Generalized Even Intuitionistic Fuzzy Number with Value Index and Ambiguity Index and its Application in Transportation problem

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Abstract.

Intuitionistic Fuzzy sets play an unique role to solve problems with state of uncertainty due to many uncontrollable factors. Intuitionistic fuzzy sets includes non membership function which extends fuzzy sets with membership function. The primary purpose is to generalize Even Intuitionistic Fuzzy number. The generalized value index and generalized ambiguity index formulas for even intuitionistic fuzzy number are defined. The generalized alpha and beta cuts corresponding to even intuitionistic fuzzy numbers are defined. Two indices ranking measure is proposed based on the generalized value index and generalized ambiguity index for even intuitionistic fuzzy number. Justification of the proposed ranking is illustrated with an example by solving a transportation problem which involves supply and demand as Octagonal Intuitionistic Fuzzy number.

Keywords: Generalized Even Intuitionistic Fuzzy Number (GEIFN), membership function (MFn), non-membership function (NMFn), generalized value index form (GVIF), generalized ambiguity index form (GAIF), alpha cut and beta cut.
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1. INTRODUCTION

The conception of fuzzy sets came in to existence in the year 1965 by Zadeh [15].Atanassov [1] extended fuzzy sets with MFn to Intuitionistic Fuzzy sets which included NMFn along with MFn. The abstraction of special Intuitionistic Fuzzy Sets was initiated by Deng-Feng Li [8] .Conceptualization of Fuzzy sets are extensively utilised in research in numerous fields such as image processing, Aerospace, criminology, antiskid braking system, transmission systems, control of subway systems, unmanned helicopter, control of automatic exposure in video camera, field of medicine to diagonize a pattern of symptoms and in copious other fields. Bellman R.E and Zadeh L.A Introduced the concept of fuzzy in decision making in the year 1970 which paved way to analyse real time problems with multiple constraints.

Chanas S, Kolodziejckzy W, and Machaj A.A (1984) used the technique of parametric programming to solve transportation problem with fuzzy supply and fuzzy demand. Chanas S and Kuchta.D (1996) defined an algorithm to find an optimal solution of the transportation problem with fuzzy cost coefficients. Dinagar D.S. and Palanivel K (2009) proposed a method to solve a transportation problem in fuzzy environment. Malini P. and Ananthanarayanan M (2016) introduced a ranking method to solve the transportation problem with trapezoidal number. C. Veeramani, M. Joseph Robinson,S.Vasanthi (2020) introduced the concept of value and ambiguity based approach for solving the trapezoidal intuitionistic fuzzy transportation problem with total quantity discounts and incremental quantity discounts in which trapezoidal intuitionistic fuzzy transportation problem converted to transportation problem of parametric form based on the value and ambiguity indices.

In this paper section 1 states the applications of fuzzy theory in various fields and its contribution in the field of decision theory by other researchers,section 2 includes generalized MFn and generalized NMFn for GEIFN, section 3 defines alpha and beta cut of GEIFN, section 4 elaborates of GVIF and GAIF in measures of MFn and NMFn for GEIFN , section 5 elaborates the ranking measures to generate crisp values , section 6 elaborates on value and ambiguity index formulas of octagonal intutionistic fuzzy number and section 7 demonstrates an example on transportation problem to uphold the conception of GEIFN .

2. Generalized Even IFN:

A GEIFN $\tilde{A}_e^I = < (\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{a}_4, \tilde{a}_5, \tilde{a}_6, \tilde{a}_7, \tilde{a}_8 \ldots., \tilde{a}_n)(\tilde{b}_1, \tilde{b}_2, \tilde{b}_3, \tilde{b}_4, \tilde{b}_5, \tilde{b}_6, \tilde{b}_7, \tilde{b}_8 \ldots.., \tilde{b}_n);\ w_{\tilde{a}}, u_{\tilde{a}}$ > is a IFS on a set of real number R, where n is even and n≥4 whose MFn and NMFn are calculated as:

Generalized Non- Membership Function $\mu_{\tilde{e}^I}(x)$ for

Generalized Non- Membership Function $\vartheta_{\tilde{e}^I}(x)$ for GEIFN

$\frac{(1-w_1)(b_2-x)}{b_2-b_1}+w_1$	$b_1 \le x \le b_2$	n ≥6,n is even.
$\frac{\left(w_{\left(\left(\left\lceil\frac{n}{4}\right\rceil-1\right)-\left(\frac{j-1}{2}\right)\right)} - w_{\left(\left(\left\lceil\frac{n}{4}\right\rceil-1\right)-\left(\frac{j-3}{2}\right)\right)}\right)(b_{k+1}-x)}{b_{k+1}-b_k} + w_{\left(\left(\left\lceil\frac{n}{4}\right\rceil-1\right)-\left(\frac{j-3}{2}\right)\right)}$	$b_k \le x \le b_{k+1}$	$k_i=\left\lceil\frac{n}{2}\right\rceil-l_i$, $l_i = 3,5,7, \ldots \ldots,\ n \ge 2l_i + 4$.
$w_{\left(\left(\left\lceil\frac{n}{4}\right\rceil-1\right)-\left(\frac{j-2}{2}\right)\right)}$	$b_i \le x \le b_{i+1}$	i $=\left\lceil\frac{n}{2}\right\rceil - j$, j= 2,4,6,.. $n \ge 2j + 4$.
$\frac{\left(w_{\left(\left\lceil\frac{n}{4}\right\rceil-1\right)} - u_{\tilde{a}}\right)\left(b_{\left\lceil\frac{n}{2}\right\rceil}-x\right)}{b_{\left\lceil\frac{n}{2}\right\rceil}-b_{\left(\left\lceil\frac{n}{2}\right\rceil-1\right)}} + u_{\tilde{a}}$	$b_{\left(\left\lceil\frac{n}{2}\right\rceil-1\right)} \le x \le b_{\left\lceil\frac{n}{2}\right\rceil}$	n ≥6,n is even.
$u_{\tilde{a}}$	$b_{\left\lceil\frac{n}{2}\right\rceil} \le x \le b_{\left\lceil\frac{n}{2}\right\rceil+1}$	n ≥6,n is even.
$\frac{\left(w_{\left(\left\lceil\frac{n}{4}\right\rceil-1\right)} - u_{\tilde{a}}\right)\left(x-b_{\left(\left\lceil\frac{n}{2}\right\rceil+1\right)}\right)}{b_{\left(\left\lceil\frac{n}{2}\right\rceil+2\right)}-b_{\left(\left\lceil\frac{n}{2}\right\rceil+1\right)}} + u_{\tilde{a}}$	$b_{\left(\left\lceil\frac{n}{2}\right\rceil+1\right)} \le x \le b_{\left(\left\lceil\frac{n}{2}\right\rceil+2\right)}$	n ≥6,n is even.
$w_{\left(\left(\left\lceil\frac{n}{4}\right\rceil-1\right)-\left(\frac{j-2}{2}\right)\right)}$	$b_i \le x \le b_{i+1}$	i $=\left\lceil\frac{n}{2}\right\rceil + j$, j= 2,4,6,.. $n \ge 2j + 4$.

$\dfrac{\left(w_{\left(\left(\left[\frac{n}{4}\right]-1\right)-\left(\frac{s-1}{2}\right)\right)} - w_{\left(\left(\left[\frac{n}{4}\right]-1\right)-\left(\frac{s-3}{2}\right)\right)}\right)(x-b_r)}{b_{r+1}-b_r} + w_{\left(\left(\left[\frac{n}{4}\right]-1\right)-\left(\frac{s-3}{2}\right)\right)}$	$b_r \le x \le b_{r+1}$	$r_i = \left[\frac{n}{2}\right] + s_i$, s_i= 3,5,7,.. $n \ge 2s_i$ +4.
$\dfrac{(1-w_1)(x-b_{n-1})}{b_n - b_{n-1}} + w_1$	$b_{n-1} \le x \le b_n$	n ≥6,n is even.
Otherwise	1	

The membership degree $w_{\tilde{a}}$ to its maximum and the degree of non-membership to its minimum $u_{\tilde{a}}$ accomplish the conditions $0 \le w_{\tilde{a}} \le 1$, $0 \le u_{\tilde{a}} \le 1$ and $0 \le w_{\tilde{a}} + u_{\tilde{a}} \le 1$.The level of confidence and non-confidence are reflected by the terms $w_{\tilde{a}}$ and $u_{\tilde{a}}$ of the Generalized Even Intuitionistic Fuzzy Number(GEIFN)
$\tilde{A}_e^I$= < $(\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{a}_4, \tilde{a}_5, \tilde{a}_6, \tilde{a}_7, \tilde{a}_8 \ldots, \tilde{a}_n)(\tilde{b}_1, \tilde{b}_2, \tilde{b}_3, \tilde{b}_4, \tilde{b}_5, \tilde{b}_6, \tilde{b}_7, \tilde{b}_8 \ldots, \tilde{b}_n)$; $w_{\tilde{a}}$, $u_{\tilde{a}}$ > .
Let $\pi_{\tilde{a}^I}$(x)=1-$\mu_{\tilde{e}^I}$(x)- $\vartheta_{\tilde{e}^I}$ (x), Then an element x in $\tilde{e}^I$, an unpredictable factor index of membership degree. For any x∈ R, $\mu_{\tilde{e}^I}$(x)+$\vartheta_{\tilde{e}^I}$ (x)=1. If $w_{\tilde{a}^I}$ =1 and $u_{\tilde{a}^I}$ = 0 then $\tilde{A}_e$ =<$(\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{a}_4, \tilde{a}_5, \tilde{a}_6, \tilde{a}_7, \tilde{a}_8 \ldots, \tilde{a}_n)$ 1, 0 > is a even fuzzy number.

