# Investigating the effect of materials manufacturing grading functions on cracked FG plate stress intensity factors in XEFGM

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## Abstract

A extended element free Galerkin method (XEFGM) was used for crack analysis in functionally graded materials (FGMs). Taking into account the fact that the meshfree method does not depend on mesh, the fracture problems can be simulated easily using it instead of the traditional numerical method. The technique of the sub-triangle technique in numerical integration at discontinuity points, enrichment functions, as well as an appropriate support field to contain points and nodes from shaping functions were employed in this study. Furthermore, the stress intensity factors are calculated by integrating the incompatible interactions (SIFs). Under the influence of a tensile load, exponential, affine/linear, and hyperbola functions are adopted for grading the materials of cracked FG plates. The work was done using the MATLAB software environment.

**Keywords:**

Material grading using functionally graded materials, meshfree method, crack propagation technique, incompatible interaction integration method

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## 1. Introduction

As a result of meeting ultra-high temperatures and cryogenic requirements, FGMs originated. Initial research aimed to achieve strength, flexibility, and fatigue resistance. By gathering materials with these favorable features, you can most effectively achieve your objective of smooth and perfect spatial variation, in this way, we avoid detrimental effects such as stress concentrations and residual stresses in discrete interfaces. It can be made more efficient by gradually changing the properties of the raw materials, thus reducing failures, while preserving the intended advantages of merging different materials. Nature provides functionally graded materials in the form of bones, teeth, wood, and bamboo. These materials are used in a variety of engineering, military, medical, and space science applications due to their unique microstructures and mechanical properties. In order to solve fracture problems that arise from these materials, numerical methods have been used [2-11]. A particular type of functionally graded material was studied in terms of fatigue loads [12-13]. Recently, the digital image correlation method with numerical verification had been used to determine the path of crack growth in a glass and epoxy material stepwise functionally graded to find stress concentration values [14-15]. Meanwhile, the optical method was employed to determine the crack path and the stress concentration factor as well as the T-stress in a materials made continuously from graded materials [16]. Weak Galerkin formulation-element free method (XEFGM) is one of these methods that has been used in this study.

## 2- Field equations

In the following equation you can apply Hooke's law, that explains the relationship between stress $\boldsymbol{\sigma}$ and strain $\boldsymbol{\varepsilon}$ in elastic materials **D**.

$$\upsilon = f(x) \tag{6}$$

Functions or stresses close to the crack can be represented as follows [2, 17-18]:

$$\sigma_{11} = \frac{1}{\sqrt{2\pi r}}\left[K_I m_{11}^{I}(\theta) + K_I m_{11}^{II}(\theta)\right] \tag{7}$$

$$\sigma_{22} = \frac{1}{\sqrt{2\pi r}}\left[K_I m_{22}^{I}(\theta) + K_{II} m_{22}^{II}(\theta)\right] \tag{8}$$

$$\sigma_{12} = \frac{1}{\sqrt{2\pi r}}\left[K_I m_{12}^{I}(\theta) + K_{II} m_{12}^{II}(\theta)\right] \tag{9}$$

$$u_1 = \frac{1}{G_{tip}}\sqrt{\frac{r}{2\pi}}\left[K_I n_1^{I}(\theta) + K_{II} n_1^{II}(\theta)\right] \tag{11}$$

$$u_2 = \frac{1}{G_{tip}}\sqrt{\frac{r}{2\pi}}\left[K_I n_2^{I}(\theta) + K_{II} n_2^{II}(\theta)\right] \tag{12}$$

$$G_{tip} = \frac{E_{tip}}{\left[2(1+\nu_{tip})\right]} \tag{13}$$

The mechanical properties in equation (11) to (13) will be extracted at the tip of crack.

$$\boldsymbol{\sigma} = \mathbf{D}\boldsymbol{\varepsilon} \tag{1}$$

where

$$\mathbf{D} = \frac{E(x)}{1-\nu^2}\begin{bmatrix} 1 & \nu(x) & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu(x))/2 \end{bmatrix} \quad \text{(Plane stress)} \tag{2}$$

and

$$D = \frac{E(x)(1-\nu(x))}{(1+\nu(x))(1-2\nu(x))}\begin{bmatrix} 1 & \frac{\nu(x)}{1-\nu(x)} & 0 \\ \frac{\nu(x)}{1-\nu(x)} & 1 & 0 \\ 0 & 0 & \frac{1-2\nu(x)}{2(1-\nu(x))} \end{bmatrix} \quad \text{(Plane strain)} \tag{3}$$

