A New Generalization of Two Parametric Akash Distribution With Applications

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Abstract: In this article, a new generalization of two parameters Akash distribution have been obtained by applying the technique of length biased known as length biased weighted two parameters Akash distribution. The different mathematical and statistical properties of the newly introduced distribution have been derived and discussed. The maximum likelihood estimate is used to estimate the parameters of the proposed distribution and also the Fisher's information matrix have been obtained. Finally, an application for the proposed distribution has been used for examining the suitability of the newly introduced distribution.

Keywords: Weighted distribution, Reliability analysis, Order statistics, Entropies, Maximum likelihood estimation.

1. INTRODUCTION

The newly proposed concept of distribution known as two parameters Akash distribution (TPAD) is a newly introduced lifetime model formulated by shanker (2017) of which one parameter Akash distribution of shanker is a particular case of it. Shanker has obtained its various important mathematical and statistical properties of the distribution including its shape, moments, order statistics, Bonferroni and Lorenz curve, Skewness, Kurtosis, mean deviations, Renyi entropy, hazard function, stochastic ordering, mean residual life function and Stress strength reliability and has obtained its parameters by using the method of moments and method of maximum likelihood estimation. Shanker has also discussed a poisson mixiture of Akash distribution known as poisson Akash distribution (PAD) and obtained its important mathematical and statistical properties and has also discussed its estimation of parameters along with application for count data from different fields of knowledge. Shanker also introduced the size-biased and zero truncated version of poission Akash distribution and also obtained its various structural properties along with the estimation of parameters by using the both methods of estimation (method of moments and method of maximum likelihood estimation). Shanker also discussed on quasi Akash distribution and obtains its various Structural properties and estimate its parameters by using the both methods of estimation. Shanker and Shukla have obtained the weighted version of Akash distribution for modelling the lifetime data and has observed that it gives better fit over several one parameter and two parameter distributions.

The two parameters Akash distribution (TPAD) with parameters ϑ and α having probability density function is given by

$$f(x;\theta,\alpha)=\frac{\theta^3}{\alpha\theta^2+2}\left(\alpha+x^2\right)e^{-\theta x};\ x>0,\theta>0,\alpha>0 \qquad (1)$$

and the two parameters Akash distribution having cumulative distribution function is given by

$$F(x;\theta,\alpha) = 1 - \left[1 + \frac{\theta x(\theta x + 2)}{\alpha\theta^2 + 2}\right] e^{-\theta x} ; x > 0, \theta > 0, \alpha > 0 \quad (2)$$

2. LENGTH BIASED WEIGHTED TWO PARAMETRIC AKASH DISTRIBUTION (LBWTPAD)

The concept of newly introduced models of distribution known as weighted distribution was given by Fisher (1934) to model the ascertainment bias. Then after Rao in 1965 developed the proposed concept in an integrated manner for the purpose of modelling statistical data, when the standard distributions was not suitable recording the equal probabilities of distributions. Particularly, weighted models were changed in such situations to record the observations according to some weighted function. The weighted distribution reduces to length biased distribution as the weight function considers only the length of the units. The length biased sampling concept was firstly given by Cox (1969) and Zelen (1974). The newly proposed weighted distributions are applied when an investigator collects the observations recorded by certain stochastic model according to nature. The weighted distributions are useful in theory of distributions, because it gives a new way of the standard existing distributions for lifetime modelling of data by using of additional parameter in the model which creates flexibility in their nature. Weighted distributions are used in different fields like branching process, ecology etc. Different authors have observed and studied the various models of weighted probability and illustrated their applications in different fields. Afaq et.al (2016) have obtained the length biased weighted version of lomax distribution with properties and applications. Rather and Subramanian (2018) have obtained the Sushila distributions length biased version. Reyad et al. (2017), have obtained distribution of length biased weighted frechet with properties and estimations.Rather and Subramanian (2018) discussed the length biased weighted generalized uniform distribution. Recently, Ganaie, Rajagopalan and Rather (2019) obtained a new length biased distribution with Applications.

Suppose X is a random variable which is non-negative with probability density function $f(x)$. Let $w(x)$ be the weighted non-negative function, then, the probability density function of the weighted random variable X_w is given by

$$f_w(x) = \frac{w(x) f(x)}{E(w(x))}, \quad x > 0.$$

Where $w(x)$ be the non - negative weight function and $E(w(x)) = \int w(x) f(x) dx < \infty.$

In this Paper, we have to introduce the length biased weighted two parameters Akash distribution. We consider the weight function as *w(x)* = x^c, the resulting distribution is known as weighted distribution. By applying *c = 1* in weighted three parameters Akash distribution, we obtain thelength biased two parameters Akash distribution.

