Perturbative Generation of the Neutrino Reactor Mixing Angle (θ_{13})

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Abstract

The Standard Model of particle physics considers three fundamental forces to describe the behaviour of the elementary particles. The Standard model was quite satisfactory till the neutrino oscillation came into play. According to the Standard Model of particle physics, neutrinos are massless and chargeless particles. It is after the discovery of neutrino oscillations that leads to the hints of neutrinos having non zero mass. Earlier hints of neutrino oscillations came from the Homestake experiment when the observed flux of neutrinos detected on earth does not match with the flux of neutrinos proposed by the standard solar model. This became the key to the solar neutrino problem. The concept of neutrino oscillations was given as the resolution of the solar neutrino problem. Earlier experimental data of the neutrino oscillations was correctly explained by the Tri-Bimaximal mixing ansatz. One of the predictions of TBM ansatz was the vanishing reactor angle 13. After the measurement of non-zero reactor angle, the TBM texture does not remain experimentally viable. However, we can still use the TBM as the leading order matrix and then study its perturbations to generate the non-zero reactor angle. In the current study, we try to explore the perturbations of various terms of TBM to generate the non-vanishing reactor angle. 84

Keywords PMNS Matrix, Perturbation, CP Violation, Majorana Masses, Tribimaximal Matrix, Neutrino Oscillation.

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INTRODUCTION

While studying β - decay, the theoretically predicted energy of an electron is calculated to be far different from the data obtained experimentally. Theoretically predicted energy of an electron is calculated to be

$$\mathbf{E_e} = \left[\frac{m_A^2 - m_B^2 + m_e^2}{2m_A}\right] c^2$$

Experimental data for the β - decay reaction is shown as;

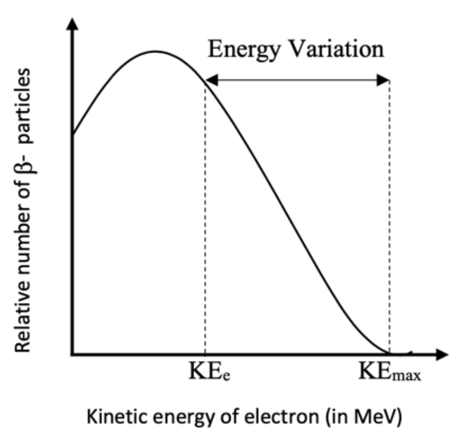

Figure 1. Kinetic Energy of electron (in MeV)

Moreover, it violates the conservation laws including linear momentum, spin angular momentum and the parity principle too. At that time it is a difficult task to explain the β - decay process, then Wolfgang Pauli came up with an idea that if we introduce another particle in β - decay reaction having very tiny mass, uncharged and must have half-spin i.e., this particle must be a fermion, we can tackle such violation.

$${}_{0}n^{1} \rightarrow {}_{1}p^{1} + {}_{-1}e^{0} \qquad (2)$$

Later on this idea was taken seriously by Enrico Fermi and gave the theoretical concept of a particle called neutrino - the little neutral particle but it was actually anti neutrino.

$${}_{0}n^{1} \rightarrow {}_{1}p^{1} + {}_{-1}e^{0} + \nu \qquad (3)$$

At the beginning of 20*th* century it has been found that neutrino exits in three flavor, it may be electron - neutrino, muon - neutrino and tau - neutrino. According to the standard model, the neutrino is considered to be a massless and chargeless particle. After the discovery of neutrino oscillation, the whole story gets changed. i.e., We come up to the conclusion that the neutrons are not massless anymore. Solar neutrino oscillation became the challenge at that time which was solved about a century later (in the Solar neutrino oscillation experiment, measured neutrino coming from the sun is much less than the calculated value at earth and is nearly $2/3^{rd}$ of theoretically predicted value). Along with the Solar Neutrino Oscillation, Atmospheric neutrino oscillation also came into play. Both of the processes are found to be consistent with the tribimaximal form of mixing matrix U of the lepton sector. Tribimaximal matrix is a specific postulate form for the PMNS laptop matrix U in which all the elements of the PMNS matrix should be taken in moduli squared form as

