Solution of Fractional Differential Equations for LC, RC and LR Circuits using Sumudu Transform Method

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Abstract

In this paper fractional LC, RC and RL electrical circuits are proposed. The current and charge at as per time are determined using Sumudu transform method. Atangana-Baleanu fractional derivatives in the Caputo sense and Caputo-Fabrizio fractional derivative operators are used.

Keywords: Caputo derivative, Caputo-Fabrizio, Atangana-Baleanu, Sumudu transform, Fractional Differential Equations.

DOI Number: 10.14704/Nq.2022.20.17.Nq880126 **Neuroquantology 2022; 20(17): 994-1002**

1. Introduction

The fractional derivative and integral definitions appeared to be a natural extension of the classical derivative and integral definitions. Because fractional orders in comparison with integer orders fractional derivative appear to have a lot of advantages. These advantages can be demonstrated in a variety of real-world modelling scenarios. It does, however, come with significant drawbacks. For example in the Riemann - Liouville definition of fractional derivative, the fractional derivative of constant is not zero which is not according to the rules of classical theory. As a result of having integer order initial conditions in various engineering problems, the Liouville – Caputo definition emerged as a choice. Singularity exists in the kernels of the Riemann – Liouville and Liouville – Caputo definitions. In some modelling issues, this condition has a considerable disadvantage. To tackle this difficulty, Caputo–Fabrizio [1] suggested an entirely new definition using non-singularity as its kernel in 2015. In addition to this new definition, Atangana–Baleanu [2] introduced a whole new fractional definition with the Atangana–Baleanu fractional derivative in 2016. Because of the fractional order, this new derivative has more unique effects than the derivative with exponential kernel, and it's thus a generalized model of the derivative with exponential kernel.

Thabet Abdeljawad [3] used the infinite binomial theorem to obtain the appropriate fractional integrals with arbitrary order, then examined their semi-group properties and their action on the ABC type fractional derivatives to prove the existence and uniqueness theorem for ABC-fractional initial value problems. The result obtained, we find that for $\mu \neq 1$, we get a nontrivial solution for the linear ABC-type initial value problem with constant coefficient and prove a certain semigroup properties simultaneously in the parameters μ and γ. For the Atangana-Baleanu (AB) fractional integrals and AB iterated fractional differ-integrals Laplace

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Relevant conflicts of interest/financial disclosures: The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest

transforms are computed.

Bahar Acaya and et. al. [4] investigated and explored specific financial challenges using non-local fractional operators such as Caputo, Caputo-Fabrizio in the spirit of Caputo (CFC), Caputo type Atangana-Baleanu (ABC), and ABC with generalized Mittag-Leffler kernel. While applying to the monetary models, the stated fractional operators allude to those beyond traditional differentiation and integration. Following that, these fractional operators work on understanding the organic market and its connections to the cost of goods under the assumption that the market is in balance. Furthermore, contrast results obtained by supporting and the reproduction inquiry in order to notice the doubtful themes in the market more specifically.

Ahmed Bokharia and et. al. [5] presented novel characteristics of the Shehu transform and solved some fractional differential equations by applying this transformation to Atangana–Baleanu derivatives in Caputo and Riemann–Liouville senses.

Andrea Giusti [6] claimed that two integro-differential operators can be viewed as basic acknowledge of a much more extensive class of fractional operators, such as the hypothesis of Prabhakar fractional integrals, after examining the meaning of two differential operators recently presented by Caputo and Fabrizio and, independently, by Atangana and Baleanu, also provided a series expansion of the Prabhakar essential in the context of Riemann-Liouville integrals of variable order. While using the novel operators within the constitutive equation of the Scott-Blair model, they do not bring any new knowledge to the linear principle of viscoelasticity.

Abdon and Dumitru [7] proposed a new fractional definition of fractional derivative having non-local and no singular kernel along with some useful properties of this derivative applied to the problems containing fractional heat transfer model.

Balocha et. al. [8] developed a section of the key relationship between fractional Laplace transform and fractional Fourier, fractional Mellin, and fractional Sumudu transforms, and the results are extremely valuable in signal processing and optics. Bodkhe et.al. [9] Obtained the solution of homogeneous and non homogeneous fractional differential equations having non-zero initial conditions by applying Sumudu transform.