3. GEIFN Alpha cut sets and Beta cut sets

Definition 1 :

An α -cut set of a GEIFN $\tilde{A}_e^I$= < $(\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{a}_4, \tilde{a}_5, \tilde{a}_6, \tilde{a}_7, \tilde{a}_8 \ldots, \tilde{a}_n)(\tilde{b}_1, \tilde{b}_2, \tilde{b}_3, \tilde{b}_4, \tilde{b}_5, \tilde{b}_6, \tilde{b}_7, \tilde{b}_8 \ldots, \tilde{b}_n)$; $w_{\tilde{a}}$, $u_{\tilde{a}}$ > is a crisp subset of R defined as $\tilde{A}_\alpha^I$ = {x / $\mu_{\tilde{e}^I}$(x) ≥ α} where 0≤ $\alpha \le w_{\tilde{a}}$.

Definition 2 :

An β -cut set of a GEIFN $\tilde{A}_e^I$= < $(\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{a}_4, \tilde{a}_5, \tilde{a}_6, \tilde{a}_7, \tilde{a}_8 \ldots, \tilde{a}_n)(\tilde{b}_1, \tilde{b}_2, \tilde{b}_3, \tilde{b}_4, \tilde{b}_5, \tilde{b}_6, \tilde{b}_7, \tilde{b}_8 \ldots, \tilde{b}_n)$; $w_{\tilde{a}}$, $u_{\tilde{a}}$ > is a crisp subset of R defined as $\tilde{A}_\beta^I$ = {x / $\vartheta_{\tilde{e}^I}$(x) ≤ β} where $u_{\tilde{a}} \le \beta \le 1$.
From these definition1 and definition 2 we observe that $\tilde{A}_\alpha^I$ and $\tilde{A}_\beta^I$ are both closed sets and are indicated by $\tilde{A}_\alpha^I$ = $\left[\tilde{A}^{I^L}_\alpha, \tilde{A}^{I^R}_\alpha\right]$ and $\tilde{A}_\beta^I$ = $\left[\tilde{A}^{I^L}_\beta, \tilde{A}^{I^R}_\beta\right]$ repectively.

The α - cut of GEIFN is defined by

$$\tilde{A}_\alpha^I=\left[\tilde{A}^{I^L}_\alpha, \tilde{A}^{I^R}_\alpha\right] = \begin{cases} \left(\tilde{A}^{I^L}_\alpha\right)_1, \left(\tilde{A}^{I^R}_\alpha\right)_1 & \alpha \in \left[0, w_{\left(\left[\frac{n}{4}\right]-1\right)}\right] \\ \left(\tilde{A}^{I^L}_\alpha\right)_2, \left(\tilde{A}^{I^R}_\alpha\right)_2 & \alpha \in \left[w_{\left(\left[\frac{n}{4}\right]-1\right)}, w_{t_1}\right] \\ \left(\tilde{A}^{I^L}_\alpha\right)_3, \left(\tilde{A}^{I^R}_\alpha\right)_3 & \alpha \in \left[w_{t_1}, w_{t_2}\right] \\ \left(\tilde{A}^{I^L}_\alpha\right)_4, \left(\tilde{A}^{I^R}_\alpha\right)_4 & \alpha \in \left[w_{t_2}, w_{t_3}\right] \\ \quad\vdots & \\ \left(\tilde{A}^{I^L}_\alpha\right)_{\left[\frac{n}{4}\right]-1}, \left(\tilde{A}^{I^R}_\alpha\right)_{\left[\frac{n}{4}\right]-1} & \alpha \in \left[w_{t_{p-1}}, w_{t_p}\right] \\ \left(\tilde{A}^{I^L}_\alpha\right)_{\left[\frac{n}{4}\right]}, \left(\tilde{A}^{I^R}_\alpha\right)_{\left[\frac{n}{4}\right]} & \alpha \in \left[w_1, w_{\tilde{a}}\right] \end{cases}$$

The β - cut of GEIFN is defined by

$$\tilde{A}^{I}_{\beta}=\left[\tilde{A}^{I}{}^{L}_{\beta}, \tilde{A}^{I}{}^{R}_{\beta}\right]=\begin{cases} \left(\tilde{A}^{I}{}^{L}_{\beta}\right)_1, \left(\tilde{A}^{I}{}^{R}_{\beta}\right)_1 & \beta \in \left[u_{\tilde{a}}, w_{\left(\left\lceil\frac{n}{4}\right\rceil-1\right)}\right] \\ \left(\tilde{A}^{I}{}^{L}_{\beta}\right)_2, \left(\tilde{A}^{I}{}^{R}_{\beta}\right)_2 & \beta \in \left[w_{\left(\left\lceil\frac{n}{4}\right\rceil-1\right)}, w_{t_1}\right] \\ \left(\tilde{A}^{I}{}^{L}_{\beta}\right)_3, \left(\tilde{A}^{I}{}^{R}_{\beta}\right)_3 & \beta \in \left[w_{t_1}, w_{t_2}\right] \\ \left(\tilde{A}^{I}{}^{L}_{\beta}\right)_4, \left(\tilde{A}^{I}{}^{R}_{\beta}\right)_4 & \beta \in \left[w_{t_2}, w_{t_3}\right] \\ \vdots & \\ \left(\tilde{A}^{I}{}^{L}_{\beta}\right)_{\left\lceil\frac{n}{4}\right\rceil-1}, \left(\tilde{A}^{I}{}^{R}_{\beta}\right)_{\left\lceil\frac{n}{4}\right\rceil-1} & \beta \in \left[w_{t_{p-1}}, w_{t_p}\right] \\ \left(\tilde{A}^{I}{}^{L}_{\beta}\right)_{\left\lceil\frac{n}{4}\right\rceil}, \left(\tilde{A}^{I}{}^{R}_{\beta}\right)_{\left\lceil\frac{n}{4}\right\rceil} & \beta \in [w_1, 1] \end{cases}$$

4. RANKING OF GENERALISED EVEN IFNS

The index of value and index of ambiguity of an even IFN are defined as those of TrIFNs by D.F.Li[8].

Definition 3: Let $\tilde{A}^{I}_{\alpha}$ and $\tilde{A}^{I}_{\beta}$ be an α-cut set and β-cut set of an Even IFN $\tilde{A}^{I}_{e}$= < $(\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{a}_4, \tilde{a}_5, \tilde{a}_6, \tilde{a}_7, \tilde{a}_8 \ldots, \tilde{a}_n)(\tilde{b}_1, \tilde{b}_2, \tilde{b}_3, \tilde{b}_4, \tilde{b}_5, \tilde{b}_6, \tilde{b}_7, \tilde{b}_8 \ldots, \tilde{b}_n)$; $w_{\tilde{a}}$, $u_{\tilde{a}}$ > respectively. Then the values of the MFn $\mu_{\tilde{A}^{I}_{e}}$ (x) and the values of the NMFn $\vartheta_{\tilde{A}^{I}_{e}}$ (x) for the Even IFN $\tilde{A}^{I}_{e}$ are formulated as follows

$$V_{\mu}\left(\tilde{A}^{I}_{e}\right) = \int_0^{w_{\tilde{a}}} \frac{L_{\tilde{A}^{I}_{e}}(\alpha)+R_{\tilde{A}^{I}_{e}}(\alpha)}{2} f(\alpha)d\alpha$$

$$V_{\vartheta}\left(\tilde{A}^{I}_{e}\right) = \int_{u_{\tilde{a}}}^{1} \frac{L_{\tilde{A}^{I}_{e}}(\beta)+R_{\tilde{A}^{I}_{e}}(\beta)}{2} g(\beta)d\beta$$

Respectively, on the interval [0, $w_{\tilde{a}}$] the function f(α) is a non-negative and non-decreasing function with $f(\alpha) = \frac{2\alpha}{w_{\tilde{a}}}$,where $\alpha \in [0, w_{\tilde{a}}]$ and f(0)=0 where $\int_0^{w_{\tilde{a}}} f(\alpha)d\alpha = w_{\tilde{a}}$. on the interval [$u_{\tilde{a}}$,1] The function g(β) is a non-negative and non-increasing function with $g(\beta) = \frac{2(1-\beta)}{1-u_{\tilde{a}}}$, where $\beta \in [u_{\tilde{a}},1]$ and g(1)=0 where $\int_{u_{\tilde{a}}}^{1} g(\beta)d\beta = 1-u_{\tilde{a}}$.