$$\begin{bmatrix} |U_{e1}|^2 & |U_{e2}|^2 & |U_{e3}|^2 \\ |U_{\mu1}|^2 & |U_{\mu2}|^2 & |U_{\mu3}|^2 \\ |U_{\tau1}|^2 & |U_{\tau2}|^2 & |U_{\tau3}|^2 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} & 0 \\ \frac{1}{6} & \frac{1}{3} & \frac{1}{2} \\ \frac{1}{6} & \frac{1}{3} & \frac{1}{2} \end{bmatrix} \qquad (4)$$

$$(U^{0})^{2} = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} & 0 \\ \frac{1}{6} & \frac{1}{3} & \frac{1}{2} \\ \frac{1}{6} & \frac{1}{3} & \frac{1}{2} \end{bmatrix} \qquad (5)$$

$$U^{0} = \begin{bmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{6}} & -\sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{bmatrix} \qquad (6)$$

This mixing is presently excluded by experiments at the level of 5σ . It is clear from the above matrix that the third mixing θ13 is completely vanishing. Tribimaximal mixing angle θ13 should be taken as zero which is quite true before the experimental results obtained from 127 days exposure of Daya Bay collaboration and 229 days data of RENO which established a non – zero value of θ13. Such form of a matrix is compatible with the older neutrino oscillation experiments used as a zeroth-order approximation to more general forms for the PMNS matrix which are also consistent with the given data. In the PDG convention for the PMNS matrix, tribimaximal mixing may be specified in terms of lepton mixing angles are, Table 1.

Table 1. Prediction of the TBM mass matrix

Parameter	TBM predictions
θ_{31}	$sin^{-1}(1/\sqrt{3}) \approx 35.3\,450$
θ_{23}	0
θ_{13}	0
δ	0

Experimentally θ_{13} was found to be non-trivial i.e., $\theta_{13} \approx 8.5$ which falsified the above pre- diction. The non - zero value of θ_{13} mixing angle is quite small compared to the other neutrino mixing angles.

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Experimental results obtained from Daya Bay and RENO experiment are as

***sin*2 2θ13 = 0.089 ± 0.010(*St. at*) ± 0.005(*syst.*) (Daya Bay)**

***sin*2 2θ13 = 0.113 ± 0.013(*St. at*) ± 0.019(*syst.*) (RENO)**

Moreover, some other experiments like Double Chooz, MINOS and T2K experiments also show the non – zero value of θ_{13} same as above mentioned experiments but with greater uncertainties. In the end, it has to be concluded that the tribimaximal matrix operation fails to explain the observed neutrino mixing. The smallness of θ_{13} compared to the other two mixing angles θ_{21} and θ_{31} encourage us to examine a small perturbation on the tribimaximal structure which leads to a realistic neutrino mixing matrix. The charged lepton mass matrix is diagonal in flavor basis. If m_1, m_2, m_3 be the left-handed neutrino Majorana masses and M^0 be the mass matrix satisfying tribimaximal mixing, then

$$M^0 = U^0 \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} (U^O)^T \qquad (7)$$

$$U^0 = \begin{bmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{6}} & -\sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{bmatrix} \qquad (8)$$

$$(U^0)^T = \begin{bmatrix} \sqrt{\frac{2}{3}} & -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{6}} \\ \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{3}} \\ 0 & \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \end{bmatrix} \qquad (9)$$

$$M^0 = \begin{bmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{6}} & -\sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{bmatrix} \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \begin{bmatrix} \sqrt{\frac{2}{3}} & -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{6}} \\ \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{3}} \\ 0 & \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \end{bmatrix} \qquad (10)$$

$$M^0 = \begin{bmatrix} \sqrt{\frac{2}{3}}m_1 + 0 + 0 & 0 + \sqrt{\frac{1}{3}}m_2 + 0 & 0 + 0 + 0 \\ -\sqrt{\frac{1}{6}}m_1 + 0 + 0 & 0 + \sqrt{\frac{1}{3}}m_2 + 0 & 0 + 0 + \sqrt{\frac{1}{2}}m_3 \\ \sqrt{\frac{1}{6}}m_1 + 0 + 0 & 0 - \sqrt{\frac{1}{3}}m_2 + 0 & 0 + 0 + \sqrt{\frac{1}{2}}m_3 \end{bmatrix} \begin{bmatrix} \sqrt{\frac{2}{3}} & -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{6}} \\ \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{3}} \\ 0 & \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \end{bmatrix} \qquad (11)$$