Bas [10] investigated a number of modelling issues, including Newton's law of cooling, population expansion, logistic equations, and the blood alcohol model, as well as the Atangana–Baleanu fractional derivative. By applying Laplace transform analytical solutions are obtained and showed that Atangana Baleanu fractional derivative delivers more precise results to exponential kernel because of having fractional order, and so it is a generalized version of the derivative with exponential kernel.

Caputo et al. [11] proposed a novel definition of fractional derivative based on a smooth kernel that can represent the temporal and spatial variables in two different ways. For this definition on the time variable the Laplace transform can be used conveniently and for the second definition it is more convenient to work with the Fourier transform.

Baleanu et. al. [12] provided fractional order model for COVID-19 using Caputo–Fabrizio derivative and solved the problems using homotopy analysis transform method (HATM) which is combination of homotopy analysis and Laplace transform gives the solution in convergent series. It is also showed mathematical results for reenacting virus transmission and comparing them to those of the Caputo derivative.

Singh et. al. [15] presented the diabetes model with the Caputo–Fabrizio fractional derivative along with its complications and by applying the homotopy analysis method, the Laplace transform and the Padé approximation, the analytical solution of the diabetes model is obtained. Many authors [17-21] presented models for electrical circuits and using different definitions of fractional derivative and transforms obtained the results. Other authors are used different mathematical modelling to solve real life problems [22-34].

Fractional calculus has a powerful ability to deal with issues in physics, signal processing, fluid dynamics, control engineering, and other fields have made it brand new mathematical equipment for solving a variety of issues in technological know-how and engineering. It mostly deals with non-integer order and complex order operators, such as fractional derivative and fractional integral. The concept has a remarkable potential to extrapolate how we perceive, model, and alter the characters around us [35].

2. Preliminaries

In this section we consider some definitions, theorems, properties and results required in further sections.

Definition 2.1 The classical Mittag-Leffler function with one parameter $E_\alpha(v)$ is

$$E_\alpha(v)=\sum_{k=0}^{\infty}\frac{v^k}{\Gamma(\alpha k+1)},\ \alpha>0$$

Definition 2.2 The classical Mittag-Leffler function with two parameter is $E_{\alpha,\beta}(v)$ is

$$E_{\alpha,\beta}(v)=\sum_{k=0}^{\infty}\frac{v^k}{\Gamma(\alpha k+\beta)},$$
$$v\in c,\beta\in c,\quad \mathrm{Re}(\alpha)>0$$

Definition 2.3 The generalized Mittag-Leffler function is given by

$$E_{\alpha,\beta}^{\eta}(v)=\sum_{k=0}^{\infty}\frac{v^k(\eta)k}{\Gamma(\alpha k+\beta)k!},$$
$$v\in c,\ \alpha,\ \beta,\ \eta\in c,\ \mathrm{Re}(\alpha)>0$$

Where $(\eta)k=\eta(\eta+1)....(\eta+k-1)$ is the pochhammer symbol introduced by Prabhakar. Note that $(1)k=k!$ and so $E_{\alpha,\beta}^{1}(v)=E_{\alpha,\beta}(v)$.

Definition 2.4 The Mittag-Leffler function for a special function is given by

$$E_\alpha(\lambda,v)=\sum_{k=0}^{\infty}\frac{\lambda^k v^{\alpha k}}{\Gamma(\alpha k+1)},$$
$$0\neq\lambda\in R,v\in c,\quad \mathrm{Re}(\alpha)>0$$

and

$$E_{\alpha,\beta}(\lambda,v)=\sum_{k=0}^{\infty}\frac{\lambda^k v^{\alpha k+\beta-1}}{\Gamma(\alpha k+\beta)},$$
$$0\neq\lambda\in R,v,\beta\in c,\ \mathrm{Re}(\alpha)>0$$

It should be noticed that $E_{\alpha,1}(\lambda,v)=E_\alpha(\lambda,v)$. Also the modified Mittag-Leffler function with three parameters can be written as

$$E_{\alpha,\beta}^{\eta}(v)=\sum_{k=0}^{\infty}\frac{\lambda^k v^{\alpha k+\beta-1}(\eta)k}{\Gamma(\alpha k+\beta)k!},$$
$$0\neq\lambda\in R,v,\beta\in c,\mathrm{Re}(\alpha)>0$$