The probability density function of length biased weighted two parameters Akash distribution is given by

$$f_l(x;\theta,\alpha) = \frac{xf(x;\theta,\alpha)}{E(x)}$$

$$f_l(x;\theta,\alpha) = \frac{x\theta^4(\alpha + x^2)e^{-\theta x}}{(\alpha\theta^2 + 6)} \quad (3)$$

Where $E(x) = \dfrac{\alpha\theta^2 + 6}{\theta(\alpha\theta^2 + 2)}$

The corresponding cumulative distribution function of length biased weighted two parameters Akash distribution is given by

$$F_l(x;\theta,\alpha) = \int_0^x f_l(x;\theta,\alpha)dx$$

$$F_l(x;\theta,\alpha) = \int_0^x \frac{x\theta^4(\alpha+x^2)e^{-\theta x}}{(\alpha\theta^2+6)}dx$$

$$F_l(x;\theta,\alpha) = \frac{1}{(\alpha\theta^2+6)}\int_0^x x\theta^4(\alpha+x^2)e^{-\theta x}dx$$

$$F_l(x;\theta,\alpha) = \frac{1}{(\alpha\theta^2+6)}\left[\alpha\theta^4\int_0^x xe^{-\theta x}dx + \theta^4\int_0^x x^3e^{-\theta x}dx\right]$$

Put $\theta x = t \Rightarrow \theta dx = dt \Rightarrow dx = \frac{dt}{\theta}$, As $x \to x, t \to \theta x,\ x \to 0, t \to 0$

After simplification, we obtain the cumulative distribution function of length biased weighted two parameters Akash distribution which is given by

$$F_l(x;\theta,\alpha) = \frac{1}{(\alpha\theta^2+6)}\left(\alpha\theta^2\gamma(2,\theta x)+\gamma(4,\theta x)\right) \qquad (4)$$

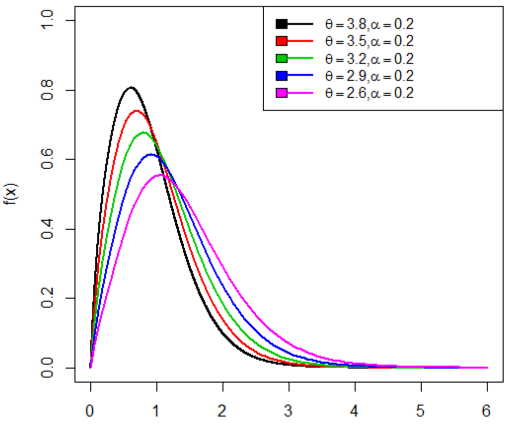

Fig.1:Pdf plot of LBWTPAD

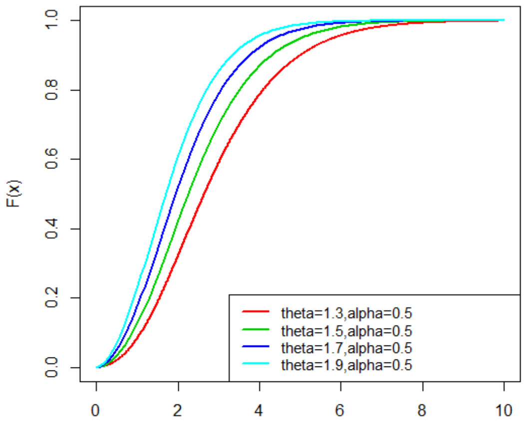

Fig.2 Cdf plot of LBWTPAD

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3. RELIABILITY MEASURES

In this portion, we have obtained the Reliability, hazard rate and Reverse hazard function of length biased weighted two parameters Akash distribution.

3.1 Reliability function

The Reliability function or the Survival function of length biased weighted two parameters Akash distribution is calculated as

$$R(x) = 1 - F_l(x;\theta,\alpha)$$

$$R(x) = 1 - \frac{1}{(\alpha\theta^2+6)}\left(\alpha\theta^2\gamma(2,\theta x)+\gamma(4,\theta x)\right)$$

3.2 Hazard function

The hazard function is also known as force of mortality and the hazard function of length biased weighted two parameters Akash distribution is given by

$$h(x)=\frac{f_l(x;\theta,\alpha)}{R(x)}$$

$$h(x)=\frac{x\theta^4(\alpha+x^2)e^{-\theta x}}{(\alpha\theta^2+6)-(\alpha\theta^2\gamma(2,\theta x)+\gamma(4,\theta x))}$$

3.3 Reverse hazard function

The reverse hazard function of length biased weighted two parameters Akash distribution is given by

$$h^r(x)=\frac{f_l(x;\theta,\alpha)}{F_l(x;\theta,\alpha)}$$

$$h^r(x)=\frac{x\theta^4(\alpha+x^2)e^{-\theta x}}{(\alpha\theta^2\gamma(2,\theta x)+\gamma(4,\theta x))}$$

