Design and Implementation of an Integrated Redundant Reliability System for k out of n Configuration

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The structure's reliability is generally, termed as a function of value in cost, but reliability will depend on a variety of factors in many real-time situations like load, size, space, volume, etc., (defined as the hidden impact of restraints on structure reliability), apart from the conventional value restraint in cost. The present paper focuses to study and analyze the impact of additional hidden restraints in optimizing the structure reliability. The integrated superfluous reliability k out of n structured structure is considered for the analysis purpose. Lagrangain multiplier method gives a solution for element, phase and structure reliability. Further Heuristic algorithm which provides a near optimum solution but not a closed bounded solution is employed to generate an integer solution, which led to the application of the Dynamic programming method. A numerical example illustrates the results obtained.

Keywords: Structure Reliability, Optimization,Lagrangian Multiplier Method,Heuristic Algorithm, Dynamic Programming Method.

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1 Introduction

Adding Superfluous units, applying the element of greater reliability, or doing both together uses more resources but increases the reliability of the structure. The optimal redundancy problem has been generalised, according to R. M. Burton and G. T. Howard (1969). Different designs for modules that worked logically in parallel were fine, and spending money on a module doesn't always mean using redundant parts. Optimal Redundancy for Reliability in Series Systems was talked about by P. M. Ghare and R. E. Taylor (1969). In particular, when dealing with complex techniques that resolve relatively large problems subject to multiple constraints without necessitating a large amount of storage space. Optimizing of structure reliability, condition to resource availability such as value, load, size is examined. Typically, reliability is evaluated in terms of its value, but when put to the test with real-world issues, the intangible influence of other constraints (such as load, size, etc.) will have a noticeable effect on enhancing the structure's reliability. An effective method for solving the redundancy-based, multi-criteria optimization problems that arise frequently in the reliability design of engineering systems is described by Krishna B. Misra and Usha Sharma (1975). Redundancy problems can be recast as integer linear programming problems, as described by Emad El-Neweihi et al. (1986) described function methods allow for deeper understanding and, in some cases, more comprehensive solutions. D. Z. Du and F. K. Hwang (1986) proposed Optimal consecutive 2 out of n structures, It could be helped to increase components in the redundancy. It could solve many design problems related to reliability because it can take into account any combination of redundancy, constraints, and individual cost functions. The unique operation of a Superfluous reliability model with numerous restraints is examined to optimize the recommended setup. Mosha Aghaei et al.(2016) suggested using an exact method based on integer programming and adding the redundancy strategy as a new decision variable to find the best answer.

The problem examines the unknowns that is, various elements (y_j), the element reliability (r_j) and the stage reliability (R_j) at a specific point for disposed multiple restraints to magnify the structure reliability which is described as a United Reliability Model (URM). In the literature, United Reliability Models are enhanced by applying value restraints where there is a fixed association between value and reliability. A consideration of the load and size as additional limitations, along with value, to form and improve the surplus reliability item for k out of n structure composition, is the distinctive pattern of the intended task. Ji Eunbyun et al. (2017) was able to generalise their work to k out of n systems by making slight modifications to the formulations of event and probability vectors. When it comes to the safe and effective development of technical systems, Krishna B. Mishra and Usha Sharma (1991) are common names. The influence of the aforementioned multiple constraints on the formulation of the integrated reliability and optimization is investigated by Pavankumar. S et al. (2020). IRRCCS is used for statistical purposes (Integrated Redundant Reliable Coherent Configuration system is considered).They provided real-world examples to demonstrate how multicriteria optimization problems can be used to solve redundancy optimization problems efficiently and effectively. An exhaustive investigation, design, analysis, and optimization of a coherent redundant reliability design have been detailed by Sridhar Akiri et al. (2021). When both technologies have parallel factors, the work of Sridhar Akiri et al. (2022) is applied to parallel-series systems. For a parallel strategy to work, all of its parts must always be up and running. The proposed method takes into account the statistical dependence between the faults of homogeneous and non-homogeneous components of k out of n systems through the use of a computed measure concerning the sensitivity of the parameters and the relative importance of the components.

2 Supposition and Notation

1. All the elements are assumed to be equally dependable at each stage. - i.e., Equal dependability applies to all constituents.
2. There should not be any correlation between the failure of one element and the performance of other elements in the same structure.