Definition 2.5 The left-sided and right-sided Caputo-fractional derivatives of order α are defined by

$${}_a^cD^\alpha v(x)=\frac{1}{\Gamma(1-\alpha)}\int_a^x(x-\tau)^{-\alpha}\,v'(\tau)\,d\tau$$

and

$${}^cD_b^\alpha v(x)=\frac{(-1)}{\Gamma(1-\alpha)}\int_x^b(\tau-x)^{\alpha}\,v'(\tau)\,d\tau,$$
$$0<\alpha<1$$

Theorem 2.1 [7] The Sumudu transform of Caputo fractional derivative is defined by

$$S\left[{}_0^cD^\alpha v(x)\right](u)=u^{-\alpha}S\left[v(x)\right]-u^{-\alpha}v(0)\qquad(2.1)$$

Definition 2.6 Let $\varphi,\psi:[0,\infty)\to R$, then classical convolution product is

$$(\phi*\psi)(x)=\int_0^x\eta(x-s)\psi(s)ds$$

Proposition 2.6 Let $\varphi,\psi:[0,\infty)\to R$, then the following property is valid

$$S\left[(\phi*\psi)(x)\right]=uS\left[\varphi(x)\right]S\left[\psi(x)\right]$$
$$=u\phi(u)\cdot\psi(u)$$

Definition 2.7 The left-sided and right-sided Caputo-Fabrizio fractional derivatives in the Caputo sense of order α are defined by

$${}_a^{CFC}D^\alpha w(x)=\frac{M(\alpha)}{(1-\alpha)}\int_a^x w'(\tau)\,e^{\mu(x-\tau)}d\tau$$

and

$${}^{CFC}D_b^\alpha w(x)=-\frac{M(\alpha)}{(1-\alpha)}\int_x^b w'(\tau)^{\mu(\tau-x)}\,d\tau$$

where $0<\alpha<1$, $M(\alpha)$ is normalization function and $\mu=-\dfrac{\alpha}{1-\alpha}$

eISSN: 1303-5150

Theorem 2.2 [13] The Sumudu transform of CFC fractional derivative $w(x)$ is

$$S\left[{}^{CFC}_{0}D_x^{\alpha} w(x)\right] = \frac{M(\alpha)}{(1-\alpha+\alpha u)}\left[S[w(x)] - w(0)\right] \quad (2.2)$$

Definition 2.8 The left-sided and right-sided Atangana-Baleanu fractional derivatives in the Caputo sense of order α are defined by

$${}^{ABC}_{a}D^{\alpha} w(x) = \frac{B(\alpha)}{(1-\alpha)}\int_a^x w'(\tau)\ E_\alpha\left[\mu(x-\tau)\right]d\tau$$

And

$${}^{ABC}D_b^{\alpha} w(x) = -\frac{B(\alpha)}{(1-\alpha)}\int_x^b w'(\tau)\ E_\alpha\left[\mu(\tau-x)\right]d\tau$$

where $0<\alpha<1$, $B(\alpha)$ is normalization function and $\mu = -\dfrac{\alpha}{1-\alpha}$

Theorem 2.3 [3] The Sumudu transform of ABC fractional derivative is defined by

$$S\left[{}^{ABC}_{0}D^{\alpha} w(x)\right] = \frac{B(\alpha)}{(1-\alpha+\alpha u^{\alpha})}\left[S[w(x) - w(0)]\right] \quad (2.3)$$

Lemma 2.1 The Sumudu transform of certain functions holds:

i. $S\left[E_\alpha\left(-ax^\alpha\right)\right] = \dfrac{1}{1+au^\alpha}$

ii. $S\left[1-E_\alpha\left(-ax^\alpha\right)\right] = \dfrac{au^\alpha}{1+au^\alpha}$

iii. $S\left[x^{\alpha-1}E_{\alpha,\alpha}\left(-ax^\alpha\right)\right] = \dfrac{u^{\alpha-1}}{1+au^\alpha}$