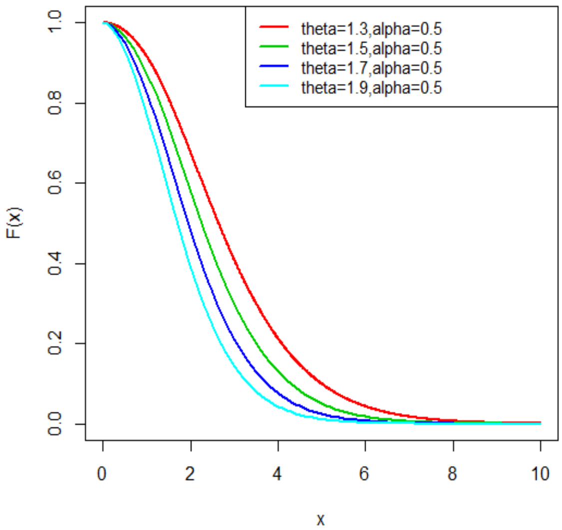

Fig.3 survival function of LBWTPAD

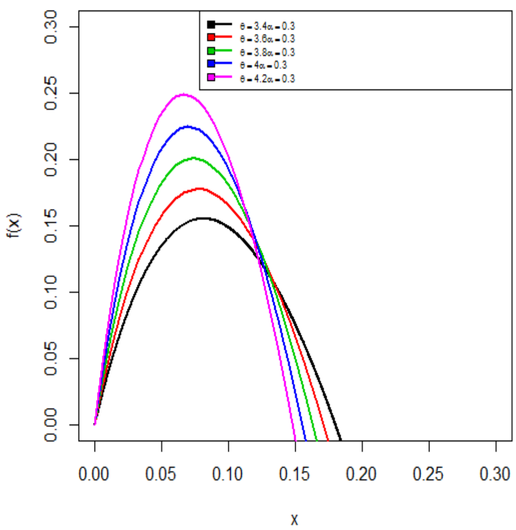

Fig.4:Showing Hazard function of LBWTPAD

4. STRUCTURAL MEASURES

In this part, we have obtained the different statistical properties of length biased weighted two parameters Akash distribution.

4.1 Moments

Let X denotes the random variable of length biased weighted two parameters Akash distribution, then the r^{th} moment of length biased weighted two parameters Akash distribution about origin is

$$E(X^r) = \int_0^\infty x^r f_l(x;\theta,\alpha)dx$$

$$E(X^r) = \int_0^\infty x^r \frac{x\theta^4(\alpha+x^2)e^{-\theta x}}{(\alpha\theta^2+6)}dx$$

$$= \frac{\theta^4}{(\alpha\theta^2+6)}\int_0^\infty x^{r+1}(\alpha+x^2)e^{-\theta x}dx$$

$$= \frac{\theta^4}{(\alpha\theta^2+6)}\left(\alpha\int_0^\infty x^{(r+2)-1}e^{-\theta x}dx + \int_0^\infty x^{(r+4)-1}e^{-\theta x}dx\right)$$

$$E(X^r) = \mu_r' = \frac{\alpha\theta^2(r+1)!+(r+3)!}{\theta^r(\alpha\theta^2+6)} \quad (5)$$

By applying r = 1,2,3 and 4 in equation (5) we get first four moments about origin of length biased weighted two parameters Akash distribution.

$$E(X) = \mu_1' = \frac{2\alpha\theta^2+24}{\theta(\alpha\theta^2+6)}$$

$$E(X^2) = \mu_2' = \frac{6\alpha\theta^2+120}{\theta^2(\alpha\theta^2+6)}$$

$$E(X^3) = \mu_3' = \frac{24\alpha\theta^2+720}{\theta^3(\alpha\theta^2+6)}$$

$$E(X^4) = \mu_4' = \frac{120\alpha\theta^2+5040}{\theta^4(\alpha\theta^2+6)}$$

$$\text{Variance}(\mu_2) = \frac{(6\alpha\theta^2+120)(\alpha\theta^2+6)-(2\alpha\theta^2+24)^2}{\theta^2(\alpha\theta^2+6)^2}$$

$$S.D(\sigma) = \sqrt{\left(\frac{(6\alpha\theta^2+120)(\alpha\theta^2+6)-(2\alpha\theta^2+24)^2}{\theta^2(\alpha\theta^2+6)^2}\right)}$$

$$\text{Coefficient of Variation} = \frac{\sigma}{\mu_1'} = \sqrt{\left(\frac{(6\alpha\theta^2+120)(\alpha\theta^2+6)-(2\alpha\theta^2+24)^2}{\theta^2(\alpha\theta^2+6)^2}\right)} \times \frac{\theta(\alpha\theta^2+6)}{2\alpha\theta^2+24}$$

$$\text{Coefficient of dispersion}(\gamma) = \frac{\sigma^2}{\mu_1'} = \frac{(6\alpha\theta^2+120)(\alpha\theta^2+6)-(2\alpha\theta^2+24)^2}{\theta^2(\alpha\theta^2+6)^2} \times \frac{\theta(\alpha\theta^2+6)}{2\alpha\theta^2+24}$$