$R_s =$ Structure Reliability
$R_j =$ Reliability of phase $j,\ 0 < R_j < 1$
$r_j =$ Reliability of each element in phase $j,\ 0 < r_j < 1$
$y_j =$ Number of elements in phase j
$v_j =$ Value coefficient of each element in phase j
$l_j =$ Load coefficient of each element in phase j
$s_j =$ Size coefficient of each element in phase j
$V_o =$ Greatest allowable structure value
$L_o =$ Greatest allowable structure load
$S_o =$ Greatest allowable structure size
$b_j,\ d_j,\ p_j,\ q_j,\ g_j,\ h_j$ are Constants

3 Analytical Miniature

Miniature constraints and the need for objectivity

$$R_s = \prod_{j=1}^{n} R_j \tag{3.1}$$

Maximize

$$R_j = \sum_{k=2}^{y_i} \binom{y_j}{k} r_j^{k} (1 - r_j)^{y_i - k}. \tag{3.2}$$

Subject to the restraints

$$\sum_{j=1}^{n} v_j y_j \le V_0 \tag{3.3}$$
$$\sum_{j=1}^{n} l_j y_j \le L_0 \tag{3.4}$$
$$\sum_{j=1}^{n} s_j y_j \le S_0 \tag{3.5}$$

That y_j is an integer is required, and this is a positive restrictions $r_j,\ R_j > 0$

4 Mathematical Operation

To initiate the mathematical design, the function most often used is torn for the reliability model and analysis. A group of researchers led by G. Sankaraiah et al. (2011) set out to investigate the effect that multiple constraints have on system dependability. An innovative approach to optimising a redundant IRM with multiple constraints was developed by Sridhar

Akiri et al. (2013). The approach takes into account the k out of n configuration system and allows the optimisation to uncover the unanticipated impact of additional constraints beyond the cost constraint. Levitin et al. (2014) said that the best way to start up system elements can lower the estimated cost of the mission while ensuring some level of system reliability. Roya Soltani (2014) looked at the literature from many different points of view, such as system structure, system performance, the state of uncertainty, and how to solve the problem. A Lagrangian multiplier is used to model and solve an integrated redundant reliability system. Because this yields a real number, we can now calculate the number of components, the reliability of each stage, and the reliability of the entire system, and we can also predict the corresponding mathematical function. 1001

$$r_j = \left[\frac{c_j}{b_j}\right]^{\frac{1}{d_j}},$$

where b_j, d_j are constants.

System reliability for the given function

$$R_s = \prod_{j=1}^{n} \quad (4.1)$$

$$R_j \quad (4.2)$$

The number of items for each phase y_j is indicated by the relation.

$$y_j = \frac{\ln(R_j)}{\ln(r_j)} \quad (4.3)$$

The method under examination is

$$\text{Maximize } R_s = \prod_{j=1}^{n} \quad (4.4)$$

$$R_j \quad (4.5)$$

Subject to the restraints

$$\sum_{j=1}^{n} \left[\left(b_j r_j^{d_j}\right) y_j\right] - V_0 \leq 0. \quad (4.6)$$

$$\sum_{j=1}^{n} \left[\left(p_j r_j^{q_j}\right) y_j\right] - L_0 \leq 0. \quad (4.7)$$

$$\sum_{j=1}^{n} \left[\left(g_j r_j^{h_j}\right) y_j\right] - S_0 \leq 0. \quad (4.8)$$

5 The Lagrangian Approach

Lagrangean multipliers can be used to optimise non-differentiable functions under constraints, not just differentiable ones. Hugh Everett's arguments from 1963 suggest that

Lagrangean multipliers are "fail-safe" in the sense that any solution found by using them is a true solution, but they do not guarantee that a true solution will be found for every problem. Sasikala et al. (2020) showed that a real-valued solution can be found for the number of elements, element reliability, moment reliability, and ultimately structure reliability when modelling and solving with the Lagrangean method. Solving the proposed model by using the Lagrangean method. A Lagrangian approach is considered as follows:

$$F = R_s + \lambda_1 \left[\sum_{j=1}^{n} \left\{b_j {r_j}^{d_j}\right\} y_j - V_0\right] + \lambda_2 \left[\sum_{j=1}^{n} \left\{p_j {r_j}^{q_j}\right\} y_j - L_0\right] + \lambda_3 \left[\sum_{j=1}^{n} \left\{g_j {r_j}^{h_j}\right\} y_j - S_0\right]. \quad (5.1)$$