4.1 The generalized value index of MFn for GEIFN

$$V_{\mu}\left(\tilde{A}^{I}_{e}\right) = \frac{\left(w_{\left(\left\lceil\frac{n}{4}\right\rceil-1\right)}\right)^2\left(\tilde{a}_1+2\tilde{a}_2+2\tilde{a}_{n-1}+\tilde{a}_n\right)}{6w_{\tilde{a}}}$$

$$+\frac{3\left(\left(w_{t_1}\right)^2-\left(w_{\left(\left\lceil\frac{n}{4}\right\rceil-1\right)}\right)^2\right)\left(\tilde{a}_{k_1} w_{\left(\left\lceil\frac{k_1+1}{k_1}\right\rceil+\left(\frac{l_1-3}{2}\right)\right)}-\tilde{a}_{k_1+1} w_{\left(\left\lceil\frac{k_1+1}{k_1}\right\rceil+\left(\frac{l_1-2}{2}\right)\right)}\right)+2\left(\left(w_{t_1}\right)^3-\left(w_{\left(\left\lceil\frac{n}{4}\right\rceil-1\right)}\right)^3\right)\left(\tilde{a}_{k_1+1}-\tilde{a}_{k_1}\right)}{6w_{\tilde{a}}\left(w_{\left(\left\lceil\frac{k_1+1}{k_1}\right\rceil+\left(\frac{l_1-3}{2}\right)\right)}-w_{\left(\left\lceil\frac{k_1+1}{k_1}\right\rceil+\left(\frac{l_1-1}{2}\right)\right)}\right)}$$

$$+\frac{3\left(\left(w_{t_1}\right)^2-\left(w_{\left(\left\lceil\frac{n}{4}\right\rceil-1\right)}\right)^2\right)\left(\tilde{a}_{r_1+1} w_{\left(\left\lceil\frac{r_1+1}{r_1}\right\rceil+\left(\frac{s_1-3}{2}\right)\right)}-\tilde{a}_{r_1} w_{\left(\left\lceil\frac{r_1+1}{r_1}\right\rceil+\left(\frac{s_1-1}{2}\right)\right)}\right)-2\left(\left(w_{t_1}\right)^3-\left(w_{\left(\left\lceil\frac{n}{4}\right\rceil-1\right)}\right)^3\right)\left(\tilde{a}_{r_1+1}-\tilde{a}_{r_1}\right)}{6w_{\tilde{a}}\left(w_{\left(\left\lceil\frac{r_1+1}{r_1}\right\rceil+\left(\frac{s_1-3}{2}\right)\right)}-w_{\left(\left\lceil\frac{r_1+1}{r_1}\right\rceil+\left(\frac{s_1-1}{2}\right)\right)}\right)}$$

$$+\frac{3\left(\left(w_{t_2}\right)^2-\left(w_{t_1}\right)^2\right)\left(\tilde{a}_{k_2} w_{\left(\left\lceil\frac{k_2+1}{k_2}\right\rceil+\left(\frac{l_2-3}{2}\right)\right)}-\tilde{a}_{k_2+1} w_{\left(\left\lceil\frac{k_2+1}{k_2}\right\rceil+\left(\frac{l_2-2}{2}\right)\right)}\right)+2\left(\left(w_{t_2}\right)^3-\left(w_{t_1}\right)^3\right)\left(\tilde{a}_{k_2+1}-\tilde{a}_{k_2}\right)}{6w_{\tilde{a}}\left(w_{\left(\left\lceil\frac{k_2+1}{k_2}\right\rceil+\left(\frac{l_2-3}{2}\right)\right)}-w_{\left(\left\lceil\frac{k_2+1}{k_2}\right\rceil+\left(\frac{l_2-1}{2}\right)\right)}\right)}$$

$$+\frac{3\left(\left(w_{t_2}\right)^2-\left(w_{t_1}\right)^2\right)\left(\tilde{a}_{r_2+1} w_{\left(\left\lceil\frac{r_2+1}{r_2}\right\rceil+\left(\frac{s_2-3}{2}\right)\right)}-\tilde{a}_{r_2} w_{\left(\left\lceil\frac{r_2+1}{r_2}\right\rceil+\left(\frac{s_2-1}{2}\right)\right)}\right)-2\left(\left(w_{t_2}\right)^3-\left(w_{t_1}\right)^3\right)\left(\tilde{a}_{r_2+1}-\tilde{a}_{r_2}\right)}{6w_{\tilde{a}}\left(w_{\left(\left\lceil\frac{r_2+1}{r_2}\right\rceil+\left(\frac{s_2-3}{2}\right)\right)}-w_{\left(\left\lceil\frac{r_2+1}{r_2}\right\rceil+\left(\frac{s_2-1}{2}\right)\right)}\right)}$$

$$+\frac{3\left((w_{t_3})^2-(w_{t_2})^2\right)\left(\tilde{a}_{k_3} w_{\left(\left|\frac{k_3+1}{k_3}\right|+\left(\frac{l_3-3}{2}\right)\right)}-\tilde{a}_{k_3+1} w_{\left(\left|\frac{k_3+1}{k_3}\right|+\left(\frac{l_3-1}{2}\right)\right)}\right)+2\left((w_{t_3})^3-(w_{t_2})^3\right)(\tilde{a}_{k_3+1}-\tilde{a}_{k_3})}{6w_{\tilde{a}}\left(w_{\left(\left|\frac{k_3+1}{k_3}\right|+\left(\frac{l_3-2}{2}\right)\right)}-w_{\left(\left|\frac{k_3+1}{k_3}\right|+\left(\frac{l_3-1}{2}\right)\right)}\right)}$$

$$+\frac{3\left((w_{t_3})^2-(w_{t_2})^2\right)\left(\tilde{a}_{r_3+1} w_{\left(\left|\frac{r_3+1}{r_3}\right|+\left(\frac{s_3-3}{2}\right)\right)}-\tilde{a}_{r_3} w_{\left(\left|\frac{r_3+1}{r_3}\right|+\left(\frac{s_3-1}{2}\right)\right)}\right)-2\left((w_{t_3})^3-(w_{t_2})^3\right)(\tilde{a}_{r_3+1}-\tilde{a}_{r_3})}{6w_{\tilde{a}}\left(w_{\left(\left|\frac{r_3+1}{r_3}\right|+\left(\frac{s_3-3}{2}\right)\right)}-w_{\left(\left|\frac{r_3+1}{r_3}\right|+\left(\frac{s_3-1}{2}\right)\right)}\right)}$$

………

$$+\frac{3\left((w_{t_p})^2-(w_{t_{p-1}})^2\right)\left(\tilde{a}_{k_{\left[\frac{n}{4}\right]-2}} w_{\left(\left|\frac{k_{\left(\left[\frac{n}{4}\right]-2\right)+1}}{k_{\left[\frac{n}{4}\right]-2}}\right|+\left(\frac{l_{\left(\left[\frac{n}{4}\right]-2\right)-3}}{2}\right)\right)}-\tilde{a}_{k_{\left(\left[\frac{n}{4}\right]-2\right)+1}} w_{\left(\left|\frac{k_{\left(\left[\frac{n}{4}\right]-2\right)+1}}{k_{\left[\frac{n}{4}\right]-2}}\right|+\left(\frac{l_{\left(\left[\frac{n}{4}\right]-2\right)-1}}{2}\right)\right)}\right)+2\left((w_{t_p})^3-(w_{t_{p-1}})^3\right)\left(\tilde{a}_{k_{\left(\left[\frac{n}{4}\right]-2\right)+1}}-\tilde{a}_{k_{\left[\frac{n}{4}\right]\ldots}}\right.}{6w_{\tilde{a}}\left(w_{\left(\left|\frac{k_{\left(\left[\frac{n}{4}\right]-2\right)+1}}{k_{\left[\frac{n}{4}\right]-2}}\right|+\left(\frac{l_{\left(\left[\frac{n}{4}\right]-2\right)-3}}{2}\right)\right)}-w_{\left(\left|\frac{k_{\left(\left[\frac{n}{4}\right]-2\right)+1}}{k_{\left[\frac{n}{4}\right]-2}}\right|+\left(\frac{l_{\left(\left[\frac{n}{4}\right]-2\right)-1}}{2}\right)\right)}\right)}$$

$$+\frac{3\left((w_{t_p})^2-(w_{t_{p-1}})^2\right)\left(\tilde{a}_{r_{\left(\left[\frac{n}{4}\right]-2\right)+1}} w_{\left(\left|\frac{r_{\left(\left[\frac{n}{4}\right]-2\right)+1}}{r_{\left(\left[\frac{n}{4}\right]-2\right)}}\right|+\left(\frac{s_{\left(\left[\frac{n}{4}\right]-2\right)-3}}{2}\right)\right)}-\tilde{a}_{r_{\left[\frac{n}{4}\right]-2}} w_{\left(\left|\frac{r_{\left(\left[\frac{n}{4}\right]-2\right)+1}}{r_{\left(\left[\frac{n}{4}\right]-2\right)}}\right|+\left(\frac{s_{\left(\left[\frac{n}{4}\right]-2\right)-1}}{2}\right)+1\right)}\right)-2\left((w_{t_p})^3-(w_{t_{p-1}})^3\right)\left(\tilde{a}_{r_{\left(\left[\frac{n}{4}\right]-2\right)+1}}-\tilde{a}_{r\ldots}\right.}{6w_{\tilde{a}}\left(w_{\left(\left|\frac{r_{\left(\left[\frac{n}{4}\right]-2\right)+1}}{r_{\left(\left[\frac{n}{4}\right]-2\right)}}\right|+\left(\frac{s_{\left(\left[\frac{n}{4}\right]-2\right)-3}}{2}\right)\right)}-w_{\left(\left|\frac{r_{\left(\left[\frac{n}{4}\right]-2\right)+1}}{r_{\left(\left[\frac{n}{4}\right]-2\right)}}\right|+\left(\frac{s_{\left(\left[\frac{n}{4}\right]-2\right)-1}}{2}\right)\right)}\right)}$$

