



Using Cooperative Learning to Overcome Students' Misconceptions about Fractions

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ABSTRACT

The purpose of this study was to identify misconceptions about Fractions among Year 4 students. In addition, the study also investigated effectiveness of Cooperative Learning in correcting students' misconceptions about Fractions. This study was conducted among Year 4 students from two classes in a primary school in Johor Bahru. Sixty students (30 students for experimental group and 30 students for control group) of heterogeneous academic levels were involved in the study. Descriptive quantitative design was used in this study. A pre-test was given to students before treatment to investigate their misconceptions about Fractions. Students in the experimental group were taught for four weeks using STAD strategy which is a Cooperative Learning approach. However, students in the control group were taught for four weeks using a traditional method. A post-test was given after the treatment to investigate the effectiveness of Cooperative Learning in correcting students' misconception about Fractions. The quantitative data were presented in percentages. Results of the study show that there are four misconceptions about Fractions among Year 4 students. First, the bigger the number of denominators, the bigger the Fractions; second, the students viewed whole number as numerator in Fractions; third, the students viewed numerator and denominator as separate numbers and fourth, students failed to find a common denominator. In addition, the findings of this study also show that cooperative learning is effective in correcting students' misconceptions about Fractions. Therefore, Cooperative Learning is recommended to be used as a teaching strategy to overcome students' misconceptions about Fractions.

Key Words: Misconception, Cooperative Learning, Fractions

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Introduction

Students will not function effectively in their learning process if they do not possess strong foundation of knowledge, skills and dispositions (Siemon, 2003). 21st century learning require students to deal with texts which require some degrees of quantitative and spatial reasoning. Learning of Mathematics at primary level involves understanding of rational numbers and proportional reasoning such as Fractions, decimal, percentage, ratio and proportion. Thus, a strong sense of Fractions is fundamental to develop students' conceptual understanding of rational

numbers (Way, 2011). Students' conceptual understanding includes relationship between numbers, a sense of quantities represented by Fractions and flexibility with visual representations of Fractions. The term Fractions in research and educational literature refers to a collection of perceptions and conceptions about Fractions rather than a single Mathematical idea. However, Fractions are often associated with conceptual understanding rather than procedural understanding.

Learning is a process which builds upon students' prior knowledge. Throughout the

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learning process, children learn to understand, interpret and incorporate new information. Therefore, teachers have to observe and investigate students' mathematical thinking in solving fractional problems in order to improve their knowledge and understanding. If students go to school with knowledge of certain concepts, there is a possibility that the concepts that they have developed are partially developed or misconceived. Martinie, (2005) revealed that some students have misconception in Fractions which are derived from their previous knowledge and this interferes their understanding of rational number. In fact, most of the concepts which work with whole numbers can actually interfere how students think about Fractions.

When students exhibit misconceptions in Fractions, teachers need to adapt their lessons to correct students' misconceptions and guide their mathematical thinking in order to improve understanding (Wong and Evans, 2011). If teachers are unaware and have no attempt to teach and uncover students' misconceptions, chances are that students will leave their classroom with the same misconceptions that they have had when they entered the classroom. Unfortunately, misconceptions in Mathematics always go undetected in since there are teachers who receive little or no professional development in addressing and identifying misconceptions. Thus, addressing misconceptions is an important part in the process of teaching and learning.

Problem Background

Misconceptions always occur in the process of learning Mathematics because of the nature of Mathematics (Ojose, 2015). Misconceptions can persist over a long period of time which is not good for students and must be overcome. Teachers must be aware of this phenomenon and ensure that students do not have misconceptions for a long term. Research suggested that misconceptions which persist for years without realization would negatively affect students in learning Mathematics in the future. Luneta and Makonye, (2010) found that teaching and learning of Mathematics is so difficult and ineffective, they suspected that poor performance in Mathematics is correlated with students' errors and misconceptions. Battista, (2001) pointed out that the way in which students construct knowledge is dependent on the cognitive structures that the students have previously developed. This means that there are preconceptions and conceptions

that students of different backgrounds and ages bring with them to the Mathematics classrooms. If their preconceptions are misconceptions, teachers play an important role to reorganize students' conception about a particular topic.

Misconceptions are misunderstandings or misinterpretations due to naive theory that impeded the rational reasoning of students (Ojose, 2015). The nature of Mathematics is that the rules keep changing from a concept to another and from a mathematical operation to another. For example, when decimals introduce with addition, $0.6 + 0.7$ is 1.3 which has one decimal place, but when decimals introduce with multiplication, 0.6×0.7 is 0.42 which has 2 decimal places. This discrepancy from addition operation to multiplication operation in decimals can be for the cause of students' misconceptions. However, Mathematics textbooks do not treat this issue directly. Thus, these misconceptions occur over and over again in each topic. It is important that concerted efforts are made by Mathematics educators in correcting students' misconceptions. Teachers' awareness on this issue and effective corrective strategies will help in leading students to the correct way of thinking about Mathematical concepts.