Proof. Since

$$\sum_{p=0}^{\infty}\frac{(p+m)!}{p!}x^p = \frac{m!}{(1-x)^{(m+1)}}$$

i.
$$S\left[E_\alpha\left(-ax^\alpha\right)\right] = \sum_{p=0}^{\infty}\frac{(p+m)!}{p!}(-a)^p\frac{S\left[x^{\alpha p}\right]}{\Gamma(\alpha p+1)}$$

$$= \sum_{p=0}^{\infty}\frac{(p+m)!}{p!}(-a)^p\frac{\Gamma(\alpha p+1)}{\Gamma(\alpha p+1)}u^{\alpha p}$$

$$= \sum_{p=0}^{\infty}\frac{(p+m)!}{p!}(-a)^p\ u^{\alpha p}$$

$$\therefore\ S\left[E_\alpha\left(-ax^\alpha\right)\right] = \frac{1}{1+au^\alpha} \quad (\because m=0)$$

ii.
$$S\left[1-E_\alpha\left(-ax^\alpha\right)\right] = 1-\frac{1}{1+au^\alpha}$$

$$= \frac{au^\alpha}{1+au^\alpha}$$

iii.
$$S\left[x^{\alpha-1}E_{\alpha,\alpha}\left(-ax^\alpha\right)\right] = \sum_{p=0}^{\infty}\frac{(p+m)!}{p!}(-a)^p\frac{S\left[x^{\alpha p+\alpha-1}\right]}{\Gamma(\alpha p+\alpha)}$$

$$= \sum_{p=0}^{\infty}\frac{(p+m)!}{p!}(-a)^p\frac{\Gamma(\alpha p+\alpha)}{\Gamma(\alpha p+\alpha)}u^{\alpha p+\alpha-1}$$

$$= u^{\alpha-1}\sum_{p=0}^{\infty}\frac{(p+m)!}{p!}(-a)^p\ u^{\alpha p}$$

$$\therefore\ S\left[x^{\alpha-1}E_{\alpha,\alpha}\left(-ax^\alpha\right)\right] = \frac{u^{\alpha-1}}{1+au^\alpha} \quad (\because m=0)$$

3. RC Electrical circuit

Let an electric circuit containing a resistance(R) and a inductance (L) are positive constants and E is the applied electro motive force with zero. According to the Kirchoff's Voltage laws, the equation of the RC series circuit is given by

The voltage drop across the resistance(R)

$V_R(t) = Ri(t)$

The voltage drop across the capacitance(C)

$$V_C(t) = \frac{1}{C}\int_0^t i(x)dx = \frac{q(t)}{C}$$

The ordinary differential equation for RC circuit is

$$R\frac{dq}{dt} + \frac{q}{C} = 0$$

with initial condition $q(0)=1$

The fractional form of above differential equation is

$$\frac{d^\alpha q}{dt^\alpha} + \frac{q}{RC} = 0 \quad (3.1)$$

CASE-I: Caputo fractional derivative

From equation (3.1)

Applying Sumudu transform defined in Lemma (2.1)

eISSN: 1303-5150

$$S\left[{}_{0}^{C}D^{\alpha}q(t)\right]+\frac{1}{RC}S\left[q(t)\right]=0$$

$$u^{-\alpha}S\left[q(t)\right]-u^{-\alpha}q(0)+\frac{1}{RC}S\left[q(t)\right]=0$$

$$S\left[q(t)\right]\left[u^{-\alpha}+A\right]=u^{-\alpha}\qquad\left[\because\ A=\frac{1}{RC}\right]$$

$$S\left[q(t)\right]=\frac{1}{\left[1+Au^{\alpha}\right]}$$

Applying inverse Sumudu transform to the above equation yields

$$q(t)=S^{-1}\left[\frac{1}{1+Au^{\alpha}}\right]$$

$$q(t)=E_{\alpha}\left(-At^{\alpha}\right)$$

CASE-II: Caputo-Fabrizio fractional derivative
From equation (3.1)
Applying Sumudu transform defined in Lemma (2.1)

$$S\left[{}_{0}^{CFC}D^{\alpha}q(t)\right]+\frac{1}{RC}S\left[q(t)\right]=0$$

$$\frac{M(\alpha)}{(1-\alpha-\alpha u)}\left[S\left[q(t)\right]-q(0)\right]+\frac{1}{RC}S\left[q(t)\right]=0$$

$$S\left[q(t)\right]\left[\frac{M(\alpha)}{(1-\alpha-\alpha u)}+A\right]=\frac{M(\alpha)}{(1-\alpha-\alpha u)}$$

$$\left[\because\ A=\frac{1}{RC}\right]$$

$$S\left[q(t)\right]=\left[\frac{\frac{M(\alpha)}{(1-\alpha-\alpha u)}}{\frac{M(\alpha)-A(1-\alpha-\alpha u)}{(1-\alpha-\alpha u)}}\right]$$