$$+\frac{(w_{\tilde{a}}^3-w_1^3)\left(\tilde{a}_{\left(\left[\frac{n}{2}\right]\right)}-\tilde{a}_{\left(\left[\frac{n}{2}\right]-1\right)}\right)}{3w_{\tilde{a}}\left(w_{\tilde{a}}-w_{\left|\frac{\frac{n}{2}}{\frac{n}{2}+1}\right|}\right)}+\frac{(w_{\tilde{a}}^2-w_1^2)\left(\tilde{a}_{\left(\left[\frac{n}{2}\right]+2\right)}w_{\tilde{a}}-\tilde{a}_{\left(\left[\frac{n}{2}\right]+1\right)}w_{\left|\frac{\frac{n}{2}+2}{\frac{n}{2}+1}\right|}\right)}{2w_{\tilde{a}}\left(w_{\tilde{a}}-w_{\left|\frac{\frac{n}{2}+2}{\frac{n}{2}+1}\right|}\right)}+\frac{(w_{\tilde{a}}^2-w_1^2)\left(\tilde{a}_{\left(\left[\frac{n}{2}\right]-1\right)}w_{\tilde{a}}-\tilde{a}_{\left(\left[\frac{n}{2}\right]\right)}w_{\left|\frac{\frac{n}{2}}{\frac{n}{2}-1}\right|}\right)}{2w_{\tilde{a}}\left(w_{\tilde{a}}-w_{\left|\frac{\frac{n}{2}}{\frac{n}{2}-1}\right|}\right)}$$

$$-\frac{(w_{\tilde{a}}^3-w_1^3)\left(\tilde{a}_{\left(\left[\frac{n}{2}\right]+2\right)}-\tilde{a}_{\left(\left[\frac{n}{2}\right]+1\right)}\right)}{3w_{\tilde{a}}\left(w_{\tilde{a}}-w_{\left|\frac{\frac{n}{2}+2}{\frac{n}{2}+1}\right|}\right)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

4.2 The generalized value index formula (GVIF) of NMFn for GEIFN

$$V_{\vartheta}(\tilde{A}_e^I) = \frac{3\left(2w_{\left(\left[\frac{n}{4}\right]-1\right)}-\left(w_{\left(\left[\frac{n}{4}\right]-1\right)}\right)^2-2u_{\tilde{a}}+u_{\tilde{a}}^2\right)\left(\tilde{b}_{\left(\left[\frac{n}{2}\right]+1\right)}w_{\left(\left[\frac{n}{4}\right]-1\right)}+\tilde{b}_{\left[\frac{n}{2}\right]}w_{\left(\left[\frac{n}{4}\right]-1\right)}-u_{\tilde{a}}\left(\tilde{b}_{\left(\left[\frac{n}{2}\right]+2\right)}+\tilde{b}_{\left(\left[\frac{n}{2}\right]-1\right)}\right)\right)}{6(1-u_{\tilde{a}})\left(w_{\left(\left[\frac{n}{4}\right]-1\right)}-u_{\tilde{a}}\right)}$$

$$+\frac{\left(3\left(w_{\left(\left[\frac{n}{4}\right]-1\right)}\right)^2-2\left(w_{\left(\left[\frac{n}{4}\right]-1\right)}\right)^3-3u_{\tilde{a}}^2+2u_{\tilde{a}}^3\right)\left(\tilde{b}_{\left(\left[\frac{n}{2}\right]+2\right)}-\tilde{b}_{\left[\frac{n}{2}\right]}-\tilde{b}_{\left(\left[\frac{n}{2}\right]+1\right)}+\tilde{b}_{\left(\left[\frac{n}{2}\right]-1\right)}\right)}{6(1-u_{\tilde{a}})\left(w_{\left(\left[\frac{n}{4}\right]-1\right)}-u_{\tilde{a}}\right)}$$

$$+\frac{3\left(2w_{t_1}-(w_{t_1})^2-2w_{\left(\left[\frac{n}{4}\right]-1\right)}+\left(w_{\left(\left[\frac{n}{4}\right]-1\right)}\right)^2\right)\left(\tilde{b}_{r_1}w_{\left(\left(\left[\frac{n}{4}\right]-1\right)-\left(\frac{s_1-1}{2}\right)\right)}-\tilde{b}_{r_1+1}w_{\left(\left(\left[\frac{n}{4}\right]-1\right)-\left(\frac{s_1-3}{2}\right)\right)}\right)+\left(3(w_{t_1})^2-2(w_{t_1})^3-3\left(w_{\left(\left[\frac{n}{4}\right]-1\right)}\right)^2+2\left(w_{\left(\left[\frac{n}{4}\right]-1\right)}\right)^3\right)(\tilde{b}_{r_1}\ldots}{6(1-u_{\tilde{a}})\left(w_{\left(\left(\left[\frac{n}{4}\right]-1\right)-\left(\frac{s_1-1}{2}\right)\right)}-w_{\left(\left(\left[\frac{n}{4}\right]-1\right)-\left(\frac{s_1-3}{2}\right)\right)}\right)}$$

$$+\frac{3\left(2w_{t_1}-(w_{t_1})^2-2w_{(\lceil\frac{n}{4}\rceil-1)}+\left(w_{(\lceil\frac{n}{4}\rceil-1)}\right)^2\right)\left(\tilde{b}_{k_1+1}w_{\left((\lceil\frac{n}{4}\rceil-1)-(\frac{l_1-1}{2})\right)}-\tilde{b}_{k_1}w_{\left((\lceil\frac{n}{4}\rceil-1)-(\frac{l_1-3}{2})\right)}\right)-\left(3(w_{t_1})^2-2(w_{t_1})^3-3\left(w_{(\lceil\frac{n}{4}\rceil-1)}\right)^2+2\left(w_{(\lceil\frac{n}{4}\rceil-1)}\right)^3\right)(\tilde{b}_{k}}{6(1-u_{\tilde{a}})\left(w_{\left((\lceil\frac{n}{4}\rceil-1)-(\frac{l_1-1}{2})\right)}-w_{\left((\lceil\frac{n}{4}\rceil-1)-(\frac{l_1-3}{2})\right)}\right)}$$

$$+\frac{3\left(2w_{t_2}-(w_{t_2})^2-2w_{t_1}+(w_{t_1})^2\right)\left(\tilde{b}_{r_2}w_{\left((\lceil\frac{n}{4}\rceil-1)-(\frac{s_1-1}{2})\right)}-\tilde{b}_{r_2+1}w_{\left((\lceil\frac{n}{4}\rceil-1)-(\frac{s_2-3}{2})\right)}\right)+\left(3(w_{t_2})^2-2(w_{t_2})^3-3(w_{t_1})^2+2(w_{t_1})^3\right)(\tilde{b}_{r_2+1}-\tilde{b}_{r_2})}{6(1-u_{\tilde{a}})\left(w_{\left((\lceil\frac{n}{4}\rceil-1)-(\frac{s_2-1}{2})\right)}-w_{\left((\lceil\frac{n}{4}\rceil-1)-(\frac{s_2-3}{2})\right)}\right)}$$

$$+\frac{3\left(2w_{t_2}-(w_{t_2})^2-2w_{t_1}+(w_{t_1})^2\right)\left(\tilde{b}_{k_2+1}w_{\left((\lceil\frac{n}{4}\rceil-1)-(\frac{l_2-1}{2})\right)}-\tilde{b}_{k_2}w_{\left((\lceil\frac{n}{4}\rceil-1)-(\frac{l_2-3}{2})\right)}\right)-\left(3(w_{t_2})^2-2(w_{t_2})^3-3(w_{t_1})^2+2(w_{t_1})^3\right)(\tilde{b}_{k_2+1}-\tilde{b}_{k_2})}{6(1-u_{\tilde{a}})\left(w_{\left((\lceil\frac{n}{4}\rceil-1)-(\frac{l_2-1}{2})\right)}-w_{\left((\lceil\frac{n}{4}\rceil-1)-(\frac{l_2-3}{2})\right)}\right)}$$

$$+ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots +$$

$$+\frac{3\left(2w_{t_p}-(w_{t_p})^2-2w_{t_{p-1}}+(w_{t_{p-1}})^2\right)\left(\tilde{b}_{r_{(\lceil\frac{n}{4}\rceil-2)}}w_{\left((\lceil\frac{n}{4}\rceil-1)-\left(\frac{s_{(\lceil\frac{n}{4}\rceil-2)}-1}{2}\right)\right)}-\tilde{b}_{r_{(\lceil\frac{n}{4}\rceil-2)}+1}w_{\left((\lceil\frac{n}{4}\rceil-1)-\left(\frac{s_{(\lceil\frac{n}{4}\rceil-2)}-3}{2}\right)\right)}\right)}{6(1-u_{\tilde{a}})\left(w_{\left((\lceil\frac{n}{4}\rceil-1)-\left(\frac{s_{(\lceil\frac{n}{4}\rceil-2)}-1}{2}\right)\right)}-w_{\left((\lceil\frac{n}{4}\rceil-1)-\left(\frac{s_{(\lceil\frac{n}{4}\rceil-2)}-3}{2}\right)\right)}\right)}$$

$$+\frac{\left(3(w_{t_p})^2-2(w_{t_p})^3-3(w_{t_{p-1}})^2+2(w_{t_{p-1}})^3\right)\left(\tilde{b}_{r_{(\lceil\frac{n}{4}\rceil-2)}+1}-\tilde{b}_{r_{(\lceil\frac{n}{4}\rceil-2)}}\right)}{6(1-u_{\tilde{a}})\left(w_{\left((\lceil\frac{n}{4}\rceil-1)-\left(\frac{s_{(\lceil\frac{n}{4}\rceil-2)}-1}{2}\right)\right)}-w_{\left((\lceil\frac{n}{4}\rceil-1)-\left(\frac{s_{(\lceil\frac{n}{4}\rceil-2)}-3}{2}\right)\right)}\right)}$$

