

$$\upsilon(x,+0) = \begin{cases} 0, & |x| > \ell, \\ \dfrac{1-\nu}{\mu}\sigma_0\sqrt{\ell^2 - x^2}, & |x| < \ell. \end{cases}$$

It is not difficult to show that

$$\sigma_y(x,+0) = \sigma_x(x,+0) = \begin{cases} -\sigma_0, & |x| < \ell, \\ \sigma_0\left[\dfrac{x}{\sqrt{x^2-\ell^2}} - 1\right], & x > \ell, \\ \sigma_0\left[-\dfrac{x}{\sqrt{x^2-\ell^2}} - 1\right], & x < -\ell. \end{cases} \qquad (3.12)$$

CONCLUSIONS

It follows from (3.9) and (3.10) that if constant normal stresses act on the banks of the crack and the layer thickness is infinite, then on the banks of the crack both components of the displacement vector are different from zero. The same result was obtained by another method in [2].

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