$$M^0 = \begin{bmatrix} \sqrt{\frac{2}{3}}m_1 & \sqrt{\frac{1}{3}}m_2 & 0 \\ -\sqrt{\frac{1}{6}}m_1 & \sqrt{\frac{1}{3}}m_2 & \sqrt{\frac{1}{2}}m_3 \\ \sqrt{\frac{1}{6}}m_1 & -\sqrt{\frac{1}{3}}m_2 & \sqrt{\frac{1}{2}}m_3 \end{bmatrix} \begin{bmatrix} \sqrt{\frac{2}{3}} & -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{6}} \\ \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{3}} \\ 0 & \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \end{bmatrix} \qquad (12)$$

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$$M^0 = \begin{bmatrix} \frac{2}{3}m_1 + \frac{1}{3}m_2 & -\frac{1}{3}m_1 + \frac{1}{3}m_2 & \frac{1}{3}m_1 - \frac{1}{3}m_2 \\ -\frac{1}{3}m_1 + \frac{1}{3}m_2 & \frac{1}{6}m_1 + \frac{1}{3}m_2 + \frac{1}{2}m_3 & -\frac{1}{6}m_1 + \frac{1}{3}m_2 + \frac{1}{2}m_3 \\ \frac{1}{3}m_1 - \frac{1}{3}m_2 & -\frac{1}{6}m_1 - \frac{1}{3}m_2 + \frac{1}{2}m_3 & \frac{1}{6}m_1 + \frac{1}{3}m_2 + \frac{1}{2}m_3 \end{bmatrix} \quad (13)$$

$$M^0 = \begin{bmatrix} \frac{2m_1+m_2}{3} & \frac{m_2-m_1}{3} & \frac{m_1-m_2}{3} \\ \frac{m_2-m_1}{3} & \frac{m_1+2m_2+3m_3}{6} & \frac{-m_1+2m_2+3m_3}{6} \\ \frac{m_1-m_2}{3} & \frac{-m_1-2m_2+3m_3}{6} & \frac{m_1+2m_2+3m_3}{6} \end{bmatrix} \quad (14)$$

Let us assume that $m_0 = \left(\frac{m_1+m_2+m_3}{3}\right) \Delta_{32} \equiv (m_3 - m_2)\Delta_{31} \equiv (m_3 - m_1)$.

$$\therefore M^0 = \begin{bmatrix} m_0 - \frac{\Delta_{31}}{3} & \frac{\Delta_{31}-\Delta_{32}}{3} & -\frac{\Delta_{31}-\Delta_{32}}{3} \\ \frac{\Delta_{31}-\Delta_{32}}{3} & m_0 + \frac{\Delta_{31}}{6} & \frac{\Delta_{31}+2\Delta_{32}}{6} \\ -\frac{\Delta_{31}-\Delta_{32}}{3} & \frac{\Delta_{31}+2\Delta_{32}}{6} & m_0 + \frac{\Delta_{31}}{6} \end{bmatrix} \quad (15)$$

Here m_1, m_2, m_3 the mass eigenvalues can be complex and we can rendered them as real and positive by diagonal phase transformation, i.e., D = *diag.* ($e_\iota\lambda_1, e_\iota\lambda_2$,1) ; where λ_ι are the Majorana phases which do not affect the neutrino oscillations.

Now we approximate $\Delta_{32} \approx \Delta_{31} \equiv \Delta$ because of the fact that $|\Delta_{32}| >> \Delta_{31} \equiv (m_2 - m_1)$. Here Δ sets the scale for atmospheric neutrino oscillations.