$$+\frac{3\left(2w_{t_p}-(w_{t_p})^2-2w_{t_{p-1}}+(w_{t_{p-1}})^2\right)\left(\tilde{b}_{k_{(\lceil\frac{n}{4}\rceil-2)}+1}w_{\left((\lceil\frac{n}{4}\rceil-1)-\left(\frac{l_{(\lceil\frac{n}{4}\rceil-2)}-1}{2}\right)\right)}-\tilde{b}_{k_{(\lceil\frac{n}{4}\rceil-2)}}w_{\left((\lceil\frac{n}{4}\rceil-1)-\left(\frac{l_{(\lceil\frac{n}{4}\rceil-2)}-3}{2}\right)\right)}\right)}{6(1-u_{\tilde{a}})\left(w_{\left((\lceil\frac{n}{4}\rceil-1)-\left(\frac{l_{(\lceil\frac{n}{4}\rceil-2)}-1}{2}\right)\right)}-w_{\left((\lceil\frac{n}{4}\rceil-1)-\left(\frac{l_{(\lceil\frac{n}{4}\rceil-2)}-3}{2}\right)\right)}\right)}$$

$$-\frac{\left(3(w_{t_p})^2-2(w_{t_p})^3-3(w_{t_{p-1}})^2+2(w_{t_{p-1}})^3\right)\left(\tilde{b}_{k_{(\lceil\frac{n}{4}\rceil-2)}+1}-\tilde{b}_{k_{(\lceil\frac{n}{4}\rceil-2)}}\right)}{6(1-u_{\tilde{a}})\left(w_{\left((\lceil\frac{n}{4}\rceil-1)-\left(\frac{l_{(\lceil\frac{n}{4}\rceil-2)}-1}{2}\right)\right)}-w_{\left((\lceil\frac{n}{4}\rceil-1)-\left(\frac{l_{(\lceil\frac{n}{4}\rceil-2)}-3}{2}\right)\right)}\right)}+\frac{(1-w_1)\left(\tilde{b}_{n-1}-w_1(\tilde{b}_n+\tilde{b}_1)+\tilde{b}_2\right)}{2(1-u_{\tilde{a}})}$$

$$+\frac{(1-3w_1^2+2w_1^3)(\tilde{b}_n-\tilde{b}_{n-1}-\tilde{b}_2+\tilde{b}_1)}{6(1-u_{\tilde{a}})((1-w_1))} \qquad \ldots\ldots\ldots\ldots (2)$$

With $0\leq w_{\tilde{a}}+u_{\tilde{a}}\leq 1$ and $V_\mu(\tilde{A}_e^I)\leq V_\vartheta(\tilde{A}_e^I)$ the measures of the MFn and NMFn of an even IFN $\tilde{A}_e^I$ can be expressed in an interval $[V_\mu(\tilde{A}_e^I)\,,\ V_\vartheta(\tilde{A}_e^I)]$.

Definition 4: Let $\tilde{A}^I_{e_\alpha}$ and $\tilde{A}^I_{e_\beta}$ represent an α-cut set and β-cut set of an even IFN $\tilde{A}_e^I=<(\tilde{a}_1,\tilde{a}_2,\tilde{a}_3,\tilde{a}_4,\tilde{a}_5,\tilde{a}_6,\tilde{a}_7,\tilde{a}_8\ldots,\tilde{a}_n)(\tilde{b}_1,\tilde{b}_2,\tilde{b}_3,\tilde{b}_4,\tilde{b}_5,\tilde{b}_6,\tilde{b}_7,\tilde{b}_8\ldots,\tilde{b}_n);w_{\tilde{a}},u_{\tilde{a}}>$ respectively. Then the ambiguities which corresponds to the MFn $\mu_{\tilde{A}_e^I}$ (x) and the ambiguities which corresponds to the NMFn $\vartheta_{\tilde{A}_e^I}$ (x) for the GEIFN are defined below

$A_\mu(\tilde{A}_e^I) = \int_0^{w_{\tilde{a}}} R_{\tilde{A}_e^I}(\alpha)-L_{\tilde{A}_e^I}(\alpha)\, f(\alpha)d\alpha$

$A_\nu(\tilde{A}_e^I) = \int_{u_{\tilde{a}}}^{1} R_{\tilde{A}_e^I}(\beta)-L_{\tilde{A}_e^I}(\beta)\, g(\beta)d\beta$ respectively

From the definition of $A_\mu(\tilde{A}_e^I)$ and $A_\vartheta(\tilde{A}_e^I)$ it is followed that $A_\mu(\tilde{A}_e^I)\geq 0$, $A_\vartheta(\tilde{A}_e^I)\geq 0$.

4.3 The generalized ambiguity index formula of MFn for GEIFN

$$A_\mu\left(\tilde{A}_e^I\right) = \frac{\left(w_{\left(\left\lceil \frac{n}{4}\right\rceil -1\right)}\right)^2 (\tilde{a}_n+2\tilde{a}_{n-1}-2\tilde{a}_2-\tilde{a}_1)}{3w_{\tilde{a}}} +$$

$$\frac{3\left(\left(w_{t_1}\right)^2-\left(w_{\left(\left\lceil \frac{n}{4}\right\rceil -1\right)}\right)^2\right)\left(\tilde{a}_{r_1+1} w_{\left(\left|\frac{r_1+1}{r_1}\right|+\left(\frac{s_1-3}{2}\right)\right)} - \tilde{a}_{r_1} w_{\left(\left|\frac{r_1+1}{r_1}\right|+\left(\frac{s_1-1}{2}\right)\right)}\right) - 2\left(\left(w_{t_1}\right)^3-\left(w_{\left(\left\lceil \frac{n}{4}\right\rceil -1\right)}\right)^3\right)\left(\tilde{a}_{r_1+1}-\tilde{a}_{r_1}\right)}{3w_{\tilde{a}}\left(w_{\left(\left|\frac{r_1+1}{r_1}\right|+\left(\frac{s_1-3}{2}\right)\right)} - w_{\left(\left|\frac{r_1+1}{r_1}\right|+\left(\frac{s_1-1}{2}\right)\right)}\right)}$$

$$- \frac{3\left(\left(w_{t_1}\right)^2-\left(w_{\left(\left\lceil \frac{n}{4}\right\rceil -1\right)}\right)^2\right)\left(\tilde{a}_{k_1} w_{\left(\left|\frac{k_1+1}{k_1}\right|+\left(\frac{l_1-3}{2}\right)\right)} - \tilde{a}_{k_1+1} w_{\left(\left|\frac{k_1+1}{k_1}\right|+\left(\frac{l_1-2}{2}\right)\right)}\right) + 2\left(\left(w_{t_1}\right)^3-\left(w_{\left(\left\lceil \frac{n}{4}\right\rceil -1\right)}\right)^3\right)\left(\tilde{a}_{k_1+1}-\tilde{a}_{k_1}\right)}{3w_{\tilde{a}}\left(w_{\left(\left|\frac{k_1+1}{k_1}\right|+\left(\frac{l_1-3}{2}\right)\right)} - w_{\left(\left|\frac{k_1+1}{k_1}\right|+\left(\frac{l_1-1}{2}\right)\right)}\right)}$$

$$+ \frac{3\left(\left(w_{t_2}\right)^2-\left(w_{t_1}\right)^2\right)\left(\tilde{a}_{r_2+1} w_{\left(\left|\frac{r_2+1}{r_2}\right|+\left(\frac{s_2-3}{2}\right)\right)} - \tilde{a}_{r_2} w_{\left(\left|\frac{r_2+1}{r_2}\right|+\left(\frac{s_2-1}{2}\right)\right)}\right) - 2\left(\left(w_{t_2}\right)^3-\left(w_{t_1}\right)^3\right)\left(\tilde{a}_{r_2+1}-\tilde{a}_{r_2}\right)}{3w_{\tilde{a}}\left(w_{\left(\left|\frac{r_2+1}{r_2}\right|+\left(\frac{s_2-3}{2}\right)\right)} - w_{\left(\left|\frac{r_2+1}{r_2}\right|+\left(\frac{s_2-1}{2}\right)\right)}\right)}$$

$$- \frac{3\left(\left(w_{t_2}\right)^2-\left(w_{t_1}\right)^2\right)\left(\tilde{a}_{k_2} w_{\left(\left|\frac{k_2+1}{k_2}\right|+\left(\frac{l_2-3}{2}\right)\right)} - \tilde{a}_{k_2+1} w_{\left(\left|\frac{k_2+1}{k_2}\right|+\left(\frac{l_2-2}{2}\right)\right)}\right) + 2\left(\left(w_{t_2}\right)^3-\left(w_{t_1}\right)^3\right)\left(\tilde{a}_{k_2+1}-\tilde{a}_{k_2}\right)}{3w_{\tilde{a}}\left(w_{\left(\left|\frac{k_2+1}{k_2}\right|+\left(\frac{l_2-3}{2}\right)\right)} - w_{\left(\left|\frac{k_2+1}{k_2}\right|+\left(\frac{l_2-1}{2}\right)\right)}\right)}$$