Students around the world face difficulties in learning Fractions. Even in countries such as China and Japan where majority of their students achieve good conceptual learning, Fractions are still considered difficult for them. Alder and Setati, (2001) revealed that misconceptions arise when a teacher thinks a student is familiar with a concept whereas the student is in fact lack of understanding in certain aspects of it. For example, a student uses Fractions and able to obtain correct answers but they are not aware that Fractions are numbers. DeTurk, (2008) revealed that without a good foundation in Fractions, students who study rational expressions in Algebra will be severely handicapped. Students need to be well versed in the concepts of Fractions such as "how to read the Fractions" and "how to work with Fractions families". Students need to be taught Fractions because Fractions are important foundation underlying Algebra, and Algebra is the gateway to all higher Fractions.

Fraction is a topic which is difficult-to-teach and difficult-to-learn and thus those notions present ongoing pedagogical challenges to Mathematics education. Students have misconceptions in primary school and persist through secondary schools then into tertiary education (Empson and Levi, 2011). According to



Gould and Mitchelmore, (2006) difficulties and misunderstandings which students face in Fractions can persist into adulthood. Their misconceptions may pose problems in other wide-ranging fields such as computer programming, construction and health care. Besides that, the field of Science, Technology, Engineering and Mathematics (STEM) also demand considerable knowledge in Fractions. An unstable grounding in Fractions can prevent individuals from pursuing advanced Mathematics and shut them off from some career opportunities. Thus, helping students to achieve a solid ground in Mathematics in general and also specifically in Fractions can promise long-term high-stakes ramifications. As an educator, it is worth spending time and efforts in order to enhance student understanding in their elementary years and overcome their misconceptions to ensure their success in Mathematics, career and future undertakings.

Purnomo *et al.*, (2014) revealed that misconception happens due to students use their understanding of natural numbers to solve problems in Fractions. Locating natural numbers in the number line can be illustrated by placing dots with same distance among each other on the number line. However, it is different when it comes to Fractions. There would be infinite numbers of Fractions between two Fractions with different place values. This is the epistemological learning obstacle that has to be anticipated by Mathematics educators that the prior knowledge, which has been considered as the right concept to solve particular problems, later become inappropriate to be applied in solving another problem. Fractions are rational numbers and they do not hold place values as whole numbers (Siegler, Thompson & Schneider 2011). A rational number can be defined as number expressed by the quotient a/b of integers, where "a" is the numerator and "b" is the denominator. The denominator, b, is a non-zero integer. Children who have not yet learned about Fractions generally believe that the properties of whole numbers are the same for all numbers. However, it is a wrong conception. For example, 1 and 2 are two whole numbers yet there are infinite rational numbers within these two numbers. This is called as "whole number bias" where students use the properties of natural number to make inferences on rational number. This bias caused difficulties in conceptualizing whole number as decomposable units. There are differences between whole numbers and rational numbers. Whole numbers

form a discrete set whereas rational numbers are densely ordered set. There is infinity of rational numbers between two rational numbers but there is no other natural number between two natural numbers (Vamvakoussi and Vosniadou, 2004).

Gabriel *et al.*, (2013) pointed out that misconceptions about Fractions can happen among students due to the multi-faceted nature of Fractions. Developing a robust understanding of Fractions is an important aspect to develop Fractions sense among students. Lamon, (2007) cited that there are multiple interpretations of Fractions, such as Fractions as operators, Fractions as part-to-whole comparisons, Fractions as measures, Fractions as ratios and Fractions as quotients. These multiple interpretations and conceptual knowledge of Fractions resulted in confusion among students as they need to choose the correct notion for a specific problem. For example, a liter of cold drink shared equally among 7 students. The notion of the operator for division may be used which 1000 ml of cold drinks divided by 7 and each gets 142.85 ml. In the same problem, one can also pour cold drinks equally into 7 glasses. Students need to master the multi-faceted nature of Fractions in order to apply the correct procedures of problem solving. Siegler, Thompson and Schneider, (2013) explained that Fractions are difficult for most students because they assume that algorithms, properties and procedures of whole numbers are also properties of all other numbers. Dunzenli and Sharma, (2010) argued that the reason why students face difficulty in the addition of Fractions is due to the improper connections of mathematical problems.

Hecht and Vagi, (2012) revealed that the deficiency in the addition of Fractions is due to lack of conceptual knowledge. Besides that, Hallet *et al.*, (2012) also revealed that students always have imbalances in conceptual knowledge and procedural knowledge of Fractions. Previous studies revealed that students always perform calculations without knowing why they do it that way. In general, conceptual knowledge defined as knowledge of concept while procedural knowledge defined as the ability to execute action sequences to solve problems (Rittle-Johnson and Schneider, 2016). Students use their conceptual knowledge to acquire procedural knowledge. According to this approach, students use mathematical symbols without understanding their meanings. For example, students learn the correct procedures to multiply Fractions but they never seem to understand the underlying principles. In



mathematical teaching, teachers always focus more on procedural knowledge than conceptual knowledge. Thus, students only learn rote procedures in a repetitive way. This leads to a misunderstanding of mathematical symbols.

