Cd, Sn, Te, and Xe Isobars at (A=116) under the Framework of (IBM-1)

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Abstract

The interacting boson model (IBM-1) has been used In the study of the nuclear structure of even-even nuclei Cd, Sn, Te, and Xe Of the same mass number (A=116), the dynamical symmetry of these isobars are determined in the region SU(5)-O(6), SU(5) and O(6). The energy level and transition energy are calculated and are all agreement with experimental data, also the binding energy and potential energy are calculated.

Key Words: Potential Energy, Experimental Data, Neutron in Nucleus, Speed of Light.

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Introduction

The interacting boson model of Arima and Iachello [1–6] has become widely accepted as a tractable theoretical scheme of correlating, describing low-energy collective properties of complex nuclei .The basic idea of this model is based on the assumption which assumes that the low –lying collective states in even – even nuclei could be described as states of a given (fixed) number N of bosons. Each boson could occupy two levels; one with angular momentum L=0 (s-boson) and the other with higher energy, with L=2 (d-boson), proton and neutron-boson degree of freedom are not distinguished .Even –even Cd, Sn, Te, and Xe isobars that (A=116) are part of an interesting region beyond the closed proton shell at Z=50 while the number of neutrons in the open shell is much large. In recent there are many studies about these isotops Cd, Sn, Te, and Xe which is the subject of this work. In (2009) Aissaoui L. et al [7] studied and calculated the low spin spectra energy of the even –even Sn, Te (A=134,136) and Xe (A=136) isotops and the electric quadrupole transition probabilities for some of these nuclei; Turkan N. and Maras I in (2011) [8] study the energy level and transition probabilities B (E2) of some even –even Te (Z=52,N=68 – 80) and even –even Xe (Z=54, N=68 – 80) by using interacting boson model (IBM-1)and (IBM-2); in (2012) Hewa Y .etal [9] studied the nuclear structure of even- even Cd (A=104- 112) isotopes, the value of energy spectrum is a good agreement with experimental result; and (2012) Hossain I et. al [10] calculated the electric quadrupole transition probabilities B(E2) for even-even Cd (A=114 -122)for some transition energy ,and in(2015) Assad I[11] studied the transition region for Xe (A= 116-120) which one in the O(6)- SU(3)the energy level ,B(E2) transition probability with the interacting boson model, then in(2016) Hossain et. al [12] calculated the electric quadruple transition probabilities B(E2) for even- even nuclei Te (A=120-126) by using interacting boson model and are agree with experimental Table (1) show the number of bosons of proton and the bosons of neutron of each nuclei. And all number of bosons are equal to eight.

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Table 1: Total number of bosons for Cd, Sn, Te and Xe Isobars

Nuclei	N_π	N_ν	$N=n_\pi+n_\nu$
$^{116}_{48}Cd_{68}$	1	7	8
$^{116}_{50}Sn_{66}$	0	8	8
$^{116}_{52}Te_{64}$	1	7	8
$^{116}_{54}Xe_{62}$	2	6	8

Theory and Method of calculation

Hamiltonian Operator

The Hamiltonian operator according to IBM-1, can be written as follows [6,13-16].

$$\hat{H} = \varepsilon\hat{n}_d + a_o(\hat{P}^{+}.\hat{P}) + a_1(\hat{L}^{+}.\hat{L}) + a_3(\hat{T}_3.\hat{T}_3) + a_4(\hat{T}_4.\hat{T}_4) \quad (2\text{-}1)$$

Where $\varepsilon, a_0, a_1, a_2, a_3, a_4$ are strength of pairing, angular momentum and Multipol terms.

The Hamiltonian as given in (2-1) tends to reduces to three limits, The vibration U(5), γ-soft O(6) and the rotational SU(3) nuclei starting with the unitary group U(6) and finishing with group O(2)[17]. In U (5) limit the effective parameter is ε, in the γ-soft limit O(6), the effective parameter is the pairing a_0, and in the SU(3) limit, the effective parameter is the quadruple a_2[14].

The dynamical symmetries in the (IBM-1)

The three corresponding dynamical symmetries of unitary group chain U(6) can be written as[15]

$$U(6)\begin{cases} SU(5) \supset O(5) \supset O(3) \supset O(2) \\ SU(6) \supset O(3) \supset O(2) \\ O(6) \supset O(5) \supset O(3) \supset O(2) \end{cases} \quad (2\text{-}2)$$

Binding energy

Nuclei are made up of proton and neutrons, but the mass of nucleus is always less than the sum of the individual masses of the proton and neutrons which constitute it. The differences are measure of the nuclear binding energy which holds the nucleus together.

This binding energy can be calculated from the Einstein relationship therefor we can write the total binding energy [18].

$$B.E_{Total}(A,Z) = \left[\left(ZM_p + NM_N\right) - M(A,Z)\right]C^2 \quad (2\text{-}3)$$

M (A,Z): mass of a specific nucleus.
M_p: mass of proton.
M_n: mass of neutron.
A: mass no. of nucleus.
Z: no. of proton in nucleus.
N: no. of neutron in nucleus.
C: speed of light.