$$+ \frac{3\left(\left(w_{t_3}\right)^2-\left(w_{t_2}\right)^2\right)\left(\tilde{a}_{r_3+1} w_{\left(\left|\frac{r_3+1}{r_3}\right|+\left(\frac{s_3-3}{2}\right)\right)} - \tilde{a}_{r_3} w_{\left(\left|\frac{r_3+1}{r_3}\right|+\left(\frac{s_3-1}{2}\right)\right)}\right) - 2\left(\left(w_{t_3}\right)^3-\left(w_{t_2}\right)^3\right)\left(\tilde{a}_{r_3+1}-\tilde{a}_{r_3}\right)}{3w_{\tilde{a}}\left(w_{\left(\left|\frac{r_3+1}{r_3}\right|+\left(\frac{s_3-3}{2}\right)\right)} - w_{\left(\left|\frac{r_3+1}{r_3}\right|+\left(\frac{s_3-1}{2}\right)\right)}\right)}$$

$$- \frac{3\left(\left(w_{t_3}\right)^2-\left(w_{t_2}\right)^2\right)\left(\tilde{a}_{k_3} w_{\left(\left|\frac{k_3+1}{k_3}\right|+\left(\frac{l_3-3}{2}\right)\right)} - \tilde{a}_{k_3+1} w_{\left(\left|\frac{k_3+1}{k_3}\right|+\left(\frac{l_3-1}{2}\right)\right)}\right) + 2\left(\left(w_{t_3}\right)^3-\left(w_{t_2}\right)^3\right)\left(\tilde{a}_{k_3+1}-\tilde{a}_{k_3}\right)}{3w_{\tilde{a}}\left(w_{\left(\left|\frac{k_3+1}{k_3}\right|+\left(\frac{l_3-2}{2}\right)\right)} - w_{\left(\left|\frac{k_3+1}{k_3}\right|+\left(\frac{l_3-1}{2}\right)\right)}\right)}$$

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$$+ \frac{3\left(\left(w_{t_p}\right)^2-\left(w_{t_{p-1}}\right)^2\right)\left(\tilde{a}_{r_{\left(\left\lceil \frac{n}{4}\right\rceil -2\right)}+1} w_{\left(\left|\frac{r_{\left(\left\lceil \frac{n}{4}\right\rceil -2\right)}+1}{r_{\left(\left\lceil \frac{n}{4}\right\rceil -2\right)}}\right|+\left(\frac{s_{\left(\left\lceil \frac{n}{4}\right\rceil -2\right)}-3}{2}\right)\right)} - \tilde{a}_{r_{\left\lceil \frac{n}{4}\right\rceil -2}} w_{\left(\left|\frac{r_{\left(\left\lceil \frac{n}{4}\right\rceil -2\right)}+1}{r_{\left(\left\lceil \frac{n}{4}\right\rceil -2\right)}}\right|+\left(\frac{s_{\left(\left\lceil \frac{n}{4}\right\rceil -2\right)}-1}{2}\right)+1\right)}\right) - 2\left(\left(w_{t_p}\right)^3-\left(w_{t_{p-1}}\right)^3\right)\left(\tilde{a}_{r_{\left(\left\lceil \frac{n}{4}\right\rceil -2\right)}+1}-\tilde{a}_r\right.}{3w_{\tilde{a}}\left(w_{\left(\left|\frac{r_{\left(\left\lceil \frac{n}{4}\right\rceil -2\right)}+1}{r_{\left(\left\lceil \frac{n}{4}\right\rceil -2\right)}}\right|+\left(\frac{s_{\left(\left\lceil \frac{n}{4}\right\rceil -2\right)}-3}{2}\right)\right)} - w_{\left(\left|\frac{r_{\left(\left\lceil \frac{n}{4}\right\rceil -2\right)}+1}{r_{\left(\left\lceil \frac{n}{4}\right\rceil -2\right)}}\right|+\left(\frac{s_{\left(\left\lceil \frac{n}{4}\right\rceil -2\right)}-1}{2}\right)\right)}\right)}$$

$$- \frac{3\left(\left(w_{t_p}\right)^2-\left(w_{t_{p-1}}\right)^2\right)\left(\tilde{a}_{k_{\left\lceil \frac{n}{4}\right\rceil -2}} w_{\left(\left|\frac{k_{\left(\left\lceil \frac{n}{4}\right\rceil -2\right)}+1}{k_{\left\lceil \frac{n}{4}\right\rceil -2}}\right|+\left(\frac{l_{\left(\left\lceil \frac{n}{4}\right\rceil -2\right)}-3}{2}\right)\right)} - \tilde{a}_{k_4+1} w_{\left(\left|\frac{k_{\left(\left\lceil \frac{n}{4}\right\rceil -2\right)}+1}{k_{\left\lceil \frac{n}{4}\right\rceil -2}}\right|+\left(\frac{l_{\left(\left\lceil \frac{n}{4}\right\rceil -2\right)}-1}{2}\right)\right)}\right) + 2\left(\left(w_{t_p}\right)^3-\left(w_{t_{p-1}}\right)^3\right)\left(\tilde{a}_{k_{\left(\left\lceil \frac{n}{4}\right\rceil -2\right)}+1}-\tilde{a}_{k_{\left\lceil \frac{n}{4}\right\rceil -2}}\right)}{3w_{\tilde{a}}\left(w_{\left(\left|\frac{k_{\left(\left\lceil \frac{n}{4}\right\rceil -2\right)}+1}{k_{\left\lceil \frac{n}{4}\right\rceil -2}}\right|+\left(\frac{l_{\left(\left\lceil \frac{n}{4}\right\rceil -2\right)}-3}{2}\right)\right)} - w_{\left(\left|\frac{k_{\left(\left\lceil \frac{n}{4}\right\rceil -2\right)}+1}{k_{\left\lceil \frac{n}{4}\right\rceil -2}}\right|+\left(\frac{l_{\left(\left\lceil \frac{n}{4}\right\rceil -2\right)}-1}{2}\right)\right)}\right)}$$

$$- \frac{2\left(w_{\tilde{a}}^3-w_1^3\right)\left(\tilde{a}_{\left(\left|\frac{n}{2}\right|\right)}-\tilde{a}_{\left(\left|\frac{n}{2}\right|-1\right)}\right)}{3w_{\tilde{a}}\left(w_{\tilde{a}}-w_{\left|\frac{\frac{n}{2}}{\frac{n}{2}-1}\right|}\right)} + \frac{\left(w_{\tilde{a}}^2-w_1^2\right)\left(\tilde{a}_{\left(\left|\frac{n}{2}\right|+2\right)} w_{\tilde{a}} - \tilde{a}_{\left(\left|\frac{n}{2}\right|+1\right)} w_{\left|\frac{\frac{n}{2}+2}{\frac{n}{2}+1}\right|}\right)}{w_{\tilde{a}}\left(w_{\tilde{a}}-w_{\left|\frac{\frac{n}{2}+2}{\frac{n}{2}+1}\right|}\right)} - \frac{\left(w_{\tilde{a}}^2-w_1^2\right)\left(\tilde{a}_{\left(\left|\frac{n}{2}\right|-1\right)} w_{\tilde{a}} - \tilde{a}_{\left(\left|\frac{n}{2}\right|\right)} w_{\left|\frac{\frac{n}{2}}{\frac{n}{2}-1}\right|}\right)}{w_{\tilde{a}}\left(w_{\tilde{a}}-w_{\left|\frac{\frac{n}{2}}{\frac{n}{2}-1}\right|}\right)}$$

$$- \frac{2(w_{\tilde{a}}^3-w_1^3)\left(\tilde{a}_{\left(\left|\frac{n}{2}\right|+2\right)}-\tilde{a}_{\left(\left|\frac{n}{2}\right|+1\right)}\right)}{3w_{\tilde{a}}\left(w_{\tilde{a}}-w_{\left|\frac{\frac{n}{2}+2}{\frac{n}{2}+1}\right|}\right)} \quad .... (3)$$

4.4 The generalized ambiguity index formula (GAIF) of NMFn for GEIFN

$$A_{\vartheta}(\tilde{A}_e^I) = \frac{3\left(2w_{\left(\left|\frac{n}{4}\right|-1\right)}-\left(w_{\left(\left|\frac{n}{4}\right|-1\right)}\right)^2-2u_{\tilde{a}}+u_{\tilde{a}}^2\right)\left(\tilde{b}_{\left(\left|\frac{n}{2}\right|+1\right)}w_{\left(\left|\frac{n}{4}\right|-1\right)}-\tilde{b}_{\left|\frac{n}{2}\right|}w_{\left(\left|\frac{n}{4}\right|-1\right)}-u_{\tilde{a}}\left(\tilde{b}_{\left(\left|\frac{n}{2}\right|+2\right)}-\tilde{b}_{\left(\left|\frac{n}{2}\right|-1\right)}\right)\right)}{3(1-u_{\tilde{a}})\left(w_{\left(\left|\frac{n}{4}\right|-1\right)}-u_{\tilde{a}}\right)}$$

$$+\frac{\left(3\left(w_{\left(\left|\frac{n}{4}\right|-1\right)}\right)^2-2\left(w_{\left(\left|\frac{n}{4}\right|-1\right)}\right)^3-3u_{\tilde{a}}^2+2u_{\tilde{a}}^3\right)\left(\tilde{b}_{\left(\left|\frac{n}{2}\right|+2\right)}+\tilde{b}_{\left|\frac{n}{2}\right|}-\tilde{b}_{\left(\left|\frac{n}{2}\right|+1\right)}-\tilde{b}_{\left(\left|\frac{n}{2}\right|-1\right)}\right)}{3(1-u_{\tilde{a}})\left(w_{\left(\left|\frac{n}{4}\right|-1\right)}-u_{\tilde{a}}\right)}$$