By using such a limit we are going to unperturbed the mass matrix in the flavor basis as;

$$M^0 \approx \begin{bmatrix} m_0 - \frac{\Delta}{3} & 0 & 0 \\ 0 & m_0 + \frac{\Delta}{6} & \frac{\Delta}{6} \\ 0 & \frac{\Delta}{2} & m_0 + \frac{\Delta}{6} \end{bmatrix} \quad (16)$$

Till now $m_1^{(0)} = m_2^{(0)} = m_0 - \frac{\Delta}{3}$, $m_3^{(0)} = m_0 + \frac{2\Delta}{3}$ and the solar mass splitting is also absent.

Now our aim is to generate this splitting using the same perturbation Hamiltonian which is responsible for $\theta_{13} \neq 0$. For this we take $m_1^{(0)}$, $m_2^{(0)}$ *and* $m_3^{(0)}$ to be real and positive.

Here M^0 emerges from the fundamental model and we are considering it as the dominant part of the neutrino mass matrix. Our aim is to discuss the independence of the specific mechanism by which *M*0 arises over the other models obtained from the tribimaximal form of the mixing matrix. For this, Parameterized Pontecorvo, Maki, Nakagawa, Sakata (PMNS) by three mixing angles (θ_{12}, θ_{23} and θ_{13}) and a single-phase angle called δ_{CP}(complex phase) related C.P violation can be written as

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$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{CP}} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (17)$$

$$= \begin{bmatrix} c_{13} + 0 + 0 & 0 + 0 + 0 & s_{13}e^{-i\delta_{CP}} + 0 + 0 \\ 0 + 0 - s_{23}s_{13}e^{i\delta_{CP}} & 0 + c_{23} + 0 & 0 + 0 + s_{23}c_{13} \\ 0 + 0 - c_{23}s_{13}e^{i\delta_{CP}} & 0 - s_{23} + 0 & 0 + 0 + c_{23}c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (18)$$

$$= \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{CP}} \\ -s_{23}s_{13}e^{i\delta_{CP}} & c_{23} & s_{23}c_{13} \\ -c_{23}s_{13}e^{i\delta_{CP}} & -s_{23} & c_{23}c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (19)$$

$$= \begin{bmatrix} c_{13}c_{12} + 0 + 0 & c_{13}s_{12} + 0 + 0 & 0 + 0 + s_{13}e^{-i\,\delta_{CP}} \\ -s_{23}s_{13}c_{12}e^{i\,\delta_{CP}} - c_{13}s_{12} + 0 & -s_{23}s_{13}s_{12}e^{i\,\delta_{CP}} + c_{23}s_{12} + 0 & 0 + 0 + s_{23}c_{13} \\ -c_{23}s_{13}c_{12}e^{i\,\delta_{CP}} + s_{23}s_{12} + 0 & -c_{23}s_{13}s_{12}e^{i\,\delta_{CP}} - s_{23}c_{12} + 0 & 0 + 0 + c_{23}c_{13} \end{bmatrix} \quad (20)$$

$$= \begin{bmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\,\delta_{CP}} \\ -s_{23}s_{13}c_{12}e^{i\,\delta_{CP}} & -s_{23}s_{13}s_{12}e^{\,i\delta_{CP}} + c_{23}s_{12} & s_{23}c_{13} \\ -c_{23}s_{13}c_{12}e^{i\,\delta_{CP}} + s_{23}s_{12} & -c_{23}s_{13}s_{12}e^{i\,\delta_{CP}} - s_{23}c_{12} & c_{23}c_{13} \end{bmatrix} \quad (21)$$

It is clearly visible that the $U_{e3}{}^0$ taken as zero in the tribimaximal mixing matrix is actually non zero. The role of non-vanishing U_{e3} or equivalently θ_{13} becomes the gateway for many researchers. Its non-zero nature is essential for CP non - conservation in neutrino oscillations and may be the inspiration to explain the leptogenesis. Moreover, the non-zero nature of θ_{13} will be the same as the quark sector where mixing between all the three generations and CP violation is a well-verified result, even though the mixing angles of both the sectors are vastly different. In case of CP violation, both the θ_{13} and the complex phase δ_{CP} should be non - vanishing.