From equations (1) to (3) gives

$$\boldsymbol{\varepsilon} = \mathbf{C}\boldsymbol{\sigma} = \begin{bmatrix} C_{11} & C_{12} & C_{16} \\ C_{12} & C_{22} & C_{26} \\ C_{16} & C_{26} & C_{66} \end{bmatrix}\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} \tag{4}$$

A compliance matrix is represented by C.

In FGM (Figure 1)

$$E = f(x) \tag{5}$$

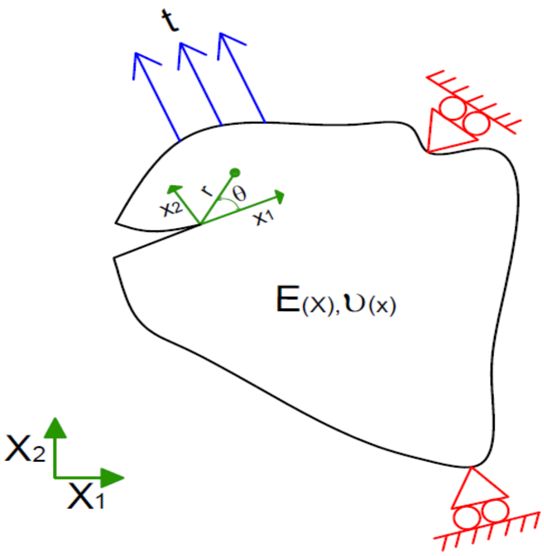


Figure 1 FGM body with crack.

$$\int_{\Omega} (\boldsymbol{L}\delta\boldsymbol{u})^T(\boldsymbol{DLu})d\Omega - \int_{\Omega} \delta\,\boldsymbol{u}^T\boldsymbol{b}d\Omega - \int_{\Gamma_t} \delta\,\boldsymbol{u}^T\bar{\boldsymbol{t}}d\Gamma - \int_{\Gamma_u} \delta\,\boldsymbol{\lambda}^T(u-\bar{u})d\Gamma - \int_{\Gamma_u} \delta\,\boldsymbol{u}^T\lambda d\Gamma = 0 \quad (19)$$

where $\boldsymbol{\lambda}$ represents the Lagrange multiplier variable. Steps to the represent and formulation this equation can be traced back to [20-25] to obtain the final discretized system equations,

$$\begin{bmatrix} \boldsymbol{K} & \boldsymbol{Q} \\ \boldsymbol{Q^T} & \boldsymbol{0} \end{bmatrix}\begin{bmatrix} \boldsymbol{U} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \boldsymbol{F} \\ \boldsymbol{q} \end{bmatrix} \quad (20)$$

The vectors in equation (26)

$$\boldsymbol{Q} = -\int_{\Gamma_u} \boldsymbol{N}^T\phi\, d\Gamma \quad (21)$$

$$\boldsymbol{q} = -\int_{\Gamma_u} \boldsymbol{N}^T\bar{\boldsymbol{u}}\, d\Gamma \quad (22)$$

$$\boldsymbol{\lambda}(\mathbf{x}) = \sum_{i=1}^{n_\lambda} N_i(\mathbf{x})\boldsymbol{\lambda}_i \quad (23)$$

U is the universal displacement vector and it can be found as:

$$U = \{\boldsymbol{u} \quad \boldsymbol{b_1} \quad \boldsymbol{b_2} \quad \boldsymbol{b_3} \quad \boldsymbol{b_4}\}^T \quad (24)$$

$\boldsymbol{b_{1-4}}$ are the enrichment functions.

