Data Analysis of Deviation in Information Networks

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ABSTRACT

In recent years, examining the information processing aspects of nature has received significant attention. In particular, entanglement has been shown to exhibit what is arguably the most distinctive and unique aspect of quantum information science. In this note, a numerical method is applied to the Bell inequalities for examining the range of nonlocality when imperfect measurements are performed at each end. A two-qubit direct protocol is also examined such that, unlike the Bell-type inequalities, the approach is not as stable as the indirect one against measurement errors in yielding the nonlocality. On the other hand, since many applications of quantum technology involve a particular type of entanglement, it is important to have a technique to manipulate correlations. In this paper, five chained 2-level entanglements are also examined with swapping protocols applied at each joint. It is numerically shown that there exists a class of states that approximate the optimal result, i.e., the weakest link.

Key Words: Data, Numerical Methods, Entanglement, Nonlocality

DOI Number: 10.14704/nq.2018.16.1.1693 **NeuroQuantology 2018; 16(7):79-86**

Introduction

Along with rapid development in the field of quantum computation (Deutsch, 1985; Shor, 1996; Kok *et al*., 2007; Ladd *et al*., 2010), researchers also have examined the possibilities of viewing nature as a computation (Hooft, 1999; Wolfram, 2002; Lloyd, 2006; Zizzi, 2006; Vedral, 2010; Schmidhuber, 2015). One may wonder what could be advantageous in this effort of treating a physical phenomenon as information processing. Traditional numerical simulation has often been regarded as an imprecise but handy tool in assisting the analytic approach. However, as far as classical natural phenomena are concerned, the more accurate description of nature may be based on a discrete model, which is better described by digital computational models. That is, a computational view of the universe may be not only a matter of convenience but in fact a more accurate description.

Another important aspect of the information theoretic approach is that it treats the observing party and the physical system on an equal footing. This is so because information always involves the data provider, the physical system, and a party that receives and interprets the data into meaningful information. Indeed, this approach of treating the observer and the object at the same time is consistent with the orthodox interpretation of quantum theory. In this paper, we wish to examine one of important aspects of the information theoretic approaches, namely, the phenomenon of entanglement. The unique nature of quantum correlations were initially discussed, along with the development of the theory, in the early 20th century (Einstein *et al*., 1935), but Bell's pioneering work (Bell, 1964; Clauser *et al*., 1969) and subsequent delicate experiments (Aspect 1982; Tittel *et al*., 1998; Yin *et al*., 2012; Yin *et al*., 2017) have put entanglement at the heart of quantum information science (Bennett *et al*., 1993; Ekert, 1991; Nielsen *et al*., 2000).

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Relevant conflicts of interest/financial disclosures: The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.
Received: 08 June 2018; **Accepted:** 12 July 2018

Stability

In Bell-type inequalities, one may consider entangled qubits, where two different choices of measurement, i.e., A_1, A_2 and B_1, B_2, are made at each end. The average value of the correlation function $\mu(A_i, B_j)$ and the locality assumption yields (Bell, 1964; Clauser *et al.*, 1969)

$$|\mu(A_1,B_1)+\mu(A_1,B_2)+\mu(A_2,B_1)-\mu(A_2,B_2)|\leq 2 \quad (1)$$

Given the inequalities, let us consider the following conditions,

$$A_2 \equiv A_1+\frac{\pi}{4}, B_2 \equiv B_1-\frac{\pi}{4} \quad (2)$$

In such cases, Fig. 1 shows the inequalities in (1) as a function of A_1 and B_1. Since the actual measurement cannot be performed with perfect precision, it is desirable to seek the range of error or deviation that will still violate the inequalities. Let us consider the case when the error may occur with A_2 and B_2 as follows,

$$A_1 = 0, A_2 = A_1+\frac{\pi}{4}-\gamma$$

$$B_1 = \frac{\pi}{8}, B_2 = B_1-\frac{\pi}{4}-\delta$$

In Fig. 2, the inequalities are shown as a function of γ and δ in the range of -1 and +1. It may be seen that there is also a significant range of freedom that yields nonlocality.

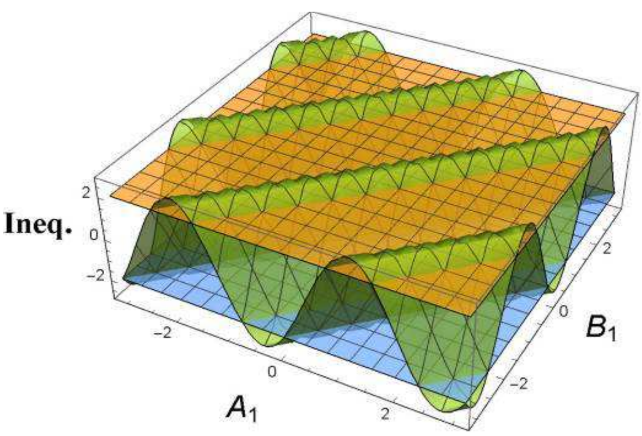

Figure 1. The Bell-type inequalities are shown as a function of A_1 and B_2, where $A_2 \equiv A_1+\frac{\pi}{4}$ and $B_2 \equiv B_1-\frac{\pi}{4}$ are assumed