PERTURBATION

Numerous attempts have been made to generate the non-zero nature of θ_{13} i.e., $\theta_{13} \neq 0$. among which we choose perturbation theory to identify the structure of Majorana Mass matrix M = M^0 + M', where $M' << M^0$, so that θ_{13} and solar mass splitting are obtained. It should be noted that both the M^0 and M' will be symmetric and complex. However we have seen in the tribimaximal mixing form, M^0 is real and symmetric i.e., Hermitian. We have to handle these two cases separately.

Moreover, Eigenstates of M^0, the unperturbed mass eigenstates, in the mass basis are simply:

$$\psi_1^{(0)} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \psi_2^{(0)} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \psi_3^{(0)} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad (22)$$

where, the basis vector $\psi_1{}^{(0)}$ and $\psi_2{}^{(0)}$ are degenerate i.e., they are not unique and are chosen to reproduce the correct solar mixing. In terms of flavor basis these eigenstates are simply the column of U 0 which can be written as:

$$\psi_1^{(0)} = \begin{pmatrix} \sqrt{\frac{2}{3}} \\ -\sqrt{\frac{1}{6}} \\ \sqrt{\frac{1}{6}} \end{pmatrix}, \ \psi_2^{(0)} = \begin{pmatrix} \sqrt{\frac{1}{3}} \\ \sqrt{\frac{1}{3}} \\ -\sqrt{\frac{1}{3}} \end{pmatrix},$$

$$\psi_3^{(0)} = \begin{pmatrix} 0 \\ \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{2}} \end{pmatrix} \quad (23)$$

2.1 First Order Perturbation

Firstly we are considering M' as symmetric, real and therefore Hermitian by which we may generate a non-zero θ_{13} without having CP - violation and hence yields $\delta = 0$.

Consider the physical system having Hamiltonian $M0$ corresponding to free system with eigen basis $|\psi_3{}^{(0)}>$ i.e., $|\psi_3{}^{(0)}>$ will be the eigenket of M^0 with eigen value $m_0{}^{(0)}$ such that

$\boldsymbol{M^0\psi_3{}^{(0)}> = m^{(0)}|\psi^{(0)}>}$ **(Time dependent Schrodinger's Equation)**

To first order we have,

$$\psi_3 = \psi_3^{(0)} + \sum_{n\neq 3} C_{n3}\psi_n^{(0)} \quad (24)$$

We know that

$<\psi_3{}^{(0)}|\psi_j{}^{(0)}> = \delta_{3j}$ (Orthonormality property)

where, $\delta_{3j} = 0$ for j = 3

= 1 for $3 \neq j$

Also, $\sum n\ |\psi_3{}^{(0)}>< \psi_3{}^{(0)}| = 1$ (Completeness property)

Now we perturbed the Majorana mass slightly such that

$M = M0 + \lambda M'$; $0 \leq \lambda \leq 1$ (25)

Here λ is an arbitrary constant (perturbation constant). It defines the strength of perturbation.
Let us consider new eigen function and eigen value as $M|\psi_3\rangle = m_0|\psi_3\rangle$
where, $|\psi_3\rangle$ (3rd new state of full Hamiltonian) = $|\psi_3^{(0)}\rangle$ (3rd old basis)+ $|\Delta\psi_j\rangle$ (same quantity which can be extended in terms of old basis)
So, the perturbation does not change the Hilbert space of the system, it means we can retain the old basis set.
Expand eigen function $|\psi_3\rangle$ and eigen value *m*0 as the power series in terms of λ , we get

$$|\psi_3\rangle = |\psi_3^{(0)}\rangle + \lambda|\psi_3^{(1)}\rangle + \lambda^2|\psi_3^{(2)}\rangle + \lambda 3|\psi_3^{(3)}\rangle + \lambda^4|\psi_3^{(4)}\rangle + - - - \quad (26)$$

$$m_0 = m_0^{(0)} + \lambda m_0^{(1)} + \lambda 2 m_0^{(2)} + \lambda 3 m_0^{(3)} + \lambda 4 m_0^{(4)} + - - -. \quad (27)$$