The average binding energy for nucleon [19].

$$B.E_{ave}(A,Z) = B.E_{total}(A,Z) / A \quad (2\text{-}4)$$

Results and Discussion

The dynamical symmetry and Energy levels for all isobars which are used in this under study (Cd, Sn, Te, and Xe) equal (A=116) by depending on the Hamiltonian equations show in table (2-2), More over table (2-3) (a,b,c,d) show the Energy and transition energy (MeV) for Cadmium, Tin, Tellurium and Xenon isobars, and compared with the experimental value.

Table (2-2): The dynamical symmetry and the Hamiltonian equation for (Cd, Sn, Te, and Xe) isobars

Isobars	DynamicalSymmetry	Hamiltonian equation
Cd	SU(5)-0(6)	$\mathcal{E}n\hat{}_d+a_0p\hat{}^{+}.p\hat{}+a_1L\hat{}. L\hat{}+a_3T\hat{}_3.T\hat{}_3+a_4T\hat{}_4.T\hat{}_4$
Sn	SU(5)	$\mathcal{E}n\hat{}_d+a_1L\hat{}. L\hat{}+a_3T\hat{}_3.T\hat{}_3+a_4T\hat{}_4.T\hat{}_4$
Te	SU(5)	$\mathcal{E}n\hat{}_d+a_1L\hat{}. L\hat{}+a_3T\hat{}_3.T\hat{}_3+a_4T\hat{}_4.T\hat{}_4$
Xe	0(6)	$a_0p\hat{}^{+}.p\hat{}+a_1L\hat{}. L\hat{}+a_3T\hat{}_3.T\hat{}_3$

Table (2-3) a: The energy and transition energy (MeV) in (IBM-1) for ^{116}Cd and compression with experimental value [20-21].

I_i^+	Energy level (MeV)		Spin sequence I_i^+-I_f^+	Energy Transition (MeV)	
	Exp.	IBM-1		Exp.	IBM-1
0_1^+	0.0000	0.0000			
2_1^+	0.5131	0.5823	2_1^+--------0_1^+	0.5131	0.5823
2_2^+	1.2130	1.2063	2_2^+--------2_1^+	0.7000	0.6240
			2_2^+--------0_1^+	1.2130	1.2063
4_1^+	1.2190	1.2285	4_1^+--------2_1^+	0.7059	0.6462
0_2^+	1.2830	1.2643	0_2^+--------2_2^+	0.0700	0.0580
			0_2^+--------2_1^+	0.7700	0.6820
0_3^+	1.3800	1.8712	0_3^+--------0_2^+	0.0974	0.6069
3_1^+	1.9170	1.8903	3_1^+--------4_1^+	0.6980	0.6618
			3_1^+--------2_1^+	1.4040	1.3080
4_2^+	1.9300	1.9030	4_2^+--------3_1^+	0.0130	0.0127
			4_2^+--------2_1^+	1.4170	1.3207
2_3^+	1.9530	1.9731	2_3^+--------0_2^+	0.6700	0.7088
			2_3^+--------2_2^+	0.7400	0.7668
6_1^+	2.0370	1.9378	6_1^+--------4_1^+	0.8180	0.7093
4_3^+	2.6000	2.6275	4_3^+--------4_2^+	0.6700	0.7245
5_1^+	2.6500	2.6434	5_1^+--------3_1^+	0.7330	0.7530
			5_1^+--------4_1^+	1.4310	1.4149
8_1^+	2.7200	2.7100	8_1^+--------6_1^+	0.6830	0.7722

Table (2-3) b: The energy and transition energy (MeV) In (IBM-1) for ^{116}Sn and comparison with experimental value [20-21]

Ii+	Energy level (MeV)		Spin sequence Ii+-If+	Energy Transition (MeV)	
	Exp.	IBM-1		Exp.	IBM-1
0_1^+	0.0000	0.0000			
2_1^+	1.2935	1.2397	2_1^+--------0_1^+	1.2935	1.2397
0_2^+	1.7569	1.8666	0_2^+--------2_1^+	0.4634	0.6269
			0_2^+--------0_1^+	1.7569	1.8666
0_3^+	2.0273	2.0058	0_3^+--------0_2^+	0.2704	0.1392
2_2^+	2.1118	2.3514	2_2^+--------2_1^+	0.8183	1.1117
			2_2^+--------0_1^+	2.1118	2.3514
2_3^+	2.2250	2.4909	2_3^+--------2_2^+	0.1130	0.1390
			2_3^+--------0_2^+	0.4681	0.6243
4_1^+	2.3904	2.3394	4_1^+--------2_1^+	1.0970	1.0997
4_2^+	2.5285	2.4789	4_2^+--------4_1^+	0.1381	0.1395
			4_2^+--------2_1^+	1.2350	1.2392
2_4^+	2.6501	2.6236	2_4^+--------2_2^+	0.5383	0.2722
			2_4^+--------2_1^+	1.3566	1.3840
3_1^+	2.9980	3.3136	3_1^+--------4_1^+	0.6076	0.9742
			3_1^+--------2_1^+	1.7045	2.0739
6_1^+	3.2790	3.2879	6_1^+--------4_1^+	0.8886	0.9485
8_1^+	3.3200	3.3169	8_1^+--------6_1^+	0.0410	0.0290