Problem Statement

Students' learning guides them to interpret, understand and incorporate new information. However, there is a possibility that the concepts that students developed are partially developed or misconceived which can lead to misconceptions. Fractions are known since ancient civilizations yet misconceptions about Fractions are still happening due to the multi-faceted nature of Fractions. In fact, many concepts which are working with whole numbers actually interfere how students think about Fractions. Misconceptions in Fractions can persist over a long period of time which is not good for students and must be overcome. Constructivism is the philosophical viewpoint about the nature of misconceptions which students have displayed in Fractions. In constructivism classrooms, students investigate and present their conclusions through exploratory activities. Consequently, the role of teachers is to encourage students to think out-of-the box by making their own connections in order to create meaningful understanding. Cooperative learning is one of the strategies utilized by constructivists whereby students work together in a small group to solve mathematical problems. In cooperative small groups, students work together, integrates new knowledge with prior knowledge, construct own meaning, explain, discuss and question new ideas. Moreover, cooperative learning also encourages students to build deeper understanding, use high-level reasoning and create supportive relationships with peer and develop self-esteem. Therefore, the first objective of this research was to investigate misconceptions of Year 4 students about Fractions. Next, the second objective was to study the effect of cooperative learning in correcting students' misconceptions about Fractions.

Student Teams Achievement Division (STAD)

Student Teams-Achievement Division (STAD) is one of the cooperative teaching models developed by (Slavin, 1980) at Johns Hopkins University. According to Slavin, (1981), by using STAD model, students are formed into groups of four or five members comprising of students with different skills and gender. Then, after each lesson, the

students work in groups to ensure that all group members have mastered the lessons given. Then, students have to answer quizzes based on the materials provided by the teachers and they have to work on their own without the help of other students. Students' scores are compared with their previous performances, and points are awarded on the basis of the degree to which students meet or exceed their earlier performances. These points are then summed to form team scores, and teams that meet certain criteria may earn certificates or other rewards. The latter element is a cardinal component of Slavin's methods that stress the significance of rewarding students in a manner calculated to improve their motivation to learn and their sense of accomplishment (Tan *et al.*, 2006).

Slavin, (2008) enumerated three main concepts of STAD as team rewards, individual accountability and equal opportunities for success. Team rewards are certificates or either rewards which are given if a STAD group achieves higher than the predetermined level. Therefore, spirit of positive competition is reinforced and all or none of the groups would be rewarded based on how they score. In terms of individual accountability, the individual learning of each group member determines the success of the teams.

STAD have been used in various subjects, from Mathematics to language, arts to social studies, and have been used from second grade through college. The STAD method is most appropriate for teaching well-defined objectives with single right answers, such as mathematical computations and applications, language usage and mechanics, geography and map skills, and science facts and concepts. According to Rai and Samsuddin, (2007), STAD is one of the many strategies in cooperative learning, which helps promote collaboration and self-regulating learning skills. STAD model encourages good interaction among students, positive attitude towards subject, better self-esteem and improves interpersonal skills (Narzoles, 2015; Khan, 2011). STAD also add an extra source of learning within the groups because some high achievers act as a role of tutor, which result in high achievements.

Slavin, (2008) revealed five stages in implementing STAD learning model which are as below:

1. Teacher Presentation. The teacher presents the material in front of the class in the classical style that focuses on the concepts of matter to be discussed.



2. **Team Study.** Students are formed into groups whose members are heterogeneous (both academic ability and gender). The trick is to rank students based on grades or the last value obtained before the student STAD cooperative learning models. The function of this grouping is to encourage cooperation in the group to study the materials and complete the tasks assigned by the teachers.
3. **Individual Quiz.** After team study, a quiz will be held with the objective to identify and to measure students' ability to learn the given topic. Students are not allowed to work with their friends. The purpose of this test is to motivate students to try and be individually responsible. Students are required to do their best as a result of group learning. In addition to individual responsibility, the students also have to realize that their success will be valuable since it contributes to the success of the group. This test is performed after one or two lessons and learning in group sessions.
4. **Individual Improvement Scores.** This is done to give the students a goal that can be achieved if they work hard and show better results as compared with previous results. In addition, students' group cooperation is determined by different methods such as looking at time of submission, test score and increase in the score of the group.
5. **Team's Achievement Recognition.** Awards will be given as appreciation to the efforts made by each group throughout the lessons.

Research Objectives

1. To identify the misconceptions of Year 4 students about Fractions.
2. To investigate the effectiveness of cooperative learning in correcting students' misconceptions about Fractions.