**3- XEFGM formulation**

From Figure (2), the equilibrium equation can write as follow:

$$\boldsymbol{L^T\sigma} + \boldsymbol{b} = 0 \quad in\ problemd\ dmain\ \Omega \quad (14)$$

And the boundary conditions

$$\boldsymbol{\sigma n} = \bar{\boldsymbol{t}} \qquad on \quad \Gamma_t \quad (15)$$

$$\boldsymbol{u} = \bar{\boldsymbol{u}} \qquad on \quad \Gamma_u \quad (16)$$

$$\boldsymbol{\sigma n} = 0 \qquad on \quad \Gamma_c \quad (17)$$

And $\boldsymbol{L}$ is

$$L = \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix} \quad (18)$$

EFGM adopted moving least squares (MLS) technique [19]. In the relevant problem, the governing equation is:

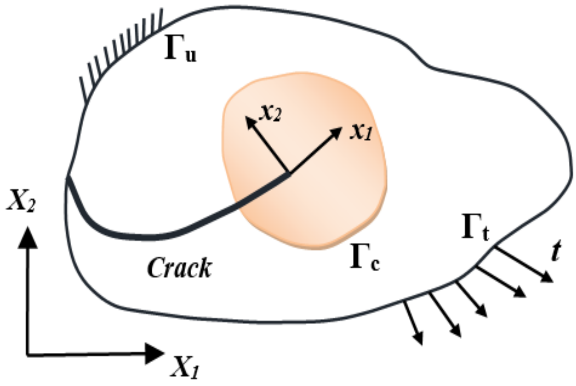




Figure 2 Two dimension cracked body.

$$w = \frac{1}{2}\sigma_{ij}\varepsilon_{ij}$$

The interaction integral and J integral can be defined [28-30]:

$$M = \int_A \left\{\sigma_{ij}u_{i,1}^{aux} + \sigma_{ij}^{aux}u_{i,1} - \frac{1}{2}(\sigma_{ik}u_{ik}^{aux} + \sigma_{ik}^{aux}u_i)\delta_{1j}\right\}q,_j\, dA + \int_A \left\{\sigma_{ij}\left(c_{ijkl}^{tip} - c_{ijkl}(x)\right)\sigma_{kl,1}^{aux}\right\}qdA$$

(30)

$$J_{local} = (K_I^2 + K_{II}^2)/E_{tip}$$
(31)

$$J_{local}^s = \frac{\left(K_I + K_I^{aux}\right)^2 + \left(K_{II} + K_{II}^{aux}\right)^2}{E_{tip}} = J_{local} + J_{local}^{aux} + M_{local}$$
(32)

## 4-Fracture analysis

In the fracture analysis, incompatibility formulation can be applied to find *J*-integral [26,27]. (29)

$$\sigma_{ij} = d_{ijkl}(x)\varepsilon_{kl}\ , \quad \varepsilon_{ij} \neq \frac{1}{2}(u_{i,j} + u_{j,i})\ , \ \sigma_{ij,j} = 0$$
(25)

where

$$\varepsilon_{ij} = C_{ijkl}(x)\sigma_{kl} \quad (i\,, j, k\,, l = 1\,, 2\,, 3)$$
(26)

with

$$\mathbf{C} = \begin{bmatrix} C_{11} & C_{12} & C_{16} \\ C_{12} & C_{22} & C_{26} \\ C_{16} & C_{26} & C_{66} \end{bmatrix} = \begin{bmatrix} C_{1111} & C_{1122} & 2C_{1112} \\ C_{2211} & C_{2222} & 2C_{2212} \\ 2C_{1211} & 2C_{1222} & 4C_{1212} \end{bmatrix}$$
(27)

From Figure 3:

$$J = \int_A (\sigma_{ij}u_{i,1} - w\delta_{1j})q,_j\ dA + \int_A \left(\sigma_{ij}u_{i,1} - w\delta_{1j}\right)_{,j} qdA \qquad (28)$$

*w* is the strain energy density:

$$K_I = M_{local}^{(1)} E_{tip} /2 , \quad (K_I^{aux} = 1.0 , K_{II}^{aux} = 0.0),$$

$$K_{II} = M_{local}^{(2)} E_{tip} /2 , \quad (K_I^{aux} = 0.0 , K_{II}^{aux} = 1.0).$$

In functionally graded materials, the last equation calculates the stress concentration values.