Table (2-3) c: The energy level and transition energy (MeV) In (IBM-1) for ^{116}Te and comparison with experimental value [20-21]

Ii+	Energy level (MeV)		Spin sequence Ii+-If+	Energy Transition (MeV)	
	Exp.	IBM-1		Exp.	IBM-1
0_1^+	0.0000	0.0000			
2_1^+	0.6788	0.6739	2_1^+--------0_1^+	0.6788	0.6739
4_1^+	1.3594	1.3571	4_1^+--------2_1^+	0.6806	0.6832
6_1^+	2.0020	2.0495	6_1^+--------4_1^+	0.6426	0.6924
8_1^+	2.7730	2.7512	8_1^+--------6_1^+	0.7710	0.7017
10^+	-------	3.4622	10^+-------8_1^+	--------	0.7110

Table (2-3) d: The Energy and Transition Energy (MeV) In (IBM-1) for ^{116}Xe and Comparison with Experimental Value [20-21]

Ii+	Energy level (MeV)		Spin sequence Ii+-If+	Energy Transition (MeV)	
	Exp.	IBM-1		Exp.	IBM-1
0_1^+	0.0000	0.0000			
2_1^+	0.3370	0.3577	2_1^+--------0_1^+	0.3370	0.3577
4_1^+	0.8080	0.8339	4_1^+--------2_1^+	0.4710	0.4762
0_2^+	--------	0.9575	0_2^+--------2_1^+	--------	0.5577
			0_2^+--------0_1^+	--------	0.9575
2_2^+	1.0158	1.0454	2_2^+--------2_1^+	0.6788	0.6877
			2_2^+--------0_1^+	1.0158	1.0454
2_3^+	1.3215	1.3386	2_3^+--------2_1^+	0.9845	0.9809
6_1^+	1.4660	1.4413	6_1^+--------4_1^+	0.6580	0.6074
3_1^+	1.4741	1.8945	3_1^+--------4_1^+	0.6662	1.0606
			3_1^+--------2_1^+	1.1372	1.5368
4_2^+	1.8388	1.7208	4_2^+--------2_1^+	1.5018	1.3631
8_1^+	2.0860	2.1899	8_1^+--------6_1^+	0.6200	0.7486
10_1^+	2.9614	3.0864	10_1^+------8_1^+	0.8754	0.8965

The theoretical and experimental Energy level (g-band) values are plotted as a function of atomic number (z) in fig (1). For each isobars the calculated energy level by using IBM-1 are compared with experimental data. The calculated data are in excellent agreement with experimental data it is shown that the energy of g-band are increase from isotope Cd that Z number equal 48 to 50 for Sn isotope that is the magic number of closed shell that the excited protons and neutrons collective are depend on it, then the energy become to decrease at isotope Te (Z=52) to Xe (Z=54).
In the figure (2) the value of E(I)/I as a function to Angular momentum I(ħ) we show that the E(I)/I begin to increase from angular momentum for each isobars except the isobar Sn begin to decrease because the Z number of it is equal to (50) of closed shell.

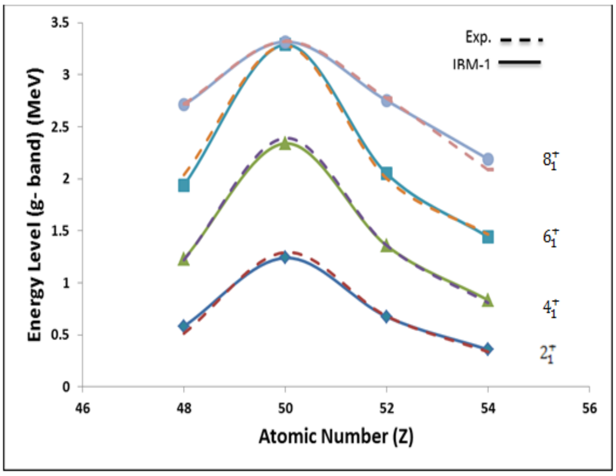

Figure 1: Energy level (g-band) (MeV) as a function to atomic number (Z) for (Cd, Sn, Te and Xe) isobars

Figure 2: E(I+)/I as a function to angular momentum for (Cd, Sn, Te and Xe) isobars

So, the Binding Energy for (Cd, Sn, Te, and Xe) are calculated by using the (2–4) equation we can see that the B.E take the positive charge at the (Z = 48) for Cadmium which it less the magic number of closed shell but this value become negative charge at closed shell and up for Z=50, 52, 54 to Sn, Te, and Xe.

Conclusion

The dynamical symmetry and energy level of isobars (Cd, Sn, Te and Xe) that Z=48 to 54, and N=68 to 62 have been studied within the frame work of interacting Boson Model 1.
We found that the energy are in good agreement with the previous experimental result. The value of g-band energy are increase in isobar (Sn) that equal to (Z= 50) and the value of E(I)/I is decrease for this isobar as a function of angular momentum.

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