We know that

$M|\psi_3\rangle = m_0|\psi_3\rangle$

But $M = M^0 + \lambda M'$; $0 \le \lambda \le 1$

Therefore, above equation becomes

$$(M^0 + \lambda M')(|\psi_3^{(0)}\rangle + \lambda|\psi_3^{(1)}\rangle + \lambda^2|\psi_3^{(2)}\rangle + \lambda^3|\psi_3^{(3)}\rangle + \lambda^4|\psi_3^{(4)}\rangle = (m_0^{(0)} + \lambda m_0^{(1)} + \lambda^2 m_0^{(2)} + \lambda^3 m_0^{(3)} + \lambda^4 m_0^{(4)})(|\psi_3^{(0)}\rangle + \lambda|\psi_3^{(1)}\rangle + \lambda^2|\psi_3^{(2)}\rangle + \lambda^3|\psi_3^{(3)}\rangle + \lambda^4|\psi_3^{(4)}\rangle) \quad (28)$$

$$\Rightarrow M^0|\psi_3^{(0)}\rangle + M^0\lambda|\psi_3^{(1)}\rangle + M^0\lambda^2|\psi_3^{(2)}\rangle + M^0\lambda^3|\psi_3^{(3)}\rangle + M^0\lambda^4|\psi_3^{(4)}\rangle + \lambda M'|\psi_3^{(0)}\rangle + \lambda^2 M'|\psi_3^{(1)} + \lambda^3 M'|\psi_3^{(2)}\rangle + \lambda^4 M'|\psi_3^{(3)}\rangle + \lambda^5 M'|\psi_3^{(4)}\rangle = m_0^{(0)}|\psi_3^{(0)}\rangle + m_0^{(0)}\lambda|\psi_3^{(1)}\rangle + m_0^{(0)}\lambda 2|\psi_3^{(2)}\rangle + m_0^{(0)}\lambda^3|\psi_3^{(3)}\rangle + m_0^{(0)}\lambda^4|\psi_3^{(4)}\rangle + \lambda m_0^{(1)}|\psi_3^{(0)}\rangle + \lambda^2 m_0^{(1)}|\psi_3^{(1)}\rangle + \lambda^3 m_0^{(1)}|\psi 3^{(2)}\rangle + \lambda^4 m_0^{(1)}|\psi 3^{(3)}\rangle + \lambda^2 m_0^{(2)}|\psi 3^{(0)}\rangle + \lambda^3 m_0^{(2)}|\psi 3^{(1)}\rangle + \lambda^4 m_0^{(2)}|\psi_3^{(2)}\rangle + \lambda^3 m_0^{(3)}|\psi_3^{(0)}\rangle + \lambda^4 m_0^{(3)}|\psi_3^{(1)}\rangle + \lambda^4 m_0^{(4)}|\psi_3^{(0)}\rangle \quad (29)$$

$$\Rightarrow M^0\lambda^1|\psi_3^{(1)}\rangle + M^0\lambda^2|\psi_3^{(2)}\rangle + M^0\lambda^3|\psi_3^{(3)}\rangle + M^0\lambda 4|\psi_3^{(4)}\rangle + \lambda M'|\psi_3^{(0)}\rangle + \lambda^2 M'|\psi_3^{(1)}\rangle + \lambda^3 M'|\psi_3^{(2)}\rangle + \lambda^4 M'|\psi_3^{(3)}\rangle + \lambda^5 M'|\psi_3^{(4)}\rangle = m_0^{(0)}\lambda|\psi_3^{(1)}\rangle + m_0^{(0)}\lambda^2|\psi_3(^{2)}\rangle + m_0^{(0)}\lambda^3|\psi_3^{(3)}\rangle + m_0^{(0)}\lambda^4|\psi_3^{(4)}\rangle + \lambda m_0^{(1)}|\psi_3^{(0)}\rangle + \lambda^2 m_0^{(1)}|\psi_3^{(1)}\rangle + \lambda^3 m_0^{(1)}|\psi_3^{(2)}\rangle + \lambda^4 m_0^{(1)}|\psi_3^{(3)}\rangle + \lambda^2 m_0^{(2)}|\psi 3^{(0)}\rangle + \lambda^3 m_0(^{2)}|\psi_3^{(1)}\rangle + \lambda^4 m_0^{(2)}|\psi_3^{(2)}\rangle + \lambda^3 m_0^{(3)}|\psi_3(^{0)}\rangle + \lambda^4 m_0^{(3)}|\psi_3(^{1)}\rangle + \lambda^4 m_0^{(4)}|\psi_3^{(0)}\rangle \quad (30)$$

Here, $m_0^{(1)}$ is the first order correction to the zeroth eigenvalue and $|\psi_3^{(1)}\rangle$ is the first order

correction to the zeroth eigen function, $m_0^{(2)}$ and $|\psi_3^{(2)}\rangle$ are the second order correction and so on.