$$J_{local}^{aux} = [(K_I^{aux})^2 + (K_{II}^{aux})^2]/E_{tip}$$

and $M_{\text{local}}$ is given by (35)



$$M_{local} = 2(K_I K_I^{aux} + K_{II} K_{II}^{aux}) /E_{tip}$$

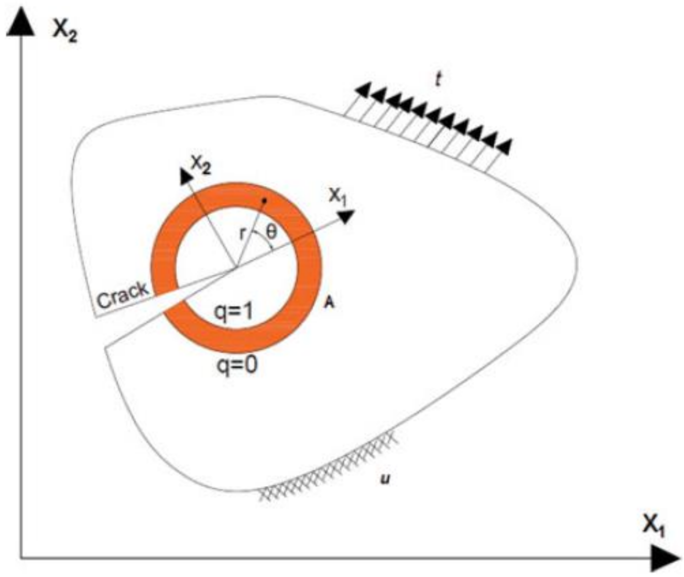


Figure 3 Cracks have an integral contour at the tip.

with XEFGM, many materials were used adopting the logarithmic function, and in this research, it will adopt other linear and trigonometric functions. Table 1 depicts the functions that will be considered in the case study, for a crack plate made from functionally graded material. Centrifugation process can be used for the Exponential function and Hyperbola function, while the linear function can be synthesized by hand layup method [31].

## 5. Materials manufacturing grading function

The functionally graded materials have a gradual manufacturing function, which affects formation, continuity, and external stresses. From Figure 1, it is very clear that the mechanical properties are variable in these materials in the direction of the dimensions. Gradual change of properties is what distinguishes this type of materials. Previously

Table 1 Functions of manufacturing that will be considered in the case study.

| No. | Manufacturing function E ($x_1$) = f ($x_1$), υ=0.25 | Previously adopted with XEFGM |
|---|---|---|
| 1 | Exponential function E ($x_1$) =98200×exp (0.002× $x_1$)-97200 | [9], [32] |
| 2 | Hyperbola function E($x_1$) = −2 × $(x_1)^2$+210 ×$x_1$+1000 | - |
| 3 | Affine/Linear function E ($x_1$) =190× $x_1$+1020 | - |



## 6. Numerical case study

A crack with material gradation occurs in this problem on an FGM rectangular plate. As illustrated in Figure 4, the specimen was subjected to a uniform tensile load. Material traits of a graduated beam are shown in Table 2.

Table 2 Data used in the numerical method.

| $X_1$ | $X_2$ | E (MPa) at $x_1$=0 – 30 | Gaussian quadrature in general cell | Gauss points near crack tips cell | Distributed nodes |
|---|---|---|---|---|---|
| 0-10 | 0-30 | 1000 ~3000 | 2×2 | 13 | 993 |

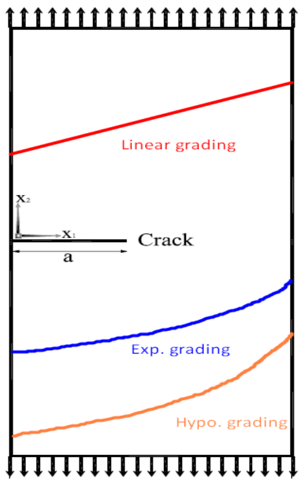


Figure 4 cracked functionally graded plate with three manufacturing grades.

Figures 5 and 6 depict the distribution of nodes and the background. As shown in Figure 7, Gauss points distribution using traditional and sub-triangle techniques. Three values are adopted to the support domain size $d_{max}$, which are 1.4, 1.7 and 2.