Methods

The purpose of this study was to identify the misconceptions encountered by Year 4 students about Fractions and also to investigate the effectiveness of cooperative learning in correcting students' misconceptions in Fractions. Hence, quantitative approach was used in this research.

Research Design

The research design used for this study was quasi-experimental design. Quasi experimental is one of the best quantitative designs to test the effect of treatment on its' target population. Two groups of respondents involved in this study which were the experimental group and control group. Figure 3.1 shows the design of this study. Pre-test with a set of questions regarding Fractions was given to the respondents in experimental group (A1) and control group (B1) before the treatment. Later, cooperative learning which was the treatment of this study was conducted to the experimental group while a traditional teaching method was given to the control group. Student Teams Achievement Division (STAD) was the treatment for this study and it was conducted to the experimental group for a duration of six weeks. This was followed by post-test with a set of questions regarding Fractions which was given to all respondents for both the experimental group (A2) and control group (B2).

Sample of Study

Due to the limitation of time and workforce, convenience sampling method was chosen for this research. Convenience sampling method is a non-probability sampling method which the sample may not represent the entire population accurately. Therefore, the results of the research cannot be used in generalizations pertaining to the entire population. By using convenience sampling method, the researcher selected target population for this research from a primary school located in Johor Bahru. A sample size of 60 respondents comprised of Year 4 students was chosen as the respondents in this research and they were equally divided to be in experimental and control groups. The respondents received three years of formal learning of Mathematics from Year 1 to Year 3 in primary school education. Although the samples were collected from two different classes, their performance in Mathematics test were at moderate level.

Research Instruments

The instruments used to collect data in this research are Student Teams Achievement Division (STAD) lesson plans, pre-test and post-test created by the researcher based on the research objectives. Four sets of lesson plans which were carried out separately throughout the six weeks of fieldwork. In addition, students were given pre-test and post-test. All respondents took pre-test



which was given before intervention while post-test was given after intervention. The pre-test was used to identify students' misconceptions about Fractions. On the other hand, post-test was used to investigate the effectiveness of Student Teams Achievement Division (STAD) in correcting students' misconceptions about Fractions. The pre-test and post-test contained 4 sections and there were 24 questions on Fractions. Items were created based on Test Specification Table and the items covered misconceptions encountered by Year 4 students in Fractions (Purnomo *et al.*, 2014; Hackenberg and Lee, 2012). All the items in pre-test and post-test had their validity check by three experts in Mathematics

Validity and Reliability

Validity and reliability of research instruments are very important to ensure trustworthiness and consistency of results. Therefore, the pre-test and post-test were checked by three experts who are teaching Year 4 Mathematics in primary schools. Items accepted by two or more experts were included in the study while items which were rejected by two or three experts were being revised as suggested by the teachers. The instruments for this study are divided into four sections namely Section A, Section B, Section C and Section D. There are six items in every section.

Table 1. Demographic Data of Participants

Demographic variables	%	N
Gender		
Male	45.0	27
Female	55.0	33
Study Groups		
Experimental group	50.0	30
Control group	50.0	30
Races		
Chinese	88.4	53
Malay	8.3	5
Indian	3.3	2

Demographic Background of Participants

Sixty students became the respondents of the study and they were divided equally to form experimental and control groups. All the participants performed averagely in their previous Mathematics tests. Among these 60 participants, 53 were Chinese; 5 Malays and 2 Indians. The numbers are shown in Table 1. They are students at one primary school in Johor Bahru. Hence, majority of the participants were Chinese. Equal number of participants was chosen for both experimental and control groups to avoid bias. The number of participants provided unique

opportunity to closely observe each individual throughout the study.

Results

This heading describes findings and discussions of this study. The data were analyzed and interpreted based on the research objectives: (1) to identify the misconceptions of Year 4 students about Fractions, and (2) to investigate the effectiveness of cooperative learning in correcting students' misconceptions about Fractions. The findings and discussion of the study are presented according to the sequence of the research questions.

Misconceptions about Fractions

There are four common misconceptions which can be usually found in the topic of addition and subtraction of Fractions [Purnomo *et al.*, 2014; Hackenberg and Lee, 2012]. Misconception I state that the bigger the number of denominator, the bigger the Fraction. Misconception II happens when students view whole number as numerator in Fractions. Misconception III is when students view numerator and denominator as separate numbers. Misconception IV happens when students fail to find a common denominator.

When comparing two Fractions, most participants compared denominator with denominator or numerator with numerator. They had misconception that a Fraction with larger number of denominator was bigger than the other Fraction with smaller number of denominator. Some participants also had misconception that a Fraction with larger number of numerator was bigger than the other Fraction with smaller number of numerator. In the pre-test, students were asked to circle the bigger Fractions for part A whereas and to circle the smaller Fractions in part B. Results showed that all participants in the experimental group and 90% of the participants in the control group had misconception I. It also concluded that most of the participants had misconception that the bigger the denominator, the bigger the Fraction.