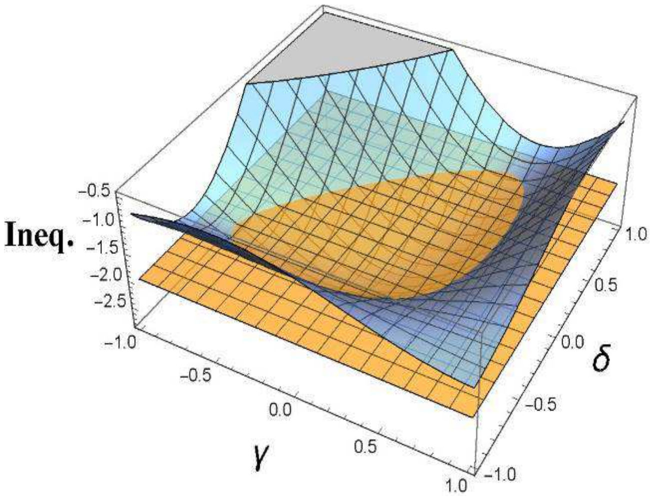

Figure 2. Given $A_1 = 0, A_2 = A_1+\frac{\pi}{4}-\gamma$, $B_1 = \frac{\pi}{8}$ and $B_2 = B_1-\frac{\pi}{4}-\delta$, the inequalities in (1) are provided with respect to γ and δ, which yield a substantial area that violate the inequalities

Let us consider another example,

$$A_1 = 0, A_2 = A_1+\gamma$$

$$B_1 = \frac{\pi}{8}, B_2 = B_1-\delta$$

In such a case, the range of measurement choice A_2 and B_2 are examined such that it still violates the inequalities to yield the nonlocality. As displayed in Fig. 3, in the range of $\gamma, \delta \in (-\pi, \pi)$, there is a wide region to violate the inequalities. Let us consider another case where $A_1 = 0, A_2 = \frac{\pi}{4}, B_1 = \frac{\pi}{8}, B_2 = B_1-\frac{\pi}{4}$. We wish to consider the following case,

$$A_1 \to A_1-\gamma_1, A_2 \to A_2-\gamma_2 \quad (3)$$

Fig. 4 shows the violation of inequalities in the case of deviated choices of the measurement for A_1 and A_2 with parameters γ_1 and γ_2. Therefore, the Bell-type inequalities, as in (1), yield the violation of locality even with a substantial deviation in measurement bases as numerically examined using figures.

*e*ISSN 1303-5150

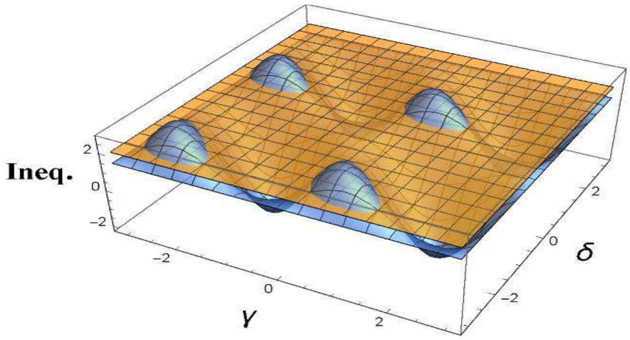

Figure 3. The inequalities are shown as a function of γ and δ where $A_1 = 0, A_2 = A_1 + \gamma, B_1 = \frac{\pi}{8}, B_2 = B_1 - \delta$ in the range of $(-\pi, \pi)$. There is a significant range of freedom in yielding nonlocality

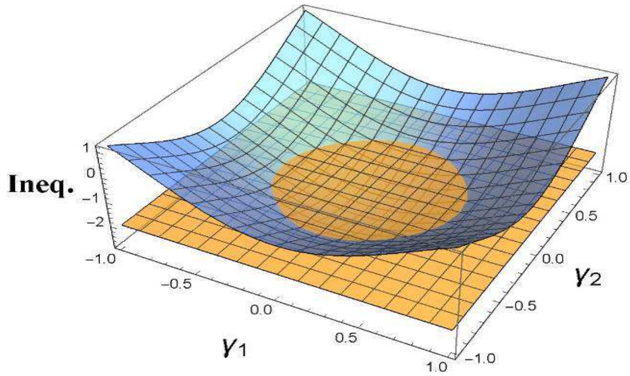

Figure 4. Given $A_1 = 0, B_1 = \frac{\pi}{8}, A_2 = \frac{\pi}{4}, B_2 = B_1 - \frac{\pi}{4}$, the range of nonlocality is shown when $A_1 \to A_1 - \gamma_1$ and $A_2 \to A_2 - \gamma_2$ are imposed

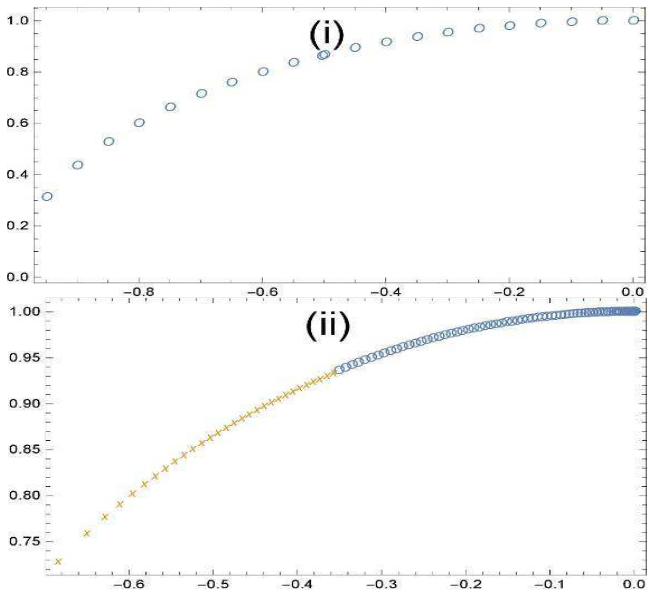

Figure 5. (i) shows the usual distribution of general two qubit correlation, whereas (ii) shows the distribution of λ_1 and λ_2 in (4) with parameter α such that *O*'s indicates when $0 < \alpha < \frac{1}{\sqrt{2}}$ and *x*'s shows when $\frac{1}{\sqrt{2}} < \alpha < 1$.