4.2 Harmonic mean

Harmonic mean is the reciprocal of the arithmetic mean of the reciprocals. Typically, it is appropriate for situations when the average of rates is desired. The harmonic mean for the proposed length biased weighted two parameters Akash distribution can be obtained as

$$H.M = E\left(\frac{1}{x}\right) = \int_0^\infty \frac{1}{x} f_l(x;\theta,\alpha)dx$$

$$= \int_0^\infty \frac{\theta^4(\alpha + x^2)e^{-\theta x}}{(\alpha\theta^2 + 6)} dx$$

$$= \frac{\theta^4}{(\alpha\theta^2+6)}\left(\alpha\int_0^\infty x^{(2-2)}e^{-\theta x}dx + \int_0^\infty x^{(3-1)}e^{-\theta x}dx\right)$$

$$= \frac{\theta^4}{(\alpha\theta^2+6)}\left(\alpha\int_0^\infty e^{-\theta x}x^{(2-2)}dx + \int_0^\infty e^{-\theta x}x^{(3-1)}dx\right)$$

$$\Rightarrow H.M = \frac{\theta^4}{(\alpha\theta^2+6)}\left(\alpha\gamma(2,\theta x) + \gamma(3,\theta x)\right)$$

4.3 Moment Generating Function

Moment generating function is defined as the function of a real valued random variable completely defines the probability distribution of a random variable. We takethe well-known definition of the moment generating function given by

$$M_X(t) = E\left(e^{tx}\right) = \int_0^\infty e^{tx} f_l(x;\theta,\alpha)dx$$

$$= \int_0^\infty \left[1 + tx + \frac{(tx)^2}{2!} + \ldots\right] f_l(x;\theta,\alpha)dx$$

$$= \int_0^\infty \sum_{j=0}^\infty \frac{t^j}{j!} x^j f_l(x;\theta,\alpha)dx$$

$$= \sum_{j=0}^\infty \frac{t^j}{j!}\mu_j'$$

$$= \sum_{j=0}^\infty \frac{t^j}{j!}\left[\frac{\alpha\theta^2(j+1)! + (j+3)!}{\theta^j(\alpha\theta^2+6)}\right]$$

$$M_X(t) = \frac{1}{(\alpha\theta^2+6)}\sum_{j=0}^\infty \frac{t^j}{j!\theta^j}\left(\alpha\theta^2(j+1)! + (j+3)!\right)$$

4.4 Characteristic function

Characteristics function is that function, if a probability density function is admitted by a random variable, then the characteristic function is the Fourier transform of the probability density function.The characteristics function of length biased weighted two parameters Akash distribution is given by

$$\varphi_x(t) = M_X(it)$$

$$M_X(it) = \frac{1}{(\alpha\theta^2 + 6)} \sum_{j=0}^{\infty} \frac{it^j}{j!\theta^j} \left(\alpha\theta^2 (j+1)! + (j+3)! \right)$$

5. ORDER STATISTICS

Order statistics are the sequence of samples arranged in an increasing order. The study of order statistics deals with the applications of these ordered sample values and their functions, then the probability density function of r^{th} order statistics $X_{(r)}$is given by

$$f_{x(r)}(x) = \frac{n!}{(r-1)!(n-r)!} f_X(x) \left(F_X(x) \right)^{r-1} \left(1 - F_X(x) \right)^{n-r} \quad (6)$$

Using equation (3) and (4) in (6), the probability density function of the r^{th} order statistics of length biased weighted two parameters Akash distribution is given by

$$f_{x(r)}(x) = \frac{n!}{(r-1)!(n-r)!} \left(\frac{x\theta^4 (\alpha + x^2) e^{-\theta x}}{(\alpha\theta^2 + 6)} \right) \left(\frac{1}{(\alpha\theta^2 + 6)} \left(\alpha\theta^2 \gamma(2, \theta x) + \gamma(4, \theta x) \right) \right)^{r-1} \times \left(1 - \frac{1}{(\alpha\theta^2 + 6)} \left(\alpha\theta^2 \gamma(2, \theta x) + \gamma(4, \theta x) \right) \right)^{n-r}$$

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The probability density function of the *ist* order statistics $X_{(1)}$ of length biased weighted two parameters Akash distribution is given by

$$f_{x(1)}(x) = \frac{nx\theta^4 (\alpha + x^2) e^{-\theta x}}{(\alpha\theta^2 + 6)} \left(1 - \frac{1}{(\alpha\theta^2 + 6)} \left(\alpha\theta^2 \gamma(2, \theta x) + \gamma(4, \theta x) \right) \right)^{n-1}$$

and the probability density function of the higher order statistics $X_{(n)}$ of length biased weighted two parameters Akash distribution is given by

$$f_{x(n)}(x) = \frac{nx\theta^4 (\alpha + x^2) e^{-\theta x}}{(\alpha\theta^2 + 6)} \left(\frac{1}{(\alpha\theta^2 + 6)} \left(\alpha\theta^2 \gamma(2, \theta x) + \gamma(4, \theta x) \right) \right)^{n-1}$$