Applying inverse Sumudu transform to the above equation yields

$$q(t)=S^{-1}\left[\frac{M(\alpha)}{M(\alpha)-A(1-\alpha-\alpha u)}\right]$$

$$q(t)=\frac{M(\alpha)\,e^{\frac{A\alpha t}{-M(\alpha)-A(\alpha-1)}}}{M(\alpha)-A(\alpha-1)}$$

CASE-III: Atangana-Baleanu fractional derivative
From equation (3.1)
Applying Sumudu transform defined in Lemma (2.1)

$$S\left[{}_{0}^{ABC}D^{\alpha}q(t)\right]+\frac{1}{RC}S\left[q(t)\right]=0$$

$$\frac{B(\alpha)}{\left(1-\alpha-\alpha u^{\alpha}\right)}\left[S\left[q(t)\right]-q(0)\right]+\frac{1}{RC}S\left[q(t)\right]=0$$

$$S\left[q(t)\right]\left[\frac{B(\alpha)}{\left(1-\alpha-\alpha u^{\alpha}\right)}+A\right]=\frac{B(\alpha)}{\left(1-\alpha-\alpha u^{\alpha}\right)}$$

$$\left[\because\ A=\frac{1}{RC}\right]$$

$$S\left[q(t)\right]=\left[\frac{\frac{B(\alpha)}{\left(1-\alpha-\alpha u^{\alpha}\right)}}{\frac{B(\alpha)-A\left(1-\alpha-\alpha u^{\alpha}\right)}{\left(1-\alpha-\alpha u^{\alpha}\right)}}\right]$$

$$=\frac{B(\alpha)}{B(\alpha)+A(1-\alpha)}\left[\frac{1}{1+\frac{A\alpha u^{\alpha}}{B(\alpha)+A(1-\alpha)}}\right]$$

Applying inverse Sumudu transform to the above equation yields

$$q(t)=\frac{B(\alpha)}{B(\alpha)+A(1-\alpha)}S^{-1}\left[\frac{1}{1+\frac{A\alpha u^{\alpha}}{B(\alpha)+A(1-\alpha)}}\right]$$

$$q(t)=\frac{B(\alpha)}{B(\alpha)+A(1-\alpha)}\left[E_{\alpha}\left(-\frac{A\alpha t^{\alpha}}{B(\alpha)+A\left(1-\alpha\right)}\right)\right]$$

4. LC electrical circuit

Let an electric circuit containing a inductance (L) and a capacitance(C) are positive constants and E is the applied electro motive force with zero. According to the Kirchoff's Voltage laws, the equation of the LR series circuit is given by
The voltage drop across resistance (R)

$$V_R(t)=Ri(t)\qquad(4.1)$$

The voltage drop across capacitance (C)

$$V_C(t)=\frac{1}{C}\int_0^t i(x)dx=\frac{q(t)}{C}\qquad(4.2)$$

The ordinary differential equation for LC circuit is

$$L\frac{d^2q}{dt^2}+\frac{q}{C}=0 \qquad (4.3)$$

with initial condition $q(0)=1$

The fractional form of above differential equation is

$$L\frac{d^{2\alpha}q}{dt^{2\alpha}}+\frac{q}{C}=0 \qquad (4.4)$$

CASE I: Caputo fractional derivative

$$S\left[D^{2\alpha}q(t)\right]+\frac{1}{LC}S\left[q(t)\right]=0 \qquad (4.5)$$

$$u^{-2\alpha}S\left[q(t)-u^{2\alpha}q(0)\right]+\frac{1}{LC}S\left[q(t)\right]=0 \qquad (4.6)$$