Let us consider another type of nonlocality, namely the direct approach with the following state (Hardy, 1992),

$$|\Psi\rangle = \kappa\left((1-\alpha^4)|00\rangle - (\alpha^3\sqrt{1-\alpha^2})(|01\rangle + |10\rangle) - (\alpha^2(1-\alpha^2))|11\rangle\right) \quad (4)$$

where

$$\kappa = \frac{1}{\sqrt{(1-\alpha^4)^2 + 2\alpha^6(1-\alpha^2) + \alpha^4(1-\alpha^2)^2}} \quad (5)$$

and $0 < \alpha < 1$. It was shown that when the measurement is made with bases $M_1 \equiv \{|0\rangle, |1\rangle\}$ and $M_2 \equiv \{\alpha|0\rangle + \sqrt{1-\alpha^2}|1\rangle, \sqrt{1-\alpha^2}|0\rangle - \alpha|1\rangle\}$ at each end, the nonlocal element in (4) may be directly argued. The proof goes as when it is measured with basis M_2 at both ends, there is a cancellation with a term which contradicts a locality condition. The diagonalized elements of (4) may be computed as follows,

$$\lambda_{1,2} = \frac{1}{2(1-\alpha^4)}\left(\frac{1-\alpha^2-\alpha^4+\alpha^6}{\sqrt{1-\alpha^4}} \mp \sqrt{\frac{1+2\alpha^2-5\alpha^4-4\alpha^6+7\alpha^8+2\alpha^{10}-3\alpha^{12}}{1-\alpha^4}}\right) \quad (6)$$

In Fig. 5, the comparison of the state in (4) ((ii)) and the general correlated states ((i)) are provided. In particular, (ii) shows the distribution of λ_1 and λ_2 with a parameter α such that $0 < \alpha < \frac{1}{\sqrt{2}}$ is indicated with *O*'s, whereas $\frac{1}{\sqrt{2}} < \alpha < 1$ is shown with *x*'s. It can be checked that, unlike the Bell-type inequalities which rely on repeated runs, a deviation of measuring basis of the direct approach with (4) would ruin the nonlocal aspect.

Numerical Approach

Quantum theory is often considered to have revolutionized the way science is practiced (Feynman, 1970; Peres, 1997). While previous attempts have often thought to reveal the property of physical systems, particularly macroscopic ones, such as how the moon evolves around the earth, the mass of the sun and so forth, quantum theory started to reveal the relationship between the measuring and the object systems, particularly microscopic ones, such as if the spin of an electron is up or down when it is measured. Since macroscopic objects are composed of microscopic physical systems, such as electrons, protons and so forth, it may be thought that the previous rules of describing the patterns of physical systems ought to be replaced by more precise relationship between the observing party and the objects.

While discussions of quantum theory, such as the one described above, often remain in the domain of philosophical thought, a physically realizable protocol involving the issues in quantum foundations has been proposed (Bennett and Brassard, 1984; Bennett *et al.*, 1993; Cleve, 1997; Kok *et al*, 2007; Ladd *et al.*, 2010). In 1964, in a groundbreaking paper (Bell, 1964), Bell presented a method that would provide an important clue with regard to the completeness of quantum theory (Einstein *et al.*, 1935). Moreover, the much-debated quantum parallelism has been utilized to provide powerful computational and communication models (Deutsch, 1985). In (Hardy *et al.*, 2000), N-chained general correlations have been considered, wherein entanglement swapping protocols yielded optimal results for certain classes of coefficients. In this paper, we wish to numerically examine entanglement swapping schemes (Zukowski *et al.*, 1993) for five 2-level states wherein a new class of coefficients that approximate the weakest link is introduced.

Let us consider the following states,

$$\left|\phi^{(l)}\right\rangle = \sum_{k=0}^{1}\sqrt{\kappa_k}\left|k\right\rangle_{2l-1}\left|k\right\rangle_{2l} \quad (7)$$

where $l = 1,2,3,4,5$, i.e., five 2-level states, and the coefficients are assumed to be ordered Schmidt coefficients. Moreover, throughout the section, $\left|\phi^{(1)}\right\rangle$ will be considered to have the weakest entanglement in the 5-chain of correlations. Entanglement swapping may be applied by measuring the four joints which would yield the new entangled state between 1 and 10. This process may be completed by yielding a combination of different coefficients (see (Hardy *et al.*, 2000)). The following 16 conditions may be considered:

$$(c1)\,\kappa_0^{(1)}\kappa_0^{(2)}\kappa_0^{(3)}\kappa_0^{(4)}\kappa_0^{(5)} \geq \kappa_1^{(1)}\kappa_1^{(2)}\kappa_1^{(3)}\kappa_1^{(4)}\kappa_1^{(5)}$$

$$(c2)\,\kappa_0^{(1)}\kappa_0^{(2)}\kappa_0^{(3)}\kappa_0^{(4)}\kappa_1^{(5)} \geq \kappa_1^{(1)}\kappa_1^{(2)}\kappa_1^{(3)}\kappa_1^{(4)}\kappa_0^{(5)}$$

$$(c3)\,\kappa_0^{(1)}\kappa_0^{(2)}\kappa_0^{(3)}\kappa_1^{(4)}\kappa_0^{(5)} \geq \kappa_1^{(1)}\kappa_1^{(2)}\kappa_1^{(3)}\kappa_0^{(4)}\kappa_1^{(5)}$$