6. LIKELIHOOD RATIO TEST

The sample of size *n* randomly drawn from the two parameters Akash distribution or length biased weighted two parameters Akash distribution. We set up the null and alternative hypothesis for testing.

$$H_o : f(x) = f(x;\theta,\alpha) \quad \text{against} \quad \mathrm{H}_1 : f(x) = f_l(x;\theta,\alpha)$$

Thus for testing the hypothesis,whether the random sample of size *n* comes from the two parameters Akash distribution or length biased weighted two parameters Akash distribution, the following test statistic is used.

$$\Delta = \frac{L_1}{L_o} = \prod_{i=1}^{n} \frac{f_l(x;\theta,\alpha)}{f(x;\theta,\alpha)}$$

$$\Delta = \frac{L_1}{L_o} = \prod_{i=1}^{n} \left[\frac{x_i \theta(\alpha\theta^2 + 2)}{(\alpha\theta^2 + 6)} \right]$$

$$\Delta = \frac{L_1}{L_o} = \left(\frac{\theta(\alpha\theta^2 + 2)}{(\alpha\theta^2 + 6)} \right)^n \prod_{i=1}^{n} x_i$$

We reject the null hypothesis if

$$\Delta = \left(\frac{\theta(\alpha\theta^2 + 2)}{(\alpha\theta^2 + 6)} \right)^n \prod_{i=1}^{n} x_i > k$$

Or equivalently, we reject the null hypothesis, when

$$\Delta^* = \prod_{i=1}^{n} x_i > k \left(\frac{(\alpha\theta^2 + 6)}{\theta(\alpha\theta^2 + 2)} \right)^n$$

$$\Delta^* = \prod_{i=1}^{n} x_i > k^*, \text{Where } \mathrm{k}^* = k \left(\frac{(\alpha\theta^2 + 6)}{\theta(\alpha\theta^2 + 2)} \right)^n$$

2log Δ is distributed as chi-square distribution with one degree of freedom, when sample of size n is large and also p- value is obtained from the chi-square distribution. Also, we reject the null hypothesis, when the value of probability is given by

$p(\Delta^* > \beta^*)$, Where $\beta^* = \prod_{i=1}^{n} x_i$ is lower than a particular level of significan ce and $\prod_{i=1}^{n} x_i$ is the observed value of the statistic Δ^*.

7. BONFERRONI AND LORENZ CURVES

The Bonferroni and Lorenz curves are nowadays used in different fields like reliability, medicine, insurance and demography, but previously its scope was only limited to economics. The bonferroni and Lorenz curves are also one of the important indicators of wealth distribution. The bonferroni and Lorenz curves are given by

$$B(p) = \frac{1}{p\mu_1'} \int_0^q x f_l(x;\theta,\alpha)dx$$

and $$L(p) = pB(p) = \frac{1}{\mu_1'} \int_0^q x f_l(x;\theta,\alpha)dx$$

Where $\mu_1' = E(X) = \dfrac{2\alpha\theta^2 + 24}{\theta(\alpha\theta^2 + 6)}$ and $q = F^{-1}(p)$

$$B(p) = \frac{\theta(\alpha\theta^2 + 6)}{p(2\alpha\theta^2 + 24)} \int_0^q \frac{x^2\theta^4(\alpha + x^2)e^{-\theta x}}{(\alpha\theta^2 + 6)} dx$$

$$B(p) = \frac{\theta^5}{p(2\alpha\theta^2 + 24)} \int_0^q x^2(\alpha + x^2)e^{-\theta x} dx$$

$$B(p) = \frac{\theta^5}{p(2\alpha\theta^2 + 24)} \left(\alpha \int_0^q e^{-\theta x} x^{(3-1)} dx + \int_0^q e^{-\theta x} x^{(5-1)} dx \right)$$

$$B(p) = \frac{\theta^5}{p(2\alpha\theta^2 + 24)} (\alpha\gamma(3, \theta q) + \gamma(5, \theta q))$$

$$L(p) = \frac{\theta^5}{(2\alpha\theta^2 + 24)} (\alpha\gamma(3, \theta q) + \gamma(5, \theta q))$$

8. ENTROPIES

The concept of entropies is used in different fields such as probability and statistics, physics, communication theory and economics. Entropies also discover the diversity, uncertainty, or randomness of a system. The Entropy of a random variable *X* is a measure of variation of the uncertainty. 851