$$S\left[q(t)\right]\left\{u^{-2\alpha}+\omega\right\}=u^{2\alpha}\left[\because\ \omega=\frac{1}{LC}\right] \qquad (4.7)$$

$$S\left[q(t)\right]=\frac{1}{1+\omega u^{2\alpha}} \qquad (4.8)$$

Taking inverse Sumudu transform

$$q(t)=E_\alpha\left(-\omega t^{2\alpha}\right) \qquad (4.9)$$

CASE III: Atangana-Baleanu fractional derivative
from equation (4.1)
Applying Sumudu transform defined in Lemma (2.1)

$$S\left[D^{2\alpha}q(t)\right]+\frac{1}{LC}S\left[q(t)\right]=0 \qquad (4.8)$$

$$\frac{B\left(\alpha^2\right)}{\left(1-\alpha-\alpha^2\right)}\left[S(q(t))-q(0)\right]+\frac{1}{LC}S\left[q(t)\right]=0 \qquad (4.9)$$

$$S\left[q(t)\right]\left[\frac{B\left(\alpha^2\right)}{\left(1-\alpha-\alpha^2\right)}+\omega^2\right]=\frac{B\left(\alpha^2\right)}{\left(1-\alpha-\alpha^2\right)^2},$$

$$\left[\because\ \omega^2=\frac{1}{LC}\right] \qquad (4.10)$$

$$S\left[q(t)\right]=\frac{B\left(\alpha^2\right)}{\left[B(\alpha)-i\omega(1-\alpha)\right]\left[B(\alpha)+i\omega(1-\alpha)\right]}\left[\frac{1}{\left(1+\frac{i\omega\alpha u^\alpha}{B(\alpha)}+i\omega(1-\alpha)B(\alpha)+i\omega(1-\alpha)\right)}\right] \qquad (4.11)$$

Applying inverse Sumudu transform to the above equation yields

$$q(t)=\frac{B\left(\alpha^2\right)}{\left[B(\alpha)-i\omega(1-\alpha)\right]\left[B(\alpha)+i\omega(1-\alpha)\right]}S^{-1}\left[\frac{1}{\left(1+\frac{i\omega\alpha u^\alpha}{B(\alpha)}+i\omega(1-\alpha)B(\alpha)+i\omega(1-\alpha)\right)}\right]$$

(4.12)

$$q(t)=\frac{B\left(\alpha^2\right)}{\left[B(\alpha^2)+\omega^2(1-\alpha)\right]}\int_0^t (t-\tau)^{\alpha-1}E_{\alpha,-\alpha}\left[-\left(\frac{i\omega\alpha}{B(\alpha)+i\omega(1-\alpha)}\right)(t-\tau)^\alpha\right]\tau^\alpha E_{\alpha,\alpha+1}\left[\frac{i\omega\alpha}{B(\alpha)-i\omega(1-\alpha)}t^\alpha\right]d\tau \qquad (4.13)$$

5. LR Electrical circuit

Let an electric circuit containing a resistance (R) and inductance (L) are positive constants and E is the applied electromotive force with zero. According to the Kirchoff's voltage law, the equation of LR series circuit is given by
The voltage drop across the resistance (R)

$$V_R(t)=Ri(t) \qquad (5.1)$$

The voltage drop across the inductance (L)

$$V_L(t)=L\frac{di}{dt} \qquad (5.2)$$

The ordinary differential equation for LR circuit is given by

$$L\frac{di}{dt}+Ri=0 \qquad (5.3)$$

with initial condition $i(0)=1$

The fractional form of above differential equation is

$$\frac{d^\alpha i}{dt^\alpha}+\frac{R}{L}i=0 \qquad (5.4)$$

CASE I: Caputo fractional derivative
from equation (5.1)
Applying Sumudu transform defined in Lemma