$$(c4)\,\kappa_0^{(1)}\kappa_0^{(2)}\kappa_0^{(3)}\kappa_1^{(4)}\kappa_1^{(5)} \geq \kappa_1^{(1)}\kappa_1^{(2)}\kappa_1^{(3)}\kappa_0^{(4)}\kappa_0^{(5)}$$

$$(c5)\,\kappa_0^{(1)}\kappa_0^{(2)}\kappa_1^{(3)}\kappa_0^{(4)}\kappa_0^{(5)} \geq \kappa_1^{(1)}\kappa_1^{(2)}\kappa_0^{(3)}\kappa_1^{(4)}\kappa_1^{(5)}$$

$$(c6)\,\kappa_0^{(1)}\kappa_0^{(2)}\kappa_1^{(3)}\kappa_0^{(4)}\kappa_1^{(5)} \geq \kappa_1^{(1)}\kappa_1^{(2)}\kappa_0^{(3)}\kappa_1^{(4)}\kappa_0^{(5)}$$

$$(c7)\,\kappa_0^{(1)}\kappa_0^{(2)}\kappa_1^{(3)}\kappa_1^{(4)}\kappa_0^{(5)} \geq \kappa_1^{(1)}\kappa_1^{(2)}\kappa_0^{(3)}\kappa_0^{(4)}\kappa_1^{(5)}$$

$$(c8)\,\kappa_0^{(1)}\kappa_0^{(2)}\kappa_1^{(3)}\kappa_1^{(4)}\kappa_1^{(5)} \geq \kappa_1^{(1)}\kappa_1^{(2)}\kappa_0^{(3)}\kappa_0^{(4)}\kappa_0^{(5)}$$

$$(c9)\,\kappa_0^{(1)}\kappa_1^{(2)}\kappa_0^{(3)}\kappa_0^{(4)}\kappa_0^{(5)} \geq \kappa_1^{(1)}\kappa_0^{(2)}\kappa_1^{(3)}\kappa_1^{(4)}\kappa_1^{(5)}$$

$$(c10)\,\kappa_0^{(1)}\kappa_1^{(2)}\kappa_0^{(3)}\kappa_0^{(4)}\kappa_1^{(5)} \geq \kappa_1^{(1)}\kappa_0^{(2)}\kappa_1^{(3)}\kappa_1^{(4)}\kappa_0^{(5)}$$

$$(c11)\,\kappa_0^{(1)}\kappa_1^{(2)}\kappa_0^{(3)}\kappa_1^{(4)}\kappa_0^{(5)} \geq \kappa_1^{(1)}\kappa_0^{(2)}\kappa_1^{(3)}\kappa_0^{(4)}\kappa_1^{(5)}$$

$$(c12)\,\kappa_0^{(1)}\kappa_1^{(2)}\kappa_0^{(3)}\kappa_1^{(4)}\kappa_1^{(5)} \geq \kappa_1^{(1)}\kappa_0^{(2)}\kappa_1^{(3)}\kappa_0^{(4)}\kappa_0^{(5)}$$

$$(c13)\,\kappa_0^{(1)}\kappa_1^{(2)}\kappa_1^{(3)}\kappa_0^{(4)}\kappa_0^{(5)} \geq \kappa_1^{(1)}\kappa_0^{(2)}\kappa_0^{(3)}\kappa_1^{(4)}\kappa_1^{(5)}$$

$$(c14)\,\kappa_0^{(1)}\kappa_1^{(2)}\kappa_1^{(3)}\kappa_0^{(4)}\kappa_1^{(5)} \geq \kappa_1^{(1)}\kappa_0^{(2)}\kappa_0^{(3)}\kappa_1^{(4)}\kappa_0^{(5)}$$

$$(c15)\,\kappa_0^{(1)}\kappa_1^{(2)}\kappa_1^{(3)}\kappa_1^{(4)}\kappa_0^{(5)} \geq \kappa_1^{(1)}\kappa_0^{(2)}\kappa_0^{(3)}\kappa_0^{(4)}\kappa_1^{(5)}$$

$$(c16)\,\kappa_0^{(1)}\kappa_1^{(2)}\kappa_1^{(3)}\kappa_1^{(4)}\kappa_1^{(5)} \geq \kappa_1^{(1)}\kappa_0^{(2)}\kappa_0^{(3)}\kappa_0^{(4)}\kappa_0^{(5)}$$

When these conditions are fulfilled, it can be verified that the average maximal entanglement yields the weakest link in the chain, i.e., $E_{\max} = 2\kappa_1^{(1)}$ (Jonathan *et al.*, 1999; Bose *et al.*, 1998; Hardy *et al.*, 2000; Shi *et al.*, 2000). For instance, in Fig. 6, the comparison is provided for coefficients $\kappa_0^{(3)}$, $\kappa_0^{(4)}$, $\kappa_0^{(5)}$ ((i) and (ii)) and for $\kappa_1^{(3)}$, $\kappa_1^{(4)}$, $\kappa_1^{(5)}$ ((iii) and (iiii)) between the arbitrary states ((i) and (iii)) and the optimal non-maximal states ((ii) and (iiii)), which are not as widely and densely distributed as the general case.