8.1 Renyi Entropy

The newly introduced concept of entropy such as Renyi entropy is important in ecology and statistics as index of diversity. The entropy is named after Alfred Renyi. The Renyi entropy is also important in quantum information, where it can be used as a measure of entanglement. There are various entropies available but the Renyi entropy is an important one. For a given probability distribution, Renyi entropy is given by

$$e(\beta) = \frac{1}{1-\beta} \log\left(\int f_l^{\beta}(x; \theta, \alpha) dx \right)$$

Where, $\beta > 0$ and $\beta \neq 1$

$$e(\beta) = \frac{1}{1-\beta} \log \int_0^\infty \left(\frac{x\theta^4(\alpha + x^2)e^{-\theta x}}{(\alpha\theta^2 + 6)} \right)^{\beta} dx$$

$$e(\beta) = \frac{1}{1-\beta} \log\left(\left(\frac{\theta^4}{(\alpha\theta^2 + 6)} \right)^{\beta} \int_0^\infty x^{\beta} e^{-\theta\beta x} (\alpha + x^2)^{\beta} dx \right) \quad (7)$$

Using binomial expansion in equation (7), we obtain

$$e(\beta)=\frac{1}{1-\beta}\log\left(\left(\frac{\theta^4}{(\alpha\theta^2+6)}\right)^{\beta}\sum_{j=0}^{\infty}\binom{\beta}{j}\alpha^{\beta-j}x^{2j}\int_0^{\infty}e^{-\theta\beta x}x^{\beta}dx\right)$$

$$e(\beta)=\frac{1}{1-\beta}\log\left(\left(\frac{\theta^4}{(\alpha\theta^2+6)}\right)^{\beta}\sum_{j=0}^{\infty}\binom{\beta}{j}\alpha^{\beta-j}\int_0^{\infty}e^{-\theta\beta x}x^{(\beta+2j+1)-1}dx\right)$$

$$e(\beta)=\frac{1}{1-\beta}\log\left(\left(\frac{\theta^4}{(\alpha\theta^2+6)}\right)^{\beta}\sum_{j=0}^{\infty}\binom{\beta}{j}\alpha^{\beta-j}\frac{\Gamma(\beta+2j+1)}{(\theta\beta)^{\beta+2j+1}}\right)$$

8.2 Tsallis Entropy

The new concept of Tsallis entropy was introduced in 1988 by constantino Tsallis as a basis for generalizing the standard statistical mechanics. The generalization of Boltzmann-Gibbs (B.G) statistical properties initiated by Tsallis has focused a great deal to attention. This generalization of B-G statistics was proposed firstly by introducing the mathematical expression of Tsallis entropy (Tsallis, 1988) for a continuous random variable is defined as follows

$$S_{\lambda}=\frac{1}{\lambda-1}\left(1-\int_0^{\infty}f_l^{\lambda}(x)dx\right)$$

$$S_{\lambda}=\frac{1}{\lambda-1}\left(1-\int_0^{\infty}\left(\frac{x\theta^4(\alpha+x^2)e^{-\theta x}}{(\alpha\theta^2+6)}\right)^{\lambda}dx\right)$$

$$S_{\lambda}=\frac{1}{\lambda-1}\left(1-\left(\frac{\theta^4}{(\alpha\theta^2+6)}\right)^{\lambda}\int_0^{\infty}x^{\lambda}e^{-\lambda\theta x}(\alpha+x^2)^{\lambda}dx\right) \quad (8)$$

Using binomial expansion in equation (8), we get

$$S_{\lambda}=\frac{1}{\lambda-1}\left(1-\left(\frac{\theta^4}{(\alpha\theta^2+6)}\right)^{\lambda}\sum_{j=0}^{\infty}\binom{\lambda}{j}\alpha^{\lambda-j}x^{2j}\int_0^{\infty}e^{-\theta\lambda x}x^{\lambda}dx\right)$$

$$S_{\lambda}=\frac{1}{\lambda-1}\left(1-\left(\frac{\theta^4}{(\alpha\theta^2+6)}\right)^{\lambda}\sum_{j=0}^{\infty}\binom{\lambda}{j}\alpha^{\lambda-j}\int_0^{\infty}e^{-\lambda\theta x}x^{(\lambda+2j+1)-1}dx\right)$$

$$S_{\lambda}=\frac{1}{\lambda-1}\left(1-\left(\frac{\theta^4}{(\alpha\theta^2+6)}\right)^{\lambda}\sum_{j=0}^{\infty}\binom{\lambda}{j}\alpha^{\lambda-j}\frac{\Gamma(\lambda+2j+1)}{(\lambda\theta)^{\lambda+2j+1}}\right)$$