$$+$$

$$\frac{3\left(2w_{t_1}-(w_{t_1})^2-2w_{\left(\left|\frac{n}{4}\right|-1\right)}+\left(w_{\left(\left|\frac{n}{4}\right|-1\right)}\right)^2\right)\left(\tilde{b}_{r_1}w_{\left(\left(\left|\frac{n}{4}\right|-1\right)-\left(\frac{s_1-1}{2}\right)\right)}-\tilde{b}_{r_1+1}w_{\left(\left(\left|\frac{n}{4}\right|-1\right)-\left(\frac{s_1-3}{2}\right)\right)}\right)+\left(3(w_{t_1})^2-2(w_{t_1})^3-3\left(w_{\left(\left|\frac{n}{4}\right|-1\right)}\right)^2+2\left(w_{\left(\left|\frac{n}{4}\right|-1\right)}\right)^3\right)(\tilde{b}_{r_1}}{3(1-u_{\tilde{a}})\left(w_{\left(\left(\left|\frac{n}{4}\right|-1\right)-\left(\frac{s_1-1}{2}\right)\right)}-w_{\left(\left(\left|\frac{n}{4}\right|-1\right)-\left(\frac{s_1-3}{2}\right)\right)}\right)}$$

$$-$$

$$\frac{3\left(2w_{t_1}-(w_{t_1})^2-2w_{\left(\left|\frac{n}{4}\right|-1\right)}+\left(w_{\left(\left|\frac{n}{4}\right|-1\right)}\right)^2\right)\left(\tilde{b}_{k_1+1}w_{\left(\left(\left|\frac{n}{4}\right|-1\right)-\left(\frac{l_1-1}{2}\right)\right)}-\tilde{b}_{k_1}w_{\left(\left(\left|\frac{n}{4}\right|-1\right)-\left(\frac{l_1-3}{2}\right)\right)}\right)-\left(3(w_{t_1})^2-2(w_{t_1})^3-3\left(w_{\left(\left|\frac{n}{4}\right|-1\right)}\right)^2+2\left(w_{\left(\left|\frac{n}{4}\right|-1\right)}\right)^3\right)(\tilde{b}_{k}}{3(1-u_{\tilde{a}})\left(w_{\left(\left(\left|\frac{n}{4}\right|-1\right)-\left(\frac{l_1-1}{2}\right)\right)}-w_{\left(\left(\left|\frac{n}{4}\right|-1\right)-\left(\frac{l_1-3}{2}\right)\right)}\right)}$$

$$+\frac{3\left(2w_{t_2}-(w_{t_2})^2-2w_{t_1}+(w_{t_1})^2\right)\left(\tilde{b}_{r_2}w_{\left(\left(\left|\frac{n}{4}\right|-1\right)-\left(\frac{s_1-1}{2}\right)\right)}-\tilde{b}_{r_2+1}w_{\left(\left(\left|\frac{n}{4}\right|-1\right)-\left(\frac{s_2-3}{2}\right)\right)}\right)+\left(3(w_{t_2})^2-2(w_{t_2})^3-3(w_{t_1})^2+2(w_{t_1})^3\right)(\tilde{b}_{r_2+1}-\tilde{b}_{r_2})}{3(1-u_{\tilde{a}})\left(w_{\left(\left(\left|\frac{n}{4}\right|-1\right)-\left(\frac{s_2-1}{2}\right)\right)}-w_{\left(\left(\left|\frac{n}{4}\right|-1\right)-\left(\frac{s_2-3}{2}\right)\right)}\right)}$$

$$-\frac{3\left(2w_{t_2}-(w_{t_2})^2-2w_{t_1}+(w_{t_1})^2\right)\left(\tilde{b}_{k_2+1}w_{\left(\left(\left|\frac{n}{4}\right|-1\right)-\left(\frac{l_2-1}{2}\right)\right)}-\tilde{b}_{k_2}w_{\left(\left(\left|\frac{n}{4}\right|-1\right)-\left(\frac{l_2-3}{2}\right)\right)}\right)-\left(3(w_{t_2})^2-2(w_{t_2})^3-3(w_{t_1})^2+2(w_{t_1})^3\right)(\tilde{b}_{k_2+1}-\tilde{b}_{k_2})}{3(1-u_{\tilde{a}})\left(w_{\left(\left(\left|\frac{n}{4}\right|-1\right)-\left(\frac{l_2-1}{2}\right)\right)}-w_{\left(\left(\left|\frac{n}{4}\right|-1\right)-\left(\frac{l_2-3}{2}\right)\right)}\right)}$$ 9143

$$+ .... +$$

$$+\frac{3\left(2w_{t_p}-(w_{t_p})^2-2w_{t_{p-1}}+(w_{t_{p-1}})^2\right)\left(\tilde{b}_{r_{\left(\left|\frac{n}{4}\right|-2\right)}}w_{\left(\left(\left|\frac{n}{4}\right|-1\right)-\left(\frac{s_{\left(\left|\frac{n}{4}\right|-2\right)}-1}{2}\right)\right)}-\tilde{b}_{r_{\left(\left|\frac{n}{4}\right|-2\right)}+1}w_{\left(\left(\left|\frac{n}{4}\right|-1\right)-\left(\frac{s_{\left(\left|\frac{n}{4}\right|-2\right)}-3}{2}\right)\right)}\right)}{3(1-u_{\tilde{a}})\left(w_{\left(\left(\left|\frac{n}{4}\right|-1\right)-\left(\frac{s_{\left(\left|\frac{n}{4}\right|-2\right)}-1}{2}\right)\right)}-w_{\left(\left(\left|\frac{n}{4}\right|-1\right)-\left(\frac{s_{\left(\left|\frac{n}{4}\right|-2\right)}-3}{2}\right)\right)}\right)}$$

$$+\frac{\left(3(w_{t_p})^2-2(w_{t_p})^3-3(w_{t_{p-1}})^2+2(w_{t_{p-1}})^3\right)\left(\tilde{b}_{r_{\left(\left|\frac{n}{4}\right|-2\right)}+1}-\tilde{b}_{r_{\left(\left|\frac{n}{4}\right|-2\right)}}\right)}{3(1-u_{\tilde{a}})\left(w_{\left(\left(\left|\frac{n}{4}\right|-1\right)-\left(\frac{s_{\left(\left|\frac{n}{4}\right|-2\right)}-1}{2}\right)\right)}-w_{\left(\left(\left|\frac{n}{4}\right|-1\right)-\left(\frac{s_{\left(\left|\frac{n}{4}\right|-2\right)}-3}{2}\right)\right)}\right)}$$

$$- \frac{3\left(2w_{t_p} - \left(w_{t_p}\right)^2 - 2w_{t_{p-1}} + \left(w_{t_{p-1}}\right)^2\right)\left(\tilde{b}_{k\left(\left[\frac{n}{4}\right]-2\right)+1} w^{\left(\left(\left[\frac{n}{4}\right]-1\right)-\left(\frac{l\left(\left[\frac{n}{4}\right]-2\right)^{-1}}{2}\right)\right)} - \tilde{b}_{k\left(\left[\frac{n}{4}\right]-2\right)} w^{\left(\left(\left[\frac{n}{4}\right]-1\right)-\left(\frac{l\left(\left[\frac{n}{4}\right]-2\right)^{-3}}{2}\right)\right)}\right)}{3(1-u_{\tilde{a}})\left(w^{\left(\left(\left[\frac{n}{4}\right]-1\right)-\left(\frac{l\left(\left[\frac{n}{4}\right]-2\right)^{-1}}{2}\right)\right)} - w^{\left(\left(\left[\frac{n}{4}\right]-1\right)-\left(\frac{l\left(\left[\frac{n}{4}\right]-2\right)^{-3}}{2}\right)\right)}\right)}$$

$$+ \frac{\left(3\left(w_{t_p}\right)^2 - 2\left(w_{t_p}\right)^3 - 3\left(w_{t_{p-1}}\right)^2 + 2\left(w_{t_{p-1}}\right)^3\right)\left(\tilde{b}_{k\left(\left[\frac{n}{4}\right]-2\right)+1} - \tilde{b}_{k\left(\left[\frac{n}{4}\right]-2\right)}\right)}{3(1-u_{\tilde{a}})\left(w^{\left(\left(\left[\frac{n}{4}\right]-1\right)-\left(\frac{l\left(\left[\frac{n}{4}\right]-2\right)^{-1}}{2}\right)\right)} - w^{\left(\left(\left[\frac{n}{4}\right]-1\right)-\left(\frac{l\left(\left[\frac{n}{4}\right]-2\right)^{-3}}{2}\right)\right)}\right)} + \frac{(1-w_1)\left(\tilde{b}_{n-1} - w_1(\tilde{b}_n - \tilde{b}_1) - \tilde{b}_2\right)}{(1-u_{\tilde{a}})}$$

$$+ \frac{(1-3w_1^2+2w_1^3)(\tilde{b}_n - \tilde{b}_{n-1} + \tilde{b}_2 - \tilde{b}_1)}{3(1-u_{\tilde{a}})((1-w_1))} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

With the condition that $0 \leq w_{\tilde{a}} + u_{\tilde{a}} \leq 1$, it follows that $A_\mu\left(\tilde{A}_e^I\right) \leq A_\vartheta\left(\tilde{A}_e^I\right)$ thus the values of the membership and non-membership function of an even IFN $\tilde{A}_e^I$ can be expressed as an interval $[A_\mu\left(\tilde{A}_e^I\right), A_\vartheta\left(\tilde{A}_e^I\right)]$.