Now equating the power of λ , we get λ^0; $M_0|\psi_3^{(0)}\rangle = m_0^{(0)}|\psi_3^{(0)}\rangle$

λ^1; $M^0|\psi_3^{(1)}\rangle + M'|\psi_3^{(0)}\rangle = m_0^{(0)}|\psi_3^{(1)}\rangle + m_0^{(1)}|\psi_3^{(0)}\rangle$

λ^2; $M^0|\psi_3^{(2)}\rangle + M'|\psi_3^{(1)}\rangle = m_0^{(0)}|\psi_3^{(2)}\rangle + m_0^{(1)}|\psi_3^{(1)}\rangle + m_0^{(2)}|\psi_3^{(0)}\rangle$

λ^3; $M^0|\psi_3^{(3)}\rangle + M'|\psi_3^{(2)}\rangle = m_0^{(0)}|\psi_3^{(3)}\rangle + m_0^{(1)}|\psi_3^{(2)}\rangle + m_0^{(2)}|\psi_3^{(1)}\rangle + m_0^{(3)}|\psi_3^{(0)}\rangle$

λ^4; $M^0|\psi_3^{(4)}\rangle + M'|\psi_3^{(3)}\rangle = m_0^{(0)}|\psi_3^{(4)}\rangle + m_0^{(1)}|\psi_3^{(3)}\rangle + m_0^{(2)}|\psi_3^{(2)}\rangle + m_0^{(3)}|\psi_3^{(1)}\rangle + m_0^{(4)}|\psi_3^{(0)}\rangle$

Let us rewrite $|\psi_3^{(1)}\rangle$ as $|\psi_3^{(1)}\rangle = 1|\psi_3^{(1)}\rangle$

But $1 = \sum_n |\psi_3^{(0)}\rangle\langle\psi_3^{(0)}|$ (Completeness property)

$\Rightarrow |\psi_3^{(1)}\rangle = \sum_n |\psi_3^{(0)}\rangle\langle\psi_3^{(0)}|\psi_3^{(1)}\rangle$

Let $\langle\psi_3^{(0)}|\psi_3^{(1)}\rangle = C_{n3}^{(1)}$

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$\Rightarrow |\psi_3(^{1)}\rangle = \sum_n C_{n3}{}^{(1)|}\psi_3{}^{(0)}\rangle$ and $|\psi_3{}^{(2)}\rangle = 1|\psi_3{}^{(2)}\rangle$

$\Rightarrow |\psi_3{}^{(2)}\rangle = \sum_n |\psi_3{}^{(0)}\rangle\langle \psi_3(^{0)|}\psi_3{}^{(2)}\rangle$

$\Rightarrow|\psi_3{}^{(2)}\rangle=\sum_n C_{n3}{}^{(2)|}\psi_3{}^{(0)}\rangle$ and so on

In general,

$$\psi_3 = \psi_3^{(0)} + \sum_{n\neq 3} C_{n3}\psi_n^{(0)} \quad (31)$$

Moreover, $M^0|\psi_3{}^{(0)}\rangle = m_0{}^{(0)|}\psi_3{}^{(0)}\rangle$

$\Rightarrow M^0|\psi_3{}^{(1)}\rangle = m_0{}^{(0)} \sum_n C_{n3}{}^{(1)|}\psi_3{}^{(0)}\rangle$