As shown in Fig. 6, the coefficients that satisfy the optimal condition are not as rich as the general states, which therefore leads us to explore the possibility of whether there are some more non-maximal initial states that will yield an outcome close to the optimal value. In order to accomplish this task, let us consider the following conditions.

$$(c16')\,\kappa_0^{(1)}\kappa_1^{(2)}\kappa_1^{(3)}\kappa_1^{(4)}\kappa_1^{(5)} < \kappa_1^{(1)}\kappa_0^{(2)}\kappa_0^{(3)}\kappa_0^{(4)}\kappa_0^{(5)}$$

It may be numerically verified that there exists a class of coefficients that satisfy conditions (c1-c15) and (c16') yet yield the average maximal entanglement that is close to the weakest link with a margin <0.01. Fig. 7 shows the average maximal entanglement with respect to $\kappa_0^{(1)}$ and $\kappa_0^{(2)}$ (i), $\kappa_0^{(3)}$ and $\kappa_0^{(4)}$ (ii), $\kappa_1^{(2)}$ and $\kappa_1^{(3)}$ (iii), and $\kappa_1^{(4)}$ and $\kappa_1^{(5)}$ (iiii), while Table 1 shows some examples of this particular class.

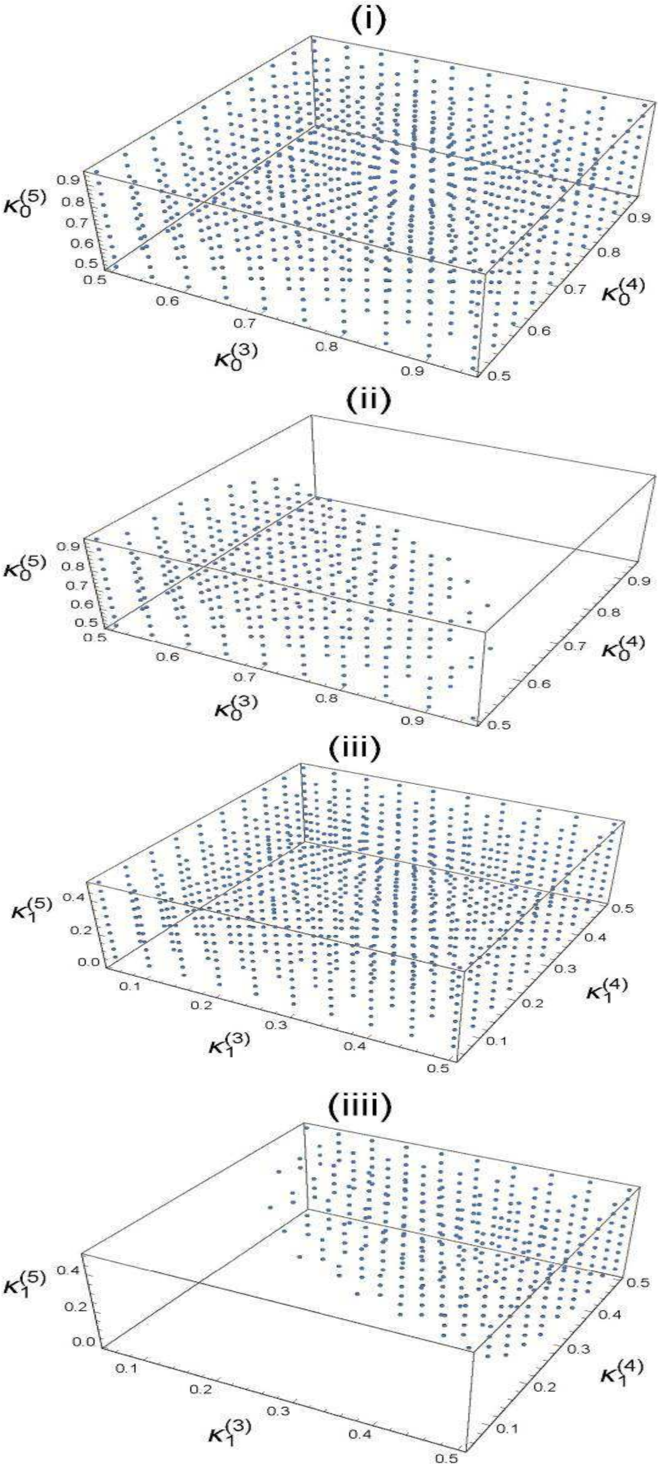

Figure 6. The coefficient distribution is displayed to compare the spreadness between the general and optimal conditions. (i) shows the plots of $\kappa_0^{(3)}$, $\kappa_0^{(4)}$, $\kappa_0^{(5)}$ for arbitrary states, while (ii) yields the outcome for optimality conditions. Similarly, (iii) and (iiii) are shown for $\kappa_1^{(3)}$, $\kappa_1^{(4)}$, $\kappa_1^{(5)}$