then,

$$F = R_s + \lambda_1 \left[\sum_{j=1}^{n} \left\{b_j {r_j}^{d_j}\right\} \frac{\ln(R_j)}{\ln(r_j)} - V_0\right] + \lambda_2 \left[\sum_{j=1}^{n} \left\{p_j {r_j}^{q_j}\right\} \frac{\ln(R_j)}{\ln(r_j)} - L_0\right] + \lambda_3 \left[\sum_{j=1}^{n} \left\{g_j {r_j}^{h_j}\right\} \frac{\ln(R_j)}{\ln(r_j)} - S_0\right]. \quad (5.2)$$

$$\frac{\partial F}{\partial R_j} = 1 + \lambda_1 \left[\sum_{j=1}^{n} \left\{b_j {r_j}^{d_j}\right\} \frac{1}{R_j} \frac{1}{\ln r_j}\right] + \lambda_2 \left[\sum_{j=1}^{n} \left\{p_j {r_j}^{q_j}\right\} \frac{1}{R_j} \frac{1}{\ln r_j}\right] + \lambda_3 \left[\sum_{j=1}^{n} \left\{g_j {r_j}^{h_j}\right\} \frac{1}{R_j} \frac{1}{\ln r_j}\right]. \quad (5.3)$$

$$\frac{\partial F}{\partial r_j} = \lambda_1 \left[\sum_{j=1}^{n} b_j \ln(R_j) \left\{d_j {r_j}^{d_j - 1} \frac{1}{\ln r_j} + {r_j}^{d_j} \frac{-1}{(\ln r_j)^2} \frac{1}{r_j}\right\}\right] + \lambda_2 \left[\sum_{j=1}^{n} p_j \ln(R_j) \left\{q_j {r_j}^{q_j - 1} \frac{1}{\ln r_j} + {r_j}^{q_j} \frac{-1}{(\ln r_j)^2} \frac{1}{r_j}\right\}\right] + \lambda_3 \left[\sum_{j=1}^{n} g_j \ln(R_j) \left\{h_j {r_j}^{v_j - 1} \frac{1}{\ln r_j} + {r_j}^{h_j} \frac{-1}{(\ln r_j)^2} \frac{1}{r_j}\right\}\right]. \quad (5.4)$$

$$\frac{\partial F}{\partial \lambda_1} = \left[\sum_{j=1}^{n} \left\{b_j {r_j}^{d_j}\right\} \frac{\ln(R_j)}{\ln(r_j)}\right] - V_0. \quad (5.5)$$

$$\frac{\partial F}{\partial \lambda_2} = \left[\sum_{j=1}^{n} \left\{p_j {r_j}^{q_j}\right\} \frac{\ln(R_j)}{\ln(r_j)}\right] - L_0. \quad (5.6)$$

$$\frac{\partial F}{\partial \lambda_3} = \left[\sum_{j=1}^{n} \left\{g_j {r_j}^{h_j}\right\} \frac{\ln(R_j)}{\ln(r_j)}\right] - S_0. \quad (5.7)$$

Where λ_1, λ_2, λ_3 are Lagrangian multipliers.

The Lagrangian method makes it possible to establish the number of elements present in each phase $((y_j))$, the optimal reliability of those elements $((r_j))$, the reliability of the phases $((R_j))$, and the reliability of the structure $((R_s))$. With this method, you get a genuine (valued) answer that takes into account price, weight, and dimensions.

6 Case Problem:

To derive the multiple parameters of a given mechanical system using optimization Techniques, where all the assumptions like value, load and size are directly proportional to the System Reliability has been considered in this research work. The same logic may not be true in the case of electronic systems. Hence, the optimal element accuracy (r_j), phase reliability (R_j), Number of elements in each phase (y_j) and structure accuracy (R_s) can be evaluated for any given mechanical system. In this work, an attempt has been made to evaluate structure accuracy of a special purpose machine that is used for Quartz watch assembly. The machine is used for the assembly of 4 components on the base of the watch plate. The machine's approximate value \$300, which has been considered as structure value, the load of the machine is 400 kg, which is the load of the structure and the space occupied by the machine is 600 cm^3, which is the volume or size of the structure. To attract the authors from different cross sections, the authors attempted to use hypothetical numbers, which can be changed according to the environment.

CONSTANTS: The data constants required for the case problem are set out below.