(2.1)

$$S\left[D^{\alpha}i(t)\right]+\frac{R}{L}S\left[i(t)\right]=0 \quad (5.5)$$

$$u^{-\alpha}S\left[i(t)\right]-u^{-\alpha}i(0)+\frac{R}{L}S\left[i(t)\right]=0 \quad (5.6)$$

$$S\left[i(t)\right]\left(u^{-\alpha}+P\right)=u^{-\alpha}, \quad \left[\because \; P=\frac{R}{L}\right] \quad (5.7)$$

$$S\left[i(t)\right]=\frac{1}{\left(1+Pu^{\alpha}\right)} \quad (5.8)$$

Applying inverse Sumudu transform yields

$$i(t)=S^{-1}\left[\frac{1}{\left(1+Pu^{\alpha}\right)}\right] \quad (5.9)$$

$$i(t)=E_{\alpha,\alpha}\left(Pt^{\alpha}\right) \quad (5.10)$$

CASE II: Caputo-Fabrizio fractional derivative from equation (5.1)

Applying Sumudu transform defined in Lemma (2.1)

$$S\left[D^{\alpha}i(t)\right]+\frac{R}{L}S\left[i(t)\right]=0 \quad (5.11)$$

$$\frac{M(\alpha)}{(1-\alpha-\alpha u)}\left[S\left[i(t)\right]-i(0)\right]+\frac{R}{L}S\left[i(t)\right]=0 \quad (5.12)$$

$$S\left[i(t)\right]\left[\frac{M(\alpha)}{(1-\alpha-\alpha u)}+P\right]=\frac{M(\alpha)}{(1-\alpha-\alpha u)} \quad (5.13)$$

$$S\left[i(t)\right]=\left[\frac{\frac{M(\alpha)}{(1-\alpha-\alpha u)}}{\frac{M(\alpha)-P(\alpha-1-\alpha u)}{(\alpha-1-\alpha u)}}\right] \quad (5.14)$$

Applying inverse Sumudu transform yields

$$i(t)=S^{-1}\left[\frac{M(\alpha)}{M(\alpha)-P(\alpha-1-\alpha u)}\right] \quad (5.15)$$

$$i(t)=\frac{M(\alpha)e^{\frac{P\alpha}{-M(\alpha)-P(\alpha-1)}}}{M(\alpha)-P(\alpha-1)} \quad (5.16)$$

CASE III: Atangana Baleanu fractional derivative

From equation (5.1)

Applying Sumudu transform defined in Lemma(2.1)

$$S\left[D^{\alpha}i(t)\right]+\frac{R}{L}S\left[i(t)\right]=0 \quad (5.17)$$

$$\frac{B(\alpha)}{1-\alpha-\alpha u^{\alpha}}\left[S\left[i(t)\right]-i(0)\right]+\frac{R}{L}S\left[i(t)\right]=0 \quad (5.18)$$

$$S\left[i(t)\right]\left[\frac{B(\alpha)}{1-\alpha-\alpha u^{\alpha}}+P\right]=\frac{B(\alpha)}{1-\alpha-\alpha u^{\alpha}},$$

$$\left[\because \; P=\frac{R}{L}\right] \quad (5.19)$$

$$S\left[i(t)\right]=\left[\frac{\frac{B(\alpha)}{1-\alpha-\alpha u^{\alpha}}}{\frac{B(\alpha)-P(\alpha-1)-\alpha u^{\alpha}}{1-\alpha-\alpha u^{\alpha}}}\right] \quad (5.20)$$

$$S\left[i(t)\right]=\frac{B(\alpha)}{B(\alpha)+P(1-\alpha)}\left[\frac{1}{1+\frac{P\alpha u^{\alpha}}{B(\alpha)+P(1-\alpha)}}\right] \quad (5.21)$$

Applying inverse Sumudu transforms to the above equation yields

$$i(t)=\frac{B(\alpha)}{B(\alpha)+P(1-\alpha)}S^{-1}\left[\frac{1}{1+\frac{P\alpha u^{\alpha}}{B(\alpha)+P(1-\alpha)}}\right] \quad (5.22)$$

$$i(t)=\frac{B(\alpha)}{B(\alpha)+P(1-\alpha)}\left[E_{\alpha}\frac{-P\alpha t^{\alpha}}{B(\alpha)+P(1-\alpha)}\right] \quad (5.23)$$

6. Conclusion

The fractional differential equations corresponding to RC, LC and LR circuit can be solved using Sumudu transform. It has been concluded that remarkable results shall be obtained using Atangana-Baleanu fractional derivatives in the Caputo sense and Caputo-Fabrizio fractional derivative operators.

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