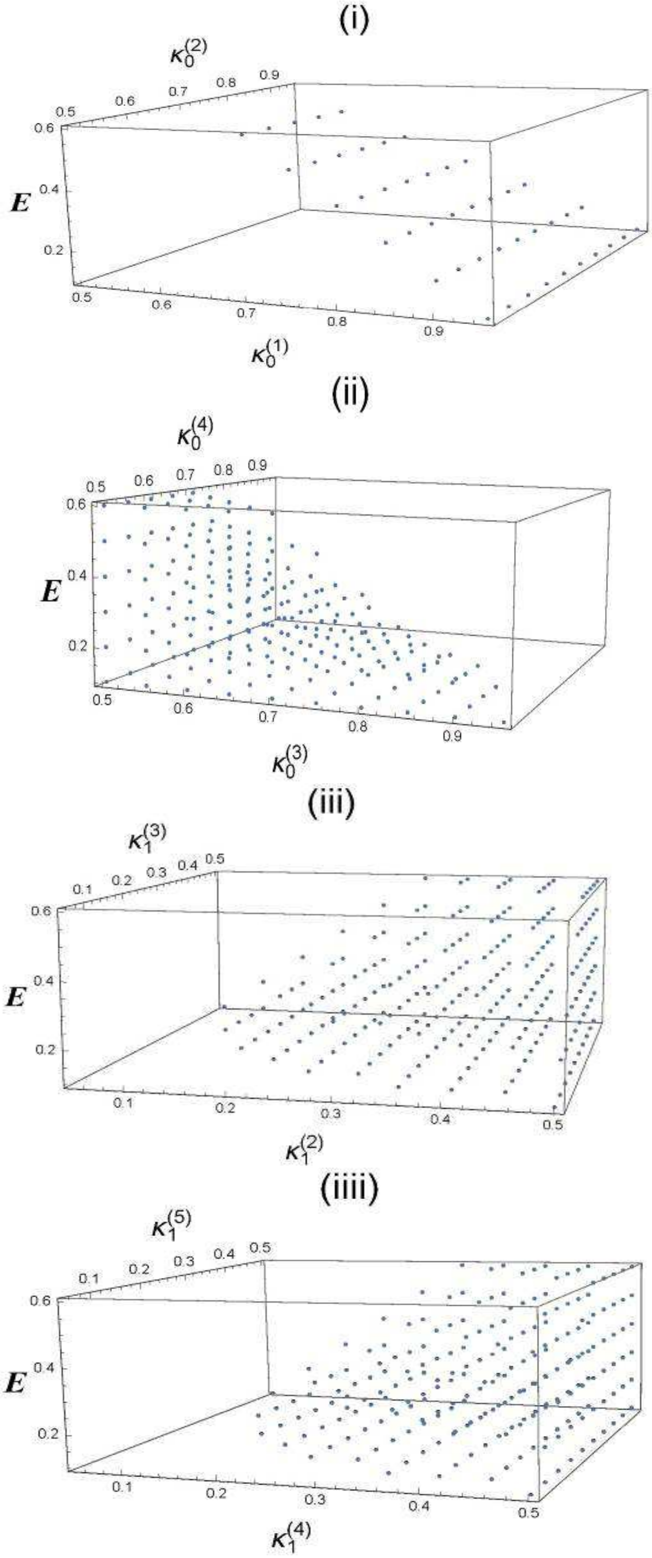

Figure 7. The distribution of coefficients that approximate the optimal result is shown. (i) shows the plots for maximal average entanglement with respect to $\kappa_0^{(1)}$ and $\kappa_0^{(2)}$, while (ii),(iii), and (iiii) display entanglement as a function of $\kappa_0^{(3)}$ and $\kappa_0^{(4)}$, $\kappa_1^{(2)}$ and $\kappa_1^{(3)}$, and $\kappa_1^{(4)}$ and $\kappa_1^{(5)}$, respectively

Table 1. Numerical examples of coefficients $\kappa_{0,1}^{(n)}$ that satisfy the conditions (c1-c15) and (c16') that approximate the optimal result

$E_{max}^{(Weakest)}$	$E_{max}^{(Approx)}$	$\kappa_0^{(1)}$	$\kappa_1^{(1)}$	$\kappa_0^{(2)}$	$\kappa_1^{(2)}$
0.5	0.49415	0.75	0.25	0.55	0.45
0.6	0.591135	0.7	0.3	0.55	0.45
0.4	0.39856	0.8	0.2	0.6	0.4
0.4	0.39817	0.8	0.2	0.6	0.4
0.3	0.29596	0.85	0.15	0.65	0.35
0.3	0.293587	0.85	0.15	0.55	0.45
0.2	0.197915	0.9	0.1	0.65	0.35
0.2	0.196175	0.9	0.1	0.6	0.4
0.1	0.09994	0.95	0.05	0.6	0.4
0.1	0.097935	0.95	0.05	0.8	0.2
$\kappa_0^{(3)}$	$\kappa_1^{(3)}$	$\kappa_0^{(4)}$	$\kappa_1^{(4)}$	$\kappa_0^{(5)}$	$\kappa_1^{(5)}$
0.6	0.4	0.6	0.4	0.55	0.45
0.6	0.4	0.55	0.45	0.55	0.45
0.6	0.4	0.6	0.4	0.55	0.45
0.55	0.45	0.55	0.45	0.65	0.35
0.6	0.4	0.6	0.4	0.6	0.4
0.55	0.45	0.6	0.4	0.75	0.25
0.6	0.4	0.65	0.35	0.65	0.35
0.55	0.45	0.65	0.35	0.75	0.25
0.7	0.3	0.7	0.3	0.7	0.3
0.7	0.3	0.55	0.45	0.65	0.35