	Value Restraints		Load Restraints		Size Restraints	
Stage	b_j	d_j	p_j	q_j	g_j	h_j
1	60	2	90	2	100	2
2	50	3	70	3	100	3
3	50	4	60	4	106	4

6.1 Design of reliability with respect to Value, Load and Size – without y_j Culminated:

(i) Reliability Design relating to Value, Load and Size

Phase	r_j	R_j	y_j	v_j	$v_j y_j$	l_j	$l_j y_j$	s_j	$s_j y_j$
01	0.9471	0.9342	1.25	54	67	87.91	110	89.70	112
02	0.9608	0.9541	1.17	44	52	62.09	73	88.70	104
03	0.9882	0.9560	3.79	48	181	57.22	217	101.09	383
			Total	Value	300	Load	400	Size	600

SYSTEM RELIABILITY = R_s = 0.8521

6.2 Accuracy Method Connected to Value, Load and Size with y_j Culminated:

The reliability form is resumed by examining the values of y_j to be integers(by bringing close to the value of y_j to the adjacent integer) and the suitable outputs relating to the value, load and size are described in the below table, further offering the data by considering the Mutation due to value, load, size and structure reliability (before and after Culminated y_j):

(i) Design of reliability in relation to Value, Load and Size.

Phase	r_j	R_j	y_j	v_j	$v_j y_j$	l_j	$l_j y_j$	s_j	$s_j y_j$
01	0.9471	0.9701	2	54	108	87.91	176	89.70	179
02	0.9608	0.9402	2	44	89	62.09	124	88.70	177
03	0.9882	0.9863	4	48	191	57.22	229	101.09	404
			Total	Value	387	Load	529	Size	761

Mutation in total value:

$$= \frac{\text{TotalvaluewithCulminated} - \text{TotalvaluewithoutCulminated}}{\text{TotalvaluewithoutCulminated}}$$

$$= 29\%$$

Mutation in total load:

$$= \frac{\text{TotalloadwithCulminated} - \text{TotalloadwithoutCulminated}}{\text{TotalloadwithoutCulminated}}$$

$$= 32.5\%$$

Mutation in total size:

$$= \frac{\text{TotalsizewithCulminated} - \text{TotalsizewithoutCulminated}}{\text{TotalsizewithoutCulminated}}$$

$$= 26.83\%$$

System Reliability = R_s = 0.8995

Mutation In System Reliability = R_s = 5.26%

7 Heuristic Method:

The Lagrangian multipliers model provides an explanation to appear at an ideal model instantly rather than any refined algorithms. This is of course done at the value of examining various elements at a specific point (y_j) as actual. This detriment can be beaten, by the heuristic method. The Heuristic method, in many situations, operates under examination and exploratory and lapse approach. A heuristic method is especially employed to immediately come to an elucidation that is fairly near to the best probable explanation or optimal clarification.

The unified reliability models for superfluous systems with multiple constraints are formed and optimized using a heuristic approach to the proposed mathematical function under consideration. The heuristic approach used in this paper to optimize the design uses the element reliability values (rj) and the number of elements in each phase (yj) as inputs to solve the problem of the mathematical function considered in this study. This method is effective in optimizing the design when (yj) are integers, making it suitable for direct application to real-world problems. The different values in respect of value, load and size for the proposed united superfluous reliability model through the heuristics are analyzed and presented for the considered mathematical function under probe is presented in section 7.

By using Heuristic Algorithm, the integer values of Value, Load and Size elements have been derived. The reliability values for the number of elements in all 3 phases are shown in the following table.

(i) Reliability Design Relating to Value, Load and Size.

Phase	r_j	R_j	y_j	v_j	$v_j y_j$	l_j	$l_j y_j$	s_j	$s_j y_j$
01	0.9471	0.9701	1	54	54	87.91	88	89.70	90
02	0.9608	0.9666	2	44.5	89	62.09	124	88.70	177
03	0.9666	0.9776	3	48.3	143	57.22	172	101.09	303
			Total	value	286	load	384	size	570

Mutation in total value $= 4.6\%$
Mutation in System Reliability=7.58%

Mutation in total load $= 4.0\%$
Mutation in System Reliability$= 7.58\%$

System Reliability(R_s) = 0.9167
Mutation in total size$= 3.83\%$
Mutation in System Reliability$= 7.58\%$

8 Dynamic Programming Algorithm :

The heuristic method generally gives a practically nearest explanation. To approve the settled Superfluous Reliability System and to procure the much needed integer solution the Dynamic Programming method is used. Dynamic programming with functional equations was developed by Richard Bellman and Stuart Dreyfus (1958) to address a specific category of issues that crop up during the assembly of multiple parts. An instance where considerations of weight and price must be taken into account when deciding how many and what kind of parts should be used to construct a device that is both lightweight and durable.To find out the phase reliability, structure reliability, phase value and System value, the Lagrangian approach can be employed as the input for the Dynamic Programming Approach. The Dynamic Programming Approach gives affability in defining the multiple aspects in every point; phase reliability and structure reliability for the given structure value. As per the method, the specification values borrowed from the Lagrangian are produced as results from the Dynamic Programming Method to procure the integer examination for the determined case problem in 3 phases.