For the first order correction, we get

$$M^0|\psi_3{}^{(1)}\rangle + M'|\psi_3{}^{(0)}\rangle = m_0{}^{(0)|}\psi_3{}^{(1)}\rangle + m_0{}^{(1)|}\psi_3{}^{(0)}\rangle \quad (32)$$

$$\Rightarrow m_0{}^{(0)} \sum_n C_{n3}{}^{(1)}|\psi_3{}^{(0)}\rangle + M'|\psi_3{}^{(0)}\rangle = m_0{}^{(0)} \sum_n C_{n3}{}^{(1)|}\psi_3{}^{(0)}\rangle + m_0{}^{(1)|}\psi_3{}^{(0)}\rangle \quad (33)$$

$$\Rightarrow m_0{}^{(0)} \sum_n C_{n3}{}^{(1)|}\psi_3{}^{(0)}\rangle - m_0{}^{(0)} \sum_n C_{n3}{}^{(1)|}\psi_3{}^{(0)}\rangle = - M'|\psi_3{}^{(0)}\rangle + m_0{}^{(1)|}\psi_3{}^{(0)}\rangle \quad (34)$$

$$\Rightarrow \sum_n C_{n3}{}^{(1)}(m_3{}^{(0)} - m_0{}^{(0)})|\psi_3{}^{(0)}\rangle = -(M' - m_0{}^{(1)})|\psi_3{}^{(0)}\rangle \quad (35)$$

$$\Rightarrow \sum_n C_{n3}{}^{(1)(}m_3{}^{(0)} - m_0{}^{(0)})|\psi_3{}^{(0)}\rangle + (M' - m_0{}^{(1)})|\psi_3{}^{(0)}\rangle = 0 \quad (36)$$

Let us take $m_0{}^{(0)} = m_n{}^{(0)}$ and from right multiply the above equation by $\langle\psi_n{}^{(0)}|$, we get

$$\Rightarrow\langle\psi_n{}^{(0)}| \sum_n C_{n3}{}^{(1)}(m_3{}^{(0)} - m_n{}^{(0)})|\psi_n{}^{(0)}\rangle + \langle\psi_n{}^{(0)}|(M' - m_0{}^{(1)})|\psi_3{}^{(0)}\rangle = 0 \quad (37)$$

$$\Rightarrow \sum_n C_{n3}{}^{(1)}(m_3{}^{(0)} - m_n{}^{(0)})\langle\psi_n{}^{(0)}|\psi_n{}^{(0)}\rangle + \langle\psi_n{}^{(0)}|M'|\psi_3{}^{(0)}\rangle - m_0{}^{(1)}) \langle\psi_n{}^{(0)|}\psi_3{}^{(0)}\rangle = 0 \quad (38)$$

Now, by using the property; $\sum n\, f(n)\delta_{n3} = \sum^3{}_n=1\, f(n)\delta_{n,2} = f(2)$

$$C_{n3}{}^{(1)}(m_3{}^{(0)} - m_n{}^{(0)}) + \langle\psi_n{}^{(0)}|M'|\psi_3{}^{(0)}\rangle - m_0{}^{(1)}\delta_{n3} = 0 \quad (39)$$

For n= 3

$$\langle\psi_n{}^{(0)}|M'|\psi_3{}^{(0)}\rangle = M_{33}' = m_0{}^{(1)} \quad (40)$$

For $n \neq 3$

$$C_{n3}{}^{(1)}(m_3{}^{(0)} - m_n{}^{(0)}) + \langle\psi_n{}^{(0)}|M'|\psi_3{}^{(0)}\rangle = 0 \quad (41)$$

$$C_{n3}^{(1)} = -\frac{\langle ?_n^{(0)}|\mathbf{M}'|?_3^{(0)}\rangle}{\mathbf{m}_3^{(0)} - \mathbf{m}_n^{(0)}} = -C_{3n}^{(1)}\ (n \neq 3) \quad (42)$$

Here the coefficient C_{n3} are real.

CONCLUSION

In conclusion, we show that the TBM ansatz can be used as the leading order mass matrix with vanishing neutrino reactor angle θ13. However we can perturb the TBM mass matrix to generate non zero reactor angle while using minimal independent parameters. Another implication of this perturbation is that the same parameters also alter the atmospheric mixing angle θ23. This way we have showed that the same parameter which generate non zero reactor angle can also decide the unknown octant of the atmospheric mixing angle.

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