Some participants have the misconception that a whole number was the numerator in Fractions. When participants were asked to convert a mixed Fraction to an improper Fraction in pre-test, they added the whole number with the numerator. They added the whole number 5 with the numerator $1(5+1=6)$, thus they chose the $6/8$ as the answer for the question. This misconception happened because participants had misconception that a whole number was also a numerator which



could be added. The proper procedure participants should just convert the whole number to a Fraction before added it with the other Fraction.

Most participants in the experimental group and control group had misconception III since they viewed numerator and denominator as separate numbers in addition and subtraction of Fractions. The participants did not know the meaning of different denominator. Participants assumed numerator and denominator as whole numbers which can be added. Thus, they added the numerator and denominator separately in the pre-test. This showed that most participants failed to recognize the relationship between denominators. Denominator is the number of equal parts which one whole is divided while numerator signifies the number of those parts.

Some of the participants had misconception in finding common denominator before adding or subtracting Fractions. This misconception led participants to convert denominator of Fractions wrongly thus led them to the wrong answer. Some of the participants had misconception to multiply numerator and denominator with the number of another numerator. They tried to find the common denominator by multiplying the numerator and denominator of $1/3$ with 5 while multiplying the other numerator and denominator of $5/6$ with 1. It was noticed that both 5 and 1 are the numerator for both Fractions. Participants did not understand the need of multiplication was to find a common denominator instead of multiplying the numerator.

Results of the study showed that misconception I and misconception III happened more frequently than misconception II and misconception IV. The number of participants who had misconception I and III was more than the number of participants who had misconception II and IV. All participants in the experimental group and 27 participants in the control group had misconception I (95% of the total number of participants). On the other hand, only 25 participants in the experimental group and 26 participants in the control group possessed misconception III (85% of the total number of participants). However, 8 participants in the experimental group and 14 participants in the control group had misconception II (37% of the total number of participants). Furthermore, 8 participants in the experimental group and 4 participants in the control group had

misconception IV (20% of the total number of participants).

Effectiveness of Cooperative Learning in Correcting Students' Misconceptions about Fractions

The second research question was to investigate the effectiveness of cooperative learning in correcting students' misconceptions about Fractions. Cooperative learning approach was conducted to the experimental group while traditional teaching was used in the control group. The pre-test and post-test performances of students were compared to investigate whether misconceptions of participants had been corrected.

Table 2 shows the number of participants with misconceptions in pre-test and post-test for the experimental group and control group. Results showed that 18 participants in the experimental group experienced corrected misconception I whereas only 1 participant in the control group had corrected misconception. For misconception II, 5 participants out of 8 participants in the experimental group managed to correct their misconception while 2 participants out of 14 participants in the control group corrected their misconception. In addition, 23 participants in the experimental group experienced corrected misconception III. None of the participant managed to correct misconception III but it was discovered that one more participant in the control group was examined to have misconception III. Lastly, 7 participants in the experimental group corrected misconception IV while 1 participant in the control group corrected misconception IV. It can be concluded that most participants in the experimental group corrected their misconceptions about Fractions while only a few participants in the control group corrected their misconceptions about Fractions. Therefore, cooperative learning is effective in correcting participants' misconceptions about Fractions.

The findings of the study showed that Year 4 participants had misconceptions in addition and subtraction of Fractions. There were four common misconceptions in addition and subtraction of Fractions. The four misconceptions were the bigger the number of denominator, the bigger the Fractions; viewed whole number as numerator in Fractions; viewed numerator and denominator as separate number; failed to find a common denominator. Among these four misconceptions, majority of the participants had misconceptions I and misconceptions III while minority of the



Table 2. Data Description for Experimental Group and Control Group

Misconceptions	Number of participants	
	Experimental group	Control group
I: The bigger the number of denominator, the bigger the Fraction		
With misconception	30	27
Without misconception	0	3
II: Viewed whole number as numerator in Fractions		
With misconception	8	14
Without misconception	22	16
III: Viewed numerator and denominator as separate number		
With misconception	25	26
Without misconception	5	4
IV: Failed to find a common denominator		
With misconception	8	4
Without misconception	22	26

Table 3. Comparison between Experimental Group and Control Group

Misconceptions	Number of Participants with misconception					
	Experimental Group			Control Group		
	Pre-test	Post-test	Difference	Pre-test	Post-test	Difference
Misconception I	30	12	- 18	27	26	- 1
Misconception II	8	3	- 5	14	12	- 2
Misconception III	25	2	- 23	26	27	+ 1
Misconception IV	8	1	- 7	4	3	- 1

participants had misconceptions II and misconceptions IV. The findings of this study also showed that cooperative learning helped to improve the participants' understanding about Fractions. Cooperative learning is effective in correcting participants' misconceptions in addition and subtraction of Fractions.