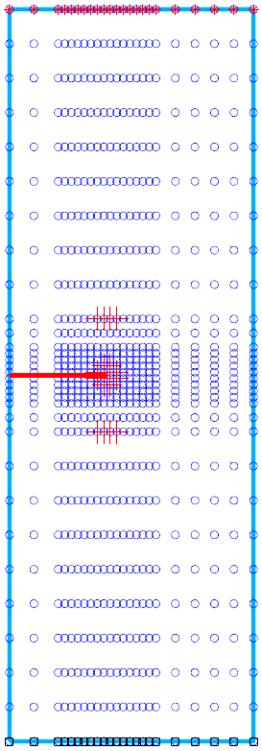

Figure 5 the distribution of nodes.

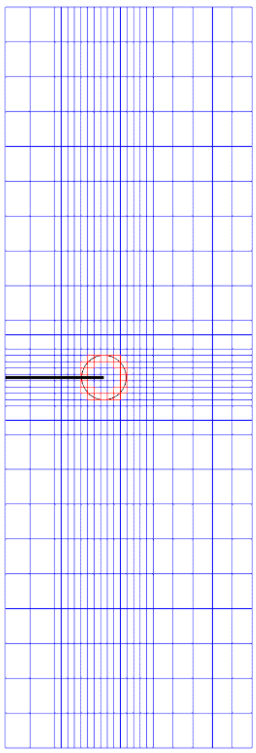

Figure 6 the distribution of the background cells.

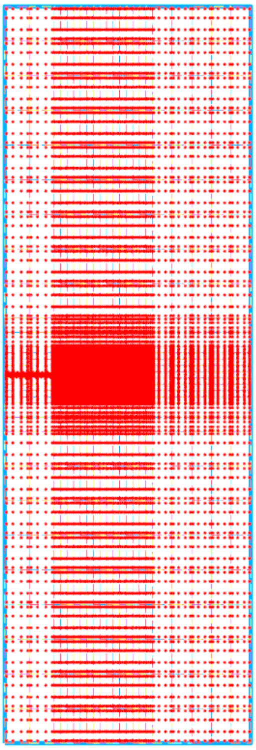

Figure 7 Gauss points distribution using traditional and sub-triangle techniques.



From Table 3, the stress concentration factor SIF values for the three manufacturing functions show a great deal of affinity.

Table 3 Normalized stress concentration factor SIF values for the three manufacturing functions with different crack length (at size of J-integral equals to 0.5).

| $d_{max}$=1.7, $r_j$=0.5 | | | |
|---|---|---|---|
| a (mm) | $K_I$ using Exponential function | $K_I$ using Affine/Linear function | $K_I$ using Hyperbola function |
| 3 | 1.4993 | 1.4978 | 1.4797 |
| 4 | 1.9256 | 1.9246 | 1.9057 |
| 5 | 2.6535 | 2.653 | 2.6318 |
| 6 | 4.0723 | 4.0724 | 4.0462 |
| 7 | 6.6353 | 6.6374 | 6.5858 |
| 8 | 12.3116 | 12.3143 | 12.2667 |

Obviously, as the crack length increases, the stress concentration factor SIF increases. It is also clear that the values of SIF when the gradient modulation of the properties is of the Hyperbola type are less than the other gradual types. This impression is very important to know, as the results obtained at a tensile stress equal to one MPa, therefore the more the applied stress increases, the more the use of the hypo hyperbolic is required. The reason for this behavior may be, the values of elastic modulus change monotonically under hyperbolic fixed, as it is found that the SIFs results are close and acceptable. The reason for this is that the numerical method XEFGM is strong for analyzing and evaluating the different fracture problems.

conditions where the hyperbolic function has a steeper trajectory than the exp function.

To verify the results (Table 4), the support domain size is changed and the contour size is



Table 4 Stress concentration factor values by changing the crack length and support domain size.

| $r_j$=0.5 (Exponential function) | | | |
|---|---|---|---|
| No. | dmax =1.7 | dmax =2 | dmax =1.4 |
| a (mm) | $K_I$ | $K_I$ | $K_I$ |
| 3 | 1.4797 | 1.531 | 1.3817 |
| 4 | 1.9057 | 1.9503 | 1.2128 |
| 5 | 2.6318 | 2.7668 | 2.5781 |
| 6 | 4.0462 | 4.0163 | 3.8335 |
| 7 | 6.5858 | 6.1623 | 6.2397 |
| 8 | 12.2667 | 13.5134 | 10.8551 |

For further verification, and as shown in Table 5, the values of the $d_{max}$ were fixed and the $r_j$ changed, as well as a highly acceptable investigation are obtained.