9. MAXIMUM LIKELIHOOD ESTIMATION AND FISHER'S INFORMATION MATRIX

The maximum likelihood method of estimation is used for estimating the parameters of the newly proposed distribution known as the length biased weighted two parameters Akash distribution. Let $x_1, x_2,\ldots,x_n$ be a random sample of size *n*from the length biased weighted two parameters Akash distribution, then the corresponding likelihood function is given by

$$L(x)=\prod_{i=1}^{n} f_l(x;\theta,\alpha)$$

$$L(x)=\prod_{i=1}^{n}\left[\frac{x_i\theta^4(\alpha+x_i^{\,2})e^{-\theta x_i}}{(\alpha\theta^2+6)}\right]$$

$$L(x)=\frac{\theta^{4n}}{(\alpha\theta^2+6)^n}\prod_{i=1}^{n}\left[x_i(\alpha+x_i^{\,2})e^{-\theta x_i}\right]$$

The log likelihood function is given by

$$\log L=4n\log\theta-n\log(\alpha\theta^2+6)+\sum_{i=1}^{n}\log x_i+\sum_{i=1}^{n}\log(\alpha+x_i^{\,2})-\theta\sum_{i=1}^{n}x_i \qquad (9)$$

The parameters of the newly introduced distribution are obtained by differentiating the log likelihood

equation (9) with respect to parametersθ and α. We obtain the normal equations as

$$\frac{\partial\log L}{\partial\theta}=\frac{4n}{\theta}-n\left(\frac{2\theta\alpha}{(\alpha\theta^2+6)}\right)-\sum_{i=1}^{n}x_i=0$$

$$\frac{\partial\log L}{\partial\alpha}=-n\left(\frac{\theta^2}{(\alpha\theta^2+6)}\right)+\sum_{i=1}^{n}\left(\frac{1}{(\alpha+x_i^{\,2})}\right)=0$$

The maximum likelihood estimates of the parameters of the distribution are obtained by solving these nonlinear system ofequations. Therefore, we use R and wolfram mathematics for estimating the parameters of the newly proposed distribution.

To obtain the confidence interval we use the asymptotic normality results. We have that if $\hat{\gamma}=(\hat{\theta},\hat{\alpha})$ denoted the MLE of $\gamma=(\theta,\alpha)$. We can state the result as

$$\sqrt{n}(\hat{\gamma}-\gamma)\to N_2(0,I^{-1}(\gamma))$$

Where $I^{-1}(\gamma)$ is limiting variance - covariance Matrix of γ.

The Fisher's Information 2×2 matrix is given below as

$$I(\gamma)=-\frac{1}{n}\begin{pmatrix} E\left(\frac{\partial^2\log L}{\partial\theta^2}\right) & E\left(\frac{\partial^2\log L}{\partial\theta\partial\alpha}\right) \\ E\left(\frac{\partial^2\log L}{\partial\alpha\partial\theta}\right) & E\left(\frac{\partial^2\log L}{\partial\alpha^2}\right)\end{pmatrix}$$

Where

$$E\left(\frac{\partial^2 \log L}{\partial \theta^2}\right)=-\frac{4n}{\theta^2}-n\left(\frac{(\alpha\theta^2+6)2\alpha-4\theta^2\alpha^2}{(\alpha\theta^2+6)^2}\right)$$

$$E\left(\frac{\partial^2 \log L}{\partial \alpha^2}\right)=n\left(\frac{\theta^4}{(\alpha\theta^2+6)^2}\right)-\sum_{i=1}^{n}\left(\frac{1}{(\alpha+x_i^{\,2})^2}\right)$$

and

$$E\left(\frac{\partial^2 \log L}{\partial \theta\partial\alpha}\right)=E\left(\frac{\partial^2 \log L}{\partial \alpha\partial\theta}\right)=-n\left(\frac{2\theta(\alpha\theta^2+6)-2\theta^3\alpha}{(\alpha\theta^2+6)^2}\right)$$

Since γ being unknown, we estimate $I^{-1}(\gamma)$ by $I^{-1}(\gamma\tilde{})$ and this can be used to obtain asymptotic confidence intervals for ϑ and α.

10. APPLICATIONS

In the section, we analyze the strength of the data sets by using the length biased weighted two parameters Akash distribution in comparison with two parameters Akash , Exponential and one parameter lindley distribution. Estimates of the unknown parameters are carried out in R software along with calculation of criterion values like AIC, AICC and BIC values. Here, we have used the two realdata sets and analyzed them to show that the length biased weighted two parameters Akash distributionfits better than the two parameters Akash, exponential, Weibull and one parameter lindley distribution. The data set 1 is reported by Gross and Clark (1975) of size 20 patients receiving an analgesic of relief times (minutes) is in Table1 and the data set 2 is reported from one of the ministry of health hospitals in Saudi Arabia of 40 patients suffering from blood cancer (see Abouammah et al.) is in table 2.