Discussion

Students tend to follow methods and rules taught to them without knowing and concerning about why and how those methods work (Sarwadi and Shahrill, 2014). There is a possibility that the concepts that students developed are partially developed or misconceived which can lead to misconceptions. Fractions have been known from ancient civilizations yet misconceptions about Fraction are still happening due to the multi-faceted nature of Fractions. In fact, many concepts which are working with whole numbers actually interfere how students think about Fractions. There are four misconceptions about Fractions which are discovered in this study and they are known as misconception I, misconception II, misconception III and misconception IV.

Misconception I

Misconception I in this study was the bigger the number of denominator, the bigger the Fraction. When comparing two Fractions, most participants compared denominator with denominator or numerator with numerator. They had misconception that a Fraction with larger number of denominator was bigger than the other

Fraction with smaller number of denominator. Some students also had misconception that a Fraction with larger number of numerators was bigger than the other Fraction with smaller number of numerator. This happened due to wrongly ordering unit Fractions. The magnitudes of Fractions do not increase in any consistent way with the size of their components. We can tell that 7 is greater than 4, but a Fraction with a numerator of 7 may or may not be larger than a Fraction with a numerator of 4 (Schneider & Siegler, 2010). All of these properties seem likely to interfere with formation of a mental number line for Fractions. Students do not understand that the denominator tells how many parts the whole has been divided into. In fact, the more parts there are, the smaller each portion will be. Besides that, students have this misconception also due to whole number bias. Whole number bias is an instance of intuitive processing that predicts items incongruent with whole number reasoning. Fractions differ from whole numbers in many mathematical ways that could easily affect how they are represented. Fractions are infinitely divisible and linked by successor relations; no Fraction comes immediately before or after another Fraction. The misconception that commonly happened in the students' mind was that they used their understanding about the natural numbers in the number line to solve any Fractions problem. The nature of Fraction requires the understanding that there are infinite Fractions between two Fractions with different



proportion (Purnomo *et al.*, 2014). This makes it impossible to count Fractions directly.

Misconception II

Misconception II in this study was viewed whole number as numerator in Fractions. Students had misconceptions that a whole number could be added to numerator directly. As a result, students failed to convert a whole number to Fraction. This happened because students did not know the relationship between the whole number and Fractions. In operations with mixed numbers, many students ignore the fractional parts and focus only on the whole numbers (Fazio & Siegler, 2011). Researchers argued that understanding rational numbers required conceptual change (Christou & Vosniadou, 2014). This initial number concept encompasses a number of background assumptions and beliefs which underlie students' expectations about what counts as a number and how it is supposed to behave which need to be re-interpreted in the process of understanding rational number.

Rational number information coming from instruction violates basic principles of the whole number concept. Thus, understanding rational number requires a restructuring of the whole number concept and the construction of a new representation of rational number on the number line. This process of restructuring takes a long time to be accomplished and leads to the formation of misconceptions. It reveals students' attempts to assimilate incompatible information about rational number into their initial whole number concept (Stafylidou and Vosniadou, 2004). A new representation is eventually constructed. However, this new representation is not robust and does not replace the initial representation of whole number but co-exists with it.

Misconception III

Misconception III in this study viewed numerator and denominator as separate numbers in addition and subtraction of Fractions. Students did not know the meaning of different denominator. Students assumed numerator and denominator as a whole number that can be added up. Thus, they added numerator and denominator separately. This showed that students failed to recognize the relationship between denominators. Denominator is the number of equal parts which one whole is divided while numerator signifies the number of those parts.

Children's difficulties with Fractions have been associated with the whole number bias (Ni and Zhou, 2005). Many Fraction misconceptions have their roots in children's belief that properties of whole numbers can be applied to Fractions. For example, children do not understand fractional notation; they often treat Fractions' numerators and denominators as two separate whole numbers, believe that the value of the Fraction increases when either the numerator or the denominator increase, and think that the unit is the smallest Fraction (Stafylidou and Vosniadou, 2004).

The same discrepancy between knowledge of whole numbers and Fractions is evident in arithmetic problems: A long search would be required to find a second grader who would claim that $3 + 3 = 6$, yet middle school students frequently claim the equivalent when they write that $1/3 + 1/3 = 2/6$ [37]. In all of these cases and many others, children confuse the characteristics of Fractions with those of whole numbers, a characteristic that has been labeled the whole number bias (Ni and Zhou, 2005).

Misconception IV

Misconception IV is failed to find a common denominator. This misconception led students to convert denominator of Fractions wrongly thus led them to the wrong answer. Some of the students had misconception to multiply denominator with another denominator to get the common denominator. This happened since students did not understand the need of multiplication was to find a common denominator instead of multiply the denominator. Besides that, students often fail to convert Fractions to a common denominator before adding or subtracting them, and instead just use the larger of the 2 denominators in the answer. This is because students do not understand that different denominators reflect different-sized unit Fractions and that adding and subtracting Fractions require a common unit Fraction.