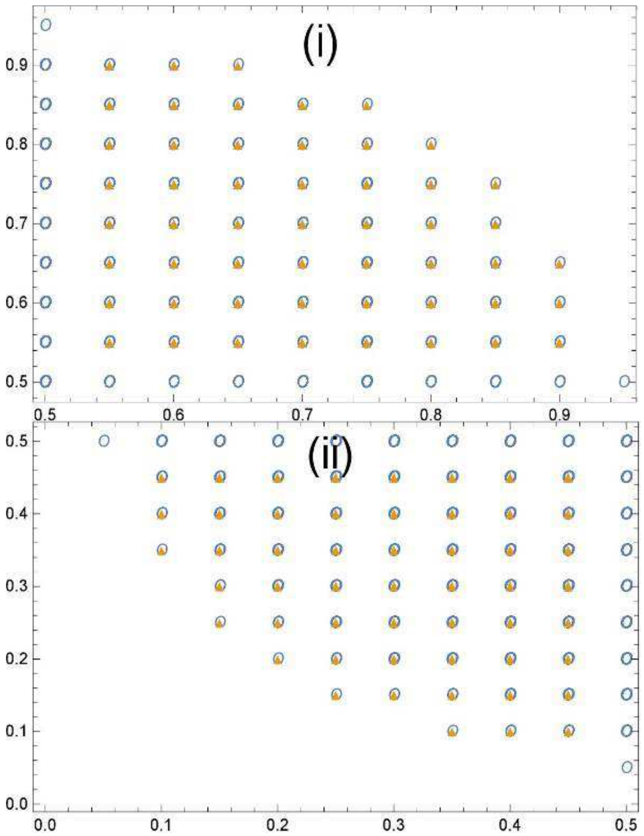

Figure 8. The comparison between general (shown with O's) and optimal (shown with ▲'s) coefficients in the case of five 2-level correlations. $\kappa_0^{(4)}$ versus $\kappa_0^{(5)}$ is shown in (i), and $\kappa_1^{(4)}$ versus $\kappa_1^{(5)}$ is shown in (ii).

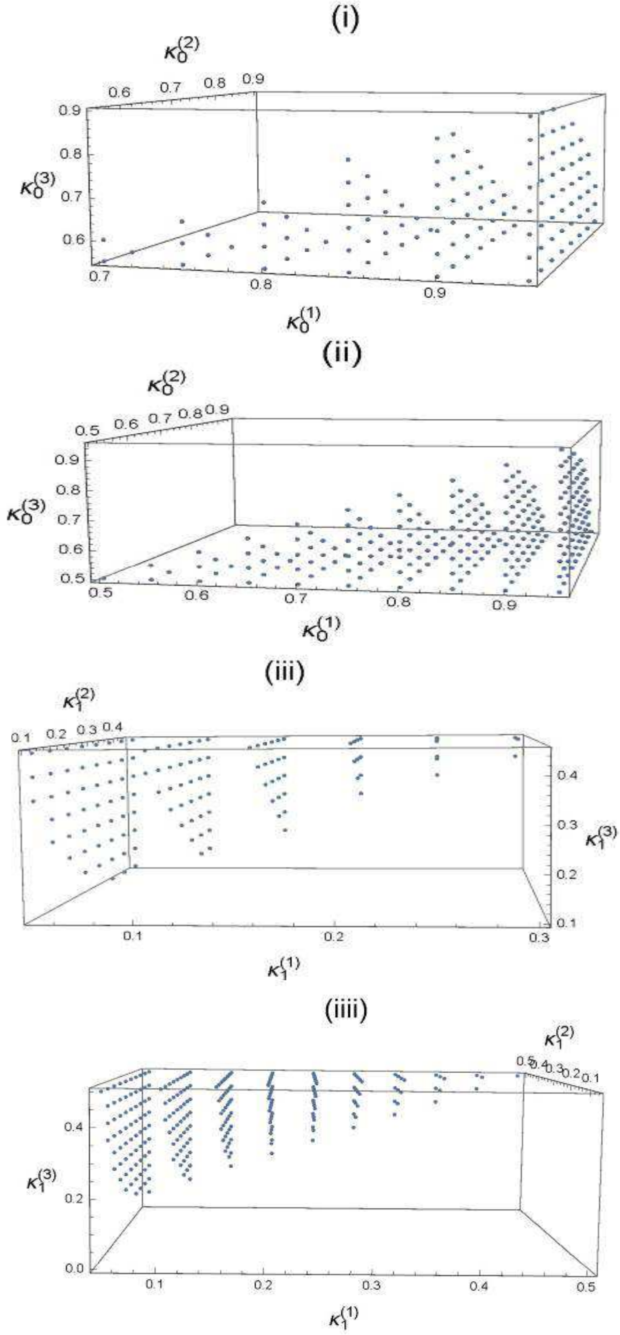

Figure 9. A comparison is provided between the states that yield the optimal result ((ii) and (iiii)) and the coefficients that approximate the weakest link ((i) and (iii)) for $\kappa_0^{(1)}$, $\kappa_0^{(2)}$, $\kappa_0^{(3)}$ ((i) and (ii)) and $\kappa_1^{(1)}$, $\kappa_1^{(2)}$, $\kappa_1^{(3)}$ ((iii) and (iiii))

Moreover, this new class of coefficients is relatively well-distributed compared to the previous case of optimal states. For instance, Fig. 8, (i) shows the coefficients of $\kappa_0^{(4)}$ and $\kappa_0^{(5)}$ in the optimal case (shown by O's) and the approximate ones shown by ▲'s. A similar comparison is displayed in (ii) for $\kappa_1^{(4)}$ and $\kappa_1^{(5)}$. Fig. 9 also provides a similar result. In (i) and (ii), the comparison is provided for coefficients, $\kappa_0^{(1)}$, $\kappa_0^{(2)}$, $\kappa_0^{(3)}$ between the approximate and the optimal cases, respectively. A similar outcome is shown in (iii) and (iiii) for coefficients $\kappa_1^{(1)}$, $\kappa_1^{(2)}$, $\kappa_1^{(3)}$.

Remarks

In this paper, numerical examination of measurement range for Bell inequalities has been provided. In particular, even the substantial range of error in measurement basis still yields the violation of inequalities; therefore, nonlocality is revealed. Moreover, a direct nonlocality proof was considered, which is advantageous in a sense that nonlocality may be shown directly, where the measurement deviation is not as tolerant as in the case of Bell inequalities.