Table 1: Data reported by Gross and Clarke (1975) of size 20 patients of relief times in minutes

1.1	1.4	1.3	1.7	1.9	1.8	1.6	2.2
1.7	4.1	1.8	1.5	1.2	1.4	3.0	1.7
2.3	1.6	2.7	2.0				

Table 2: Data of 40 Patients reported from one of ministry of health hospitals in Saudi Arabia suffering from blood cancer

0.315	0.496	0.616	1.145	1.208	1.263	1.414	2.025	2.036
2.162	2.211	2.37	2.532	2.693	2.805	2.91	2.912	3.192
3.263	3.348	3.348	3.427	3.499	3.534	3.767	3.751	3.858
3.986	4.049	4.244	4.323	4.381	4.392	4.397	4.647	4.753
4.929	4.973	5.074	5.381					

In order to compare the length biased weighted two parameter Akash distribution with two parameters Akash, Exponential and one parameter lindley distribution, we are using the criterion values AIC (Akaike information criterion) given by Akaike (1976), AICC (corrected Akaike information criterion) and BIC (Bayesian information criterion) given by Schwartz (1987). The best distribution corresponds to minimum values of AIC, AICC and BIC. The formulas for calculation of AIC, AICC andBIC values are

$$\mathrm{AIC} = 2k - 2\log L \qquad \mathrm{AICC} = AIC + \frac{2k(k+1)}{n-k-1} \quad \text{and} \quad \mathrm{BIC} = k\log n - 2\log L$$

Where k is the number of parameters in the statistical model,n is the sample size and $-2\log L$is the maximized value of the log-likelihood function under the considered model.

From results given in table 3 below, it is observed that the length biased weighted two parameters Akash distribution have the lesser AIC, AICC, BIC and -2logL values as compared to the two parameters Akash, Exponential and one parameter lindleydistribution, which indicated that the length biased weighted two parameters Akash distribution fits better than the two parameter Akash, Exponential, Weibull and one parameter lindley distribution. Hence we can conclude that the length biased weighted two parameters Akash distribution shows a better fit than the two parameters Akash, Exponential, Weibull and one parameter lindley distribution.

Table 3: Parameters Estimates and Comparison of LBWTPAD With Two Parameters Akash, Exponential, Weibull and One Parameter Lindley Distribution.

Data sets	Distribution	MLE	S.E	-2logL	AIC	BIC	AICC
1	LBWTPAD	$\hat{\alpha} = 0.0010$ $\hat{\theta} = 2.1044$	$\hat{\alpha} = 0.1540$ $\hat{\theta} = 0.1506$	41.864	45.864	47.855	46.570
	Two Parameter Akash	$\hat{\alpha} = 0.0010$ $\hat{\theta} = 1.5776$	$\hat{\alpha} = 0.1320$ $\hat{\theta} = 0.1290$	45.809	49.809	51.801	50.515
	Weibull	$\hat{\alpha} = 0.8532$ $\hat{\theta} = 0.3721$	$\hat{\alpha} = 0.8532$ $\hat{\theta} = 0.3721$	42.743	46.743	48.734	47.448
	Exponential	$\hat{\theta} = 0.5263$	$\hat{\theta} = 0.1176$	65.674	67.674	68.669	67.896
	Lindley	$\hat{\theta} = 0.8161$	$\hat{\theta} = 0.1360$	60.499	62.499	63.494	62.721
2	LBWTPAD	$\hat{\alpha} = 0.6522$ $\hat{\theta} = 1.1887$	$\hat{\alpha} = 0.6654$ $\hat{\theta} = 0.1134$	144.304	148.304	151.682	148.628
	Two parameter Akash	$\hat{\alpha} = 0.0678$ $\hat{\theta} = 0.9367$	$\hat{\alpha} = 0.1650$ $\hat{\theta} = 0.0945$	147.292	151.292	154.67	151.616
	Weibull	$\hat{\alpha} = 0.0036$ $\hat{\theta} = 0.0231$	$\hat{\alpha} = 0.1471$ $\hat{\theta} = 0.1758$	145.536	149.536	151.146	149.860
	Exponential	$\hat{\theta} = 0.3183$	$\hat{\theta} = 0.0503$	171.556	173.556	175.245	173.661
	Lindley	$\hat{\theta} = 0.5269$	$\hat{\theta} = 0.0607$	160.501	162.501	164.19	162.606

11. CONCLUSION

In the present manuscript, we introduce the length biased weighted two parameters Akash distribution as a new generalization of two parameters Akash distribution. The subject distribution is generated by using the length biased technique and taking the two parameter Akash distribution as the base distribution. We have also derived and discussed some Structural properties of the newly proposed distribution. The applications of the newly introduced distribution has also been demonstrated with real life data sets and the result of the two data sets shows that the length biased weighted two parameters Akash distribution fits better over two parameter Akash, Exponential, Weibull and one parameter lindley distribution.

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