Effectiveness of Cooperative Learning in Correcting Students' Misconceptions about Fractions

Findings from this study indicated that cooperative learning had significant effects in correcting students' misconceptions in Fractions. The experimental group showed significant improvement in students' Mathematics understanding and attitudes towards mathematics in comparison to the control group.



The result suggested that the corrected misconceptions of students about Fractions were due to the significant effects of cooperative learning. Cooperative learning promotes the move away from a passive approach to learning which allow students to become stakeholders in their learning process. Traditional classroom focuses on rote learning and memorization which is more teacher-centered method. However, cooperative learning groups are groups which work together to accomplish a shared goal. When students work together in a group, they take responsibility in their learning thus encounter meaningful learning. Artzt and Newman, (2006) revealed that cooperative learning encourages students to work together as a team to solve problems, complete tasks and accomplish the same goals. When students try to solve problem, they notice each other misconceptions and try to correct them. This enables students to work together and arrive at the final solution on the basis of teamwork.

With an abundance of standards to cover, Mathematics instruction has traditionally been paper and pencil exercises. Bernero, (2000) noted that many students find Mathematics boring because it tends to be individualized work with repetitive assignments. Bednar, Coughlin, Evans and Sievers, (2002) stated that traditional approaches to instruction do not allow every student the opportunity to reach his or her full potential in their learning. Pinzker, (2001) noted that when students are taught to follow specific procedures to solve problems, they are not learning and understanding the concepts involved. Often, teachers explain steps or a procedure to solve problems, before students fully grasp the meaning behind what they were doing. Students need to be able to understand meaning, especially with the mathematical concept of Fractions before they are able to perform algorithms with proficiency (Aksu, 1997). Teachers should be able to teach mathematical concepts in such a way that students can apply their knowledge to real life situations.

The findings of this study showed that students with misconceptions had corrected their misconception after being taught using cooperative learning. Cooperative learning is a successful teaching strategy in which small teams, each with students of different levels of ability, use a variety of learning activities to improve their understanding of a subject (Anthony, 2013). Each member of a team is responsible not only for learning what is taught but also for helping team

members in their learning, thus creating an atmosphere of achievement. In a cooperative learning classroom, the learning environment is structured in a way to ensure students work together and are able to see diverse viewpoints or ideas. Group work is not complete until everyone has mastered the concept. Thus, it provides opportunity for students who have misconceptions to correct their misconceptions in order to master it. By starting students out with small amount of time and building to longer periods of time to work as a group will help students feel more confident working with others. Training in questioning has been practiced in the STAD cooperative learning group. A study done by (Gillies and Haynes, 2011) found that students who received training in questioning had more advances in reasoning and problem-solving abilities. Talked-based activities, such as those which occur during cooperative group work, can be useful in scaffolding and development of reasoning and understanding. "There is no doubt that children are more interactive and learn more when they have been taught to communicate as they work on common tasks" (Gillies & Haynes, 2011). The study showed that students needed to be taught how to engage in meaningful arguments. The study also showed the importance of being explicit when implementing cooperative learning in the classroom. If a teacher uses specific strategies within a cooperative learning environment, students are more likely to be engaged in more elaboration and achieve better understanding about Mathematics.

This is also because students practice positive social interdependence in the cooperative group. Positive social interdependence implies that students' outcomes are affected by their own and others' actions (Johnson *et al.*, 2008). This interdependence can be structured in various ways within a group. It requires students to work towards a common goal, and they perceive that they can achieve this goal only if all the members of their group attain their individual goals. This positive goal interdependence can be defined in terms of either a joint product or the mastery learning of all members. Thus, students are able to correct their misconceptions indirectly to master the topic. Positive interdependence can be reinforced by other dimensions, such as sharing complementary resources, being responsible for a delimited part of the task, or endorsing a specific responsibility. Individual responsibility involves each member contributing and being held



accountable for his/her own learning and that of others (Kagan and Kagan, 2000). Assigning specific roles to team members, identifying each other's contributions, and assessing individual learning are some of the ways that individual responsibility can be increased (Bennett *et al.*, 1991).

When students solve problem independently, they tend to overwhelmingly rely on their own biased personal experience in order to find the solution while missing or rarely rendezvousing a different perspective on the same issue (Cheng, 2011). It is often seen that even when alternative solutions come across their minds they tend to be overlooked due to the existence of the first solution already in mind which is their misconceptions. However, in cooperative learning, a team composed of subjects with different Mathematical knowledge, experiences, background and thinking patterns. This will help each other to take advantage of the complimentary views of others in the epistemic process, benefiting from multichannel communication and further polishing their capability in problem solving. Cheng, (2011) believed that under the circumstances of cooperative learning, thinking independently and cooperative communication nurture each other. It is especially important to see the independency of thinking in the processes of students' representing, listening and discussing. In fact, teachers tend to reserve a certain amount of time for students to be able to think independently prior to communication; however, they often overlook students' independency during discussion.