5. THE RANKING METHOD [8]

Ranking is evaluated by taking the sum of value index and ambiguity index

Definition 5:

Let $\tilde{G}_e^I = < (\tilde{m}_1, \tilde{m}_2, \tilde{m}_3, \tilde{m}_4, \tilde{m}_5, \ldots\ldots \tilde{m}_{n-1}, \tilde{m}_n)(\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{a}_4, \tilde{a}_5 \ldots\ldots, \tilde{a}_{n-1}, \tilde{a}_n);\ w_{\tilde{m}}, u_{\tilde{m}} >$ be an even IFN. A value index and ambiguity index for the even IFN are defined as

$V(\tilde{G}_e^I, \lambda) = V_\mu\left(\tilde{G}_e^I\right) + \lambda\,(V_\vartheta\left(\tilde{G}_e^I\right) - V_\mu\left(\tilde{G}_e^I\right))$

$A(\tilde{G}_e^I, \lambda) = A_\mu\left(\tilde{G}_e^I\right) + \lambda\,(A_\vartheta\left(\tilde{G}_e^I\right) - A_\mu\left(\tilde{G}_e^I\right))$

Respectively, where $\lambda \in [0,1]$ is a weight which represents the decision maker's preference and with the choice of $\lambda = \frac{1}{2}$ is a appropriate one. One can assume λ value depending on the decision maker's strategic approach towards uncertainty. With choice of $\lambda = \frac{1}{2}$, the indexed value measure and indexed ambiguity measure for even IFN is formulated as below

$V(\tilde{G}_e^I, \frac{1}{2}) = \frac{V_\mu\left(\tilde{G}_e^I\right) + V_\vartheta\left(\tilde{G}_e^I\right)}{2}$, $A(\tilde{G}_e^I, \frac{1}{2}) = \frac{A_\mu\left(\tilde{G}_e^I\right) + A_\vartheta\left(\tilde{G}_e^I\right)}{2}$

The proposed Ranking measure

$R(\tilde{G}_e^I) = V(\tilde{G}_e^I, \frac{1}{2}) + A(\tilde{G}_e^I, \frac{1}{2})$

The ranking measure of $R(\tilde{G}_e^I)$ depends on both $V(\tilde{G}_e^I)$ and $A(\tilde{G}_e^I)$.

6. Octagonal Intuitionistic Fuzzy Number

We obtain value index formula and ambiguity index formula with respect to the membership and non membership function of Octagonal Intuitionistic Fuzzy Number by substituting n=8 in equation 1,2,3 and 4 which are given below.

6.1 The Value index $V_\mu\left(\tilde{A}^I\right)$ of the MFn

$$\frac{w_1^2(\tilde{a}_1+2\tilde{a}_2+2\tilde{a}_7+\tilde{a}_8)}{6w_{\tilde{a}}} + \frac{(w_{\tilde{a}}+w_1)[w_{\tilde{a}}(\tilde{a}_3+\tilde{a}_6) - w_1(\tilde{a}_4+\tilde{a}_5)]}{2w_{\tilde{a}}} + \frac{(w_{\tilde{a}}^2+w_{\tilde{a}}w_1+w_1^2)(\tilde{a}_4-\tilde{a}_3-\tilde{a}_6+\tilde{a}_5)}{3w_{\tilde{a}}}$$

6.2 The Value index $V_\vartheta\left(\tilde{A}^I\right)$ of the NMFn

$$\frac{1}{(1-u_{\tilde{a}})(w_2-u_{\tilde{a}})}\left[\frac{(2w_1-w_1^2-2u_{\tilde{a}}+u_{\tilde{a}}^2)\left(w_1(\tilde{b}_4+\tilde{b}_5) - \mathcal{U}_{\tilde{a}}(\tilde{b}_6+\tilde{b}_3)\right)}{2} + \frac{(3w_1^2-2w_1^3-3u_{\tilde{a}}{}^2+2u_{\tilde{a}}{}^3)(\tilde{b}_6-\tilde{b}_5-\tilde{b}_4+\tilde{b}_3)}{6}\right]$$
$$+\left[\frac{(1-w_1)\left((\tilde{b}_2+\tilde{b}_7) - w_1(\tilde{b}_8+\tilde{b}_1)\right)}{2(1-u_{\tilde{a}})}\right] + \left[\frac{(1-3w_1^2+2w_1^3)(\tilde{b}_8-\tilde{b}_7-\tilde{b}_2+\tilde{b}_1)}{6(1-u_{\tilde{a}})(1-w_1)}\right]$$

6.3 The Ambiguity index $A_\mu\left(\tilde{A}^I\right)$ of the MFn

$$\frac{w_1^2(\tilde{a}_8-\tilde{a}_1+2\tilde{a}_7-2\tilde{a}_2)}{3w_{\tilde{a}}} + \frac{(w_{\tilde{a}}+w_1)[w_{\tilde{a}}(\tilde{a}_6-\tilde{a}_3) - w_1(\tilde{a}_5-\tilde{a}_4)]}{w_{\tilde{a}}} - \frac{2(w_{\tilde{a}}{}^2+w_{\tilde{a}}w_1+w_1^2)(\tilde{a}_6-\tilde{a}_5+\tilde{a}_4-\tilde{a}_3)}{3w_{\tilde{a}}}$$

6.4 The Ambiguity index $A_\vartheta\left(\tilde{A}^I\right)$ of the NMFn

$$\frac{1}{(1-u_{\tilde{a}})(w_1-u_{\tilde{a}})}\left[\frac{(2w_1-w_1^2-2u_{\tilde{a}}+u_{\tilde{a}}^2)\left(w_1(\tilde{b}_5-\tilde{b}_4)-u_{\tilde{a}}(b_6-b_3)\right)}{1}+\frac{(3w_1^2-2w_1^3-3u_{\tilde{a}}^2+2u_{\tilde{a}}^3)(\tilde{b}_6-\tilde{b}_5+\tilde{b}_4-\tilde{b}_3)}{3}\right]$$
$$+\frac{(1-w_1)[(\tilde{b}_7-\tilde{b}_2)-w_1(\tilde{b}_8-\tilde{b}_1)]}{(1-u_{\tilde{a}})}+\frac{(1-3w_1^2+2w_1^3)(\tilde{b}_8-\tilde{b}_7+\tilde{b}_2-\tilde{b}_1)}{3(1-u_{\tilde{a}})(1-w_1)}$$

7. NUMERICAL EXAMPLE

Consider 3×3 transportation problem with supply and demand as octagonal intuitionistic fuzzy number for minimization.

Table 1: Intuitionistic Octagonal Fuzzy transportation table

	B1	B2	B3	supply
A1	4.5	6.5	9.5	(3,4,5,6,7,8,9,10; 2,3,4,6,7,9,10,11; 0.9,0.1)
A2	7.5	11.5	8.5	(4,5,6,7,8,9,10,11; 3,4,5,7,8,10,11,12; 0.9,0.1)
A3	8.5	10.5	7.5	(3,4,5,6,7,8,9,10; 2,3,4,6,7,9,10,11; 0.9,0.1)
Demand	(2,3,4,5,6,7,8,9; 1,2,3,5,6,8,9,10; 0.9,0.1)	(5,6,7,8,9,10,11,12; 4,5,6,8,9,11,12,13; 0.9,0.1)	(3,4,5,6,7,8,9,10; 2,3,4,6,7,9,10,11; 0.9,0.1)	

By applying the proposed ranking method

Let $\tilde{d}_1^I$ = (2,3,4,5,6,7,8,9;1,2,3,5,6,8,9,10;0.9,0.1)

Then V($\tilde{d}_1^I$) = 4.9503 and A($\tilde{d}_1^I$) = 3.319 and R($\tilde{d}_1^I$) = 8.27

Similarly the rank of other demand and supply are obtained in the below table 9145

Table 2: Rank of the supply and demand

	B1	B2	B3	supply
A1	4.5	6.5	9.5	9.17
A2	7.5	11.5	8.5	10.07
A3	8.5	10.5	7.5	9.17
Demand	8.27	10.97	9.17	

It is a balanced transportation problem since total supply = total demand = 28.41

Table 3: VAM method

	B1	B2	B3	supply
A1	4.5	6.5 9.17	9.5	9.17
A2	7.5 8.27	11.5	8.5 1.8	10.07
A3	8.5	10.5 1.8	7.5 7.37	9.17
Demand	8.27	10.97	9.17	

Number of allocations= m+n-1=5, this is a non degeneracy transportation problem

Table 4: MODI method

	B1	B2	B3
A1	2	-	6
A2	-	0	-
A3	2	-	-

$d_{ij} \geq 0$, The optimality is obtained.

Total minimum transportation cost = 211.105/-

Table 5: Comparative Study

	Fuzzy Transportation Problem	
	VAM	MODI
Proposed ranking	211.105	211.105
Accuracy ranking[12]	279.45	279.45

8. CONCLUSION

This paper demonstrates a generalized MFn and NMFn for generalized even intuitionistic fuzzy number (GEIFN) with ranking based on generalized value index form (GVIF) and generalized ambiguity index form (GAIF) . A numerical example is demonstrated when n is assigned with value 8 which generates value index and ambiguity index of an octagonal IFN. The proposed ranking is applied to solve the transportation problem with octagonal intuitionistic fuzzy number for demand and supply to obtain optimal solution using VAM and MODI method. The proposed ranking gives minimum transportation cost when compared with the accuracy ranking function illustrating the efficacy of the proposed method.

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