A numerical approach was utilized to compare the optimal and general cases of entanglement concentration in the 5-chained 2-level states. Moreover, a new class of non-maximal states that approximates the optimal result has been presented. With the recent development of the information theoretic approach to illuminate the process of nature, a numerical approach like the one used in this paper has begun to be regarded as not only a useful tool to simulate the analytic approach, but also as an actual and more precise description of the phenomena. For instance, Wheeler coined the well-known phrase, *it from bit*, emphasizing the role of the observer and information (Wheeler, 1990). Landauer has also expressed a similar view based on a different approach, stating that *information is physical* (Landauer, 1961). More recently, the evolution of the observable universe has been considered as a process of computation and the total number of elementary computations rendered since the Big Bang has been estimated to be ~ 10^{123} (Lloyd, 2002).

References

Aspect A, Dalibard J, Roger G. Experimental test of Bell's inequalities using time-varying analyzers. Physical review letters 1982; 49(25): 1804-07.

Bell JS. On the Einstein-Podolsky-Rosen Paradox. Physics 1964; 1: 195–200. Reprinted in John Stuart Bell, Speakable and Unspeakable in Quantum Mechanics.

Bennett CH, Brassard G. Quantum cryptography: Public key distribution and coin tossing. Theoretical Computer Science 2014; 560(P1): 7-11.

Bennett CH, Brassard G, Crépeau C, Jozsa R, Peres A, Wootters WK. Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels. Physical Review Letters 1993; 70(13):1895-99.

Bose S, Vedral V, Knight PL. Multiparticle generalization of entanglement swapping. Physical Review A 1998; 57(2): 822.-29.

Clauser JF, Horne MA, Shimony A, Holt RA. Proposed experiment to test local hidden-variable theories. Physical Review Letters 1969; 23(15): 880-84.

Cleve R, Buhrman H. Substituting quantum entanglement for communication. Physical Review A 1997;56(2):1201-04.

Deutsch D. Quantum theory, the Church-Turing principle and the universal quantum computer. Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences 1985; 400(1818): 97-117.

Einstein A, Podolsky B, Rosen N. Can quantum-mechanical description of physical reality be considered complete?. Physical Review 1935; 47(10): 777-80.

Ekert AK. Quantum cryptography based on Bell's theorem. Physical review Letters.1991; 67(6):661-63.

Feynman R. Feynman Lectures on Physics. Addison Wesley Longman, 1970.

Hardy L. Quantum mechanics, local realistic theories, and Lorentz-invariant realistic theories. Physical Review Letters 1992; 68(20):2981-85.

Hardy L. Method of areas for manipulating the entanglement properties of one copy of a two-particle pure entangled state. Physical Review A 1999; 60(3): 1912-23.

Hardy L, Song DD. Entanglement-swapping chains for general pure states. Physical Review A 2000; 62(5): 052315.

Jonathan D, Plenio MB. Minimal conditions for local pure-state entanglement manipulation. Physical Review Letters 1999; Kok P, Munro WJ, Nemoto K, Ralph TC, Dowling JP, Milburn GJ. Linear optical quantum computing with photonic qubits. Reviews of Modern Physics 2007; 79(1): 135-74.

Ladd TD, Jelezko F, Laflamme R, Nakamura Y, Monroe C, O'Brien JL. Quantum computers. Nature 2010; 464(7285): 45-53.

Landauer R. Irreversibility and heat generation in the computing process. IBM Journal of Research and Development 1961; 5(3):183-191.

Lloyd S. Computational capacity of the universe. Physical Review Letters 2002; 88(23): 237901.

Lloyd S. Programming the universe. Alfred A. New York: Knopf, 2006.

Nielsen MA, Chuang I. Quantum computation and quantum information, Cambridge: Cambridge University Press, 2000.

Peres A. Quantum Theory: Concepts and Methods. Dordrecht : Kluwer, 1997.

Schmidhuber J. Computer universes and an algorithmic theory of everything. arXiv:1501.01373. 2015.

Shi BS, Jiang YK, Guo GC. Optimal entanglement purification via entanglement swapping. Physical Review A 2000; 62(5): 054301.

Shor P. Algorithms for quantum computation: discrete logarithms and factoring, in Proceedings of the 35th Annual Symposium on the Foundations of Computer Science, edited by S. Goldwasser (IEEE Computer Society, Los Alamitos, CA), 1996: 124.

Hooft GT. Quantum gravity as a dissipative deterministic system. Classical and Quantum Gravity 1999; 16(10): 3263-79.

Tittel W, Brendel J, Zbinden H, Gisin N. Violation of Bell inequalities by photons more than 10 km apart. Physical Review Letters 1998; 81(17):3563-66.

Vedral V. Decoding reality: the universe as quantum information. Oxford: Oxford University Press, 2010.

Wheeler JA. Information, physics, quantum: the search for links. In Zurek ed., Complexity, entropy, and the physics of information. Redwood City, California: Addison-Wesley, 1990.

Wolfram S. A new kind of science. Champaign: Wolfram media, 2002.

Yin J, Ren JG, Lu H, Cao Y, Yong HL, Wu YP, Liu C, Liao SK, Zhou F, Jiang Y, Cai XD. Quantum teleportation and entanglement distribution over 100-kilometre free-space channels. Nature 2012; 488(7410): 185-88.

Yin J, Cao Y, Li YH, Liao SK, Zhang L, Ren JG, Cai WQ, Liu WY, Li B, Dai H, Li GB. Satellite-based entanglement distribution over 1200 kilometers. Science 2017; 356(6343): 1140-44.

Zizzi PA. Spacetime at the Planck scale: The quantum computer view. The Foundations of Quantum Mechanics, Historical Analysis and Open Questions-Cesena 2004: Cesena, Italy 2006: 345-58.

Zukowski M, Zeilinger A, Horne MA, Ekert AK. ``Event-ready-detectors'' Bell experiment via entanglement swapping. Physical Review Letters 1993; 71: 4287-90.