Peer interaction is the key to cooperative learning. Cooperative learning deals with task structure and requires students to work together. Slavin, (1987) stated that providing time for students to study together does not increase achievement but students who give and elaborate explanation gain the most from cooperative learning. When working individually on math problems, students tend to give up when concepts and tasks become difficult (Pinkzer, 2001) If students have each other to use as resources, they are more inclined to step up to the challenge and persist with trying to work on a problem. Students can work together and share ideas in order to understand the material presented to them, as well as correct their misconceptions in Fractions.

Moreover, group goals and individual accountability are two features which make cooperative learning work. In Student Team Learning, the important thing is not doing something together but to learn something as a team. When students learn together in a team, they help each other to understand a concept better and thus correct their peers' misconceptions about Fractions. Slavin, (2010) found that students feel more successful when working in groups and working with other types of students. Those students who gain the most out of cooperative groups are those students who are willing to give and receive. The motivational aspect to cooperative learning should not be overlooked. Some students like to cooperate with their peers (Gardner, 1999). Teachers, in an attempt to provide for students' needs for affiliation, autonomy, and physical activity may also use cooperative learning strategies for students' needs to be social.

STAD is a cooperative teaching method which was developed by (Slavin, 1981) and was implemented in this study. In STAD, students are assigned to work in teams. The teams compose of high, average, and low performing students, and of boys and girls of different racial or ethnic backgrounds. Thus, each team is a microcosm of the entire class. There are five main steps implemented in STAD. The teacher first introduces new materials to be learned. The team members then study worksheets on the material until they master the material. Individual quizzes are taken on the material being studied. The teacher then combines the scores to create team scores. Members of the team with highest scores are given compliments and rewards. Students tend to put more effort when their marks are contributed to the group to make higher scores. Thus, students work hard for the group with the help of their group members. Students share their ideas to the students with misconceptions so that they can correct their misconceptions to master the topic.

In term of learning achievement using the STAD, a study of (Keramati, 2009), entitled "The effect of cooperative learning on academic achievement of physics course", it is found that experimental group students taught by cooperative learning (STAD technique) were more successful than the control group students. At this point, it was found that cooperative learning increased academic achievement of students to a higher level when compared to

conventional teaching method (Keramati, 2009). STAD stands for student team achievement divisions; it is a collaborative learning strategy in which small groups of learners with different levels of ability work together to accomplish a shared learning goal. STAD enhanced learning achievement and increased social skills of students to work together in collaborative groups. Students work together in the group to share their knowledge and make sure every member master the topic. The more they work together, the higher chances for students who have misconceptions can correct their misconceptions in Fractions. This is because the more they work, the more they understand, retain, and feel better about themselves and their peers. Moreover, working together in a collaborative environment encourages student responsibility for learning.

"Research on cooperative learning is more than sufficient to justify the practical use of these methods to accelerate student's achievement, but much work still lies ahead to understand fully why and how the methods effect students' learning" (Slavin, 1987). The importance of having independent accountability makes cooperative learning more effective. Students take responsibility in their learning thus they learn from each other in the group and correct their misconceptions indirectly. Besides that, cooperative learning enhances intergroup relationships, social acceptance, and friendship among students. "Outcomes seen in many studies of cooperative learning include gains in self-esteem, liking of school, time on-task, and attendance" (Slavin, 1990). With cooperative learning being implemented in classrooms, all students can be successful, even high achievers and students with misconceptions. Therefore, cooperative learning is beneficial for everyone.

Conclusions

Based on the present research, the researcher concluded that Year 4 participants had misconceptions in addition and subtraction of Fractions. There are four common misconceptions in addition and subtraction of Fractions such as the bigger the number of denominator, the bigger the Fractions; viewed whole number as numerator in Fractions; viewed numerator and denominator as separate number; failed to find a common denominator. Misconceptions about Fractions occurred among students due to multi-faceted nature of Fractions, wrongly ordering unit of Fractions and whole number bias. In this

research, cooperative learning showed significant and positive effects in correcting students' misconceptions about Fraction. This is because students become the centre of learning and are self-motivated and autonomous in their mathematical learning process. Through cooperative learning, knowledge is actively constructed by students and they take responsibility in their studies. Besides that, peer encouragement is also important in cooperative learning whereby students share their ideas and knowledge with each other. Therefore, students can realize about their misconception and reconstruct new understanding about Fractions. Teacher should be aware of the potential and chances that students generate misconceptions in Fractions. Therefore, hard work and interventions should be given to perceive and overcome students' misconceptions.

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