Table 5 Stress concentration factor values by changing the crack length and J-integral size.

| $d_{max}$ =1.7 | $r_j$ = 0.7 | $r_j$ = 0.9 |
|---|---|---|
| a mm | $K_I$ | $K_I$ |
| 3 | 1.4894 | 1.4982 |
| 4 | 1.9083 | 1.9246 |
| 5 | 2.6447 | 2.6899 |
| 6 | 4.0927 | 4.1004 |
| 7 | 6.7138 | 6.8652 |
| 8 | 15.0159 | 15.1461 |

gives accurate results with a smooth distribution without updating or refinement in the distribution of nodes.

Figure 8 shows the contours of the total stresses on the plate in various directions. Additionally, this depicts that XEFGM method

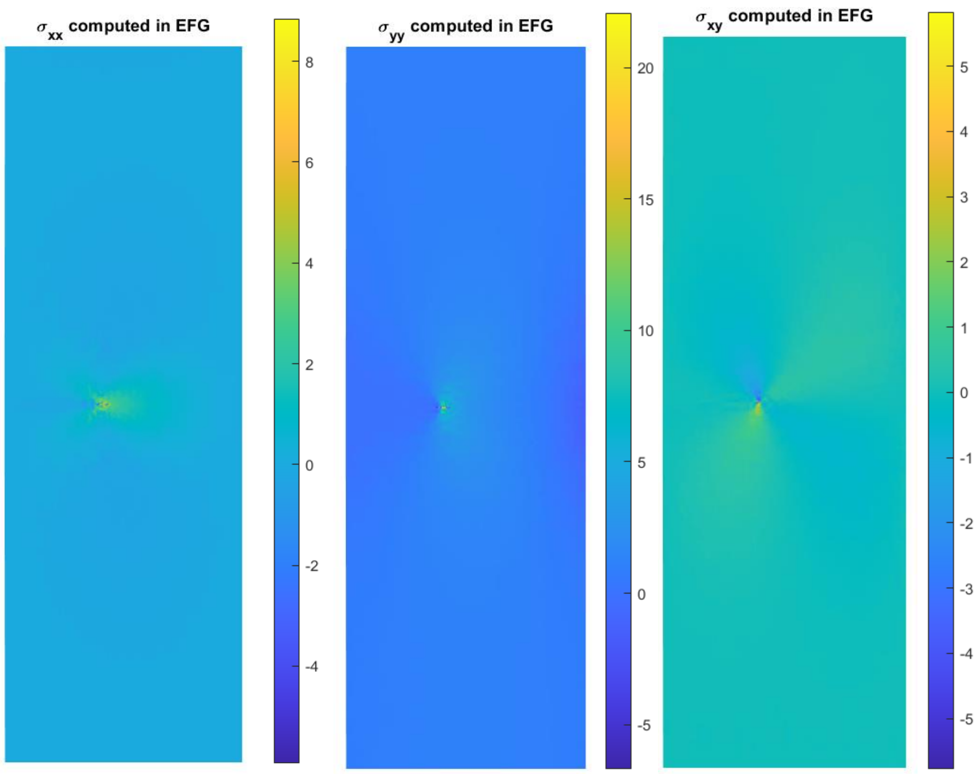


Figure 8 Contours of stress components using XEFGM.

## 7. Conclusion

Obviously the XEFGM method has also shown to be accurate even for functionally graded materials with different gradation manufacturing functions. Exponential, affine/linear, and hyperbola functions are adopted as functions of manufacturing the graded materials of cracked FG plate under the influence of a tensile load. It is obvious that the crack length affects the stress concentration factor SIF. Similarly, the values of SIF are lower when the gradient modulation of the properties is of the Hyperbola type as opposed to the other gradual types. As the results obtained at a tensile stress equal to one MPa are very important to understand, as the higher the applied stress, the more the hyperbolic rule becomes necessary. Under hyperbolic conditions where the hyperbolic function has a steeper trajectory than the exp function, the values of elastic modulus change monotonically.

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