8.1 Dynamic Programming - phase 1:

Number of Elements y_j	Phase Reliability R_j
01	0.8271
02	0.9701
03	0.9948
04	0.9991
05	0.9998
06	0.9999

8.2 Dynamic Programming - phase 2:

Number of Elements y_j	Phase Reliability R_j			
02	0.9099			
03	0.9436	0.9321		
04	0.9468	0.9666	0.9353	
05	0.8270	0.9698	0.9912	0.9394

8.3 Dynamic Programming - phase 3:

Number of Elements y_j	Phase Reliability R_j					
03	0.6927					
04	0.9211	0.8580				
05	0.9552	0.8789	0.8975			
06	0.9795	0.9114	0.9193	0.9070		
07	0.9881	0.9346	0.9534	0.9290	0.9099	
08	0.9881	0.9428	0.9776	0.9634	0.9320	0.9099

(i) Reliability Design Relating to Value, Load and Size.

phase	r_j	R_j	y_j	v_j	$v_j y_j$	l_j	$l_j y_j$	s_j	$s_j y_j$
01	0.9471	0.9701	1	54	54	87.91	88	89.70	90

02	0.9608	0.9666	2	44.5	89	62.09	124	88.70	177
03	0.9882	0.9776	3	48.6	143	57.22	172	101.09	303
			Total	Value	286	Load	384	Size	570

Mutation in Total Value =4.66%
Mutation in Total Load=4.0%
Mutation in Total Size =3.83%

System Reliability (R_s) = 0.9167
Mutation in System Reliability $= 7.58\%$

The analysis gives the following information to improve the structure reliability for the proposed combined reliability structure restrained by various constraints in the k out of n configuration.

	LAGRANGIAN METHOD									
phase	**Without X_j Culminated**					**With X_j Culminated**				
	y_j	Rs	value	load	size	y_j	Rs	value	load	size
phase 1	1.25	0.8521	300	400	600	2	0.8995	387	529	761
phase 2	1.17					2				
phase 3	3.79					4				
phase	**Heuristic Method**					**Dynamic Programming**				
	y_j	Rs	value	load	size	y_j	Rs	value	load	size
phase 1	1	0.9167	286	384	570	1	0.9167	286	384	570
phase 2	2					2				
phase 3	3					3				

(Restraints $V_0 \leq 300$, $L_0 \leq 400$, $S_0 \leq 600$)

9 Conclusion:

The results of the planned mathematical approach indicate that, despite the Lagrangian method's presentation of an explanation in real values, the multiple aspects required are being brought close to the adjacent integer in preparation for possible application to real-world problems. The structure specifications responsible for enforcing the reliability structure method will be affected in particular by this coarse method of rounding-off (y_j) values. This method also presupposes that the more valuable the occurrence, the more trustworthy it is, but it never suggests a scientific explanation.

Heuristic approaches, when applied to the appropriate methods, can be effective in determining enlarged authenticity for the organization while meeting essential restraints; this is especially true when trying to determine an integer explanation before administering the best of exact means, such as the dynamic programming method.

The Value, Load, and Size guidelines are followed without hiccup, and the desired outcomes are attained thanks to the research and implementation of a productive programming approach for the prospective mathematical method. This paper analyzed the composition and optimization of a unique numerous restraint superfluous reliability structure and then focused on stabilizing that structure.

Figuring out each restraint's component count, phase, and structure reliability in optimizing the proposed model is found to be greatly aided by the concept of deriving a sophisticated integer solution for the proposed model via a dynamic programming approach.

The following are some suggestions for future research directions:

1. Resolving the IRM using various mathematical functions that take into account the Cost, Weight and Volume
2. Considering other mathematical techniques, such as Newton-Raphson methods and Integer programming
3. Other elements into account, such as Space, Temperature, and Time.

10 Compliance with Ethical Standards:

There is no conflict of interest between any parties in this paper. Also, ethical clearance is not required as there are no humans or animals involved in this project. The consent of all the authors are taken in confidence.

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