Dynamical system induced by tensor product of composition and multiplication operators on tensor product of Bloch space of cross sections

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Abstract :

Tensor product of composition and multiplication operators induce dynamical system on tensor product of Bloch space of cross section $\mathcal{LB}_0(X)$ (or $\mathcal{LB}_b(X)$).

DOI: Number: 10.48047/nq.2022.20.22.NQ10477 **NeuroQuantology 2022; 20(22): 4727-4732**

1 Introduction

Let X and Y represents compact Hausdroff spaces and $\mathcal{C}(X,Y)$ represents the space of all continuous complex valued functions from X and Y. If X equals to Y, we write $\mathcal{C}(X)$ for $\mathcal{C}(X,Y)$. Assume that $X \otimes Y$ is the tensor product of X and Y. Then each mapping $\tau: X \to X$ produces a linear transformation C_τ from $\mathcal{C}(X,Y)$ itself denoted as $C_\tau f = f \circ \tau$ for every $f \in \mathcal{C}(X,Y)$ and it is referred to as a composition operator on $\mathcal{C}(X,Y)$ induced by τ. Consider $\vartheta: X \to C$ to be a mapping. The scalar multiplication then produces a linear transformation M_ϑ from $\mathcal{C}(X,Y)$ itself defined as $M_\vartheta f = \vartheta f$ for any $f \in \mathcal{C}(X,Y)$ where the product of functions is expressed point-wise and is known as a multiplication operator on $\mathcal{C}(X,Y)$. Let $\vartheta_t: X \to \mathbb{R}$ defined by $\vartheta_t(x) = e^{th(x)}$ for all $t \in \mathbb{R}$ and

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2000 AMS Subject Classification Code : 46MO5, 47A80, 47B37, 47B38.

Key words and Phrases : Tensor product operator, composition operators, multiplication operator, dynamical system and Bloch space.

$x \in X$ where $h \in C_b(X, \mathbb{R})$. Also $\tau_t: \mathbb{R} \to \mathbb{R}$ is defined by $\tau_t(\omega) = \tau + \omega$ the self map [9].

Let G be a topological group with e as the identity, let X be a topological space and $\vartheta: G \times X \to X$ be the continuous map such that [(i)]

1. $\vartheta(e, x) = x$ for every $x \in X$
2. $\vartheta(s + t, x) = \vartheta(s, \vartheta(t, x))$ for every $t, s \in G, x \in X$.

Then the triple (G, X, ϑ) is called a transformation group, X is a state space. If $G = (\mathbb{R}, +)$ the corresponding transformation group is called a dynamical system. The transformation group $(\mathbb{R}, X, \vartheta)$ is known as continuous dynamical system. If X is a Banach space and $\vartheta(t, \alpha x + \beta y) = \alpha\vartheta(t, x) + \beta\vartheta(t, y)$, for $t \in \mathbb{R}, \alpha, \beta \in C$, $x, y \in X$ the $(\mathbb{R}^+, X, \vartheta)$ is called a linear dynamical system [1].

The Bloch space, named for French mathematician Andre Bloch and abbreviated $\mathcal{B}$ in complex analysis, is the space of holomorphic functions f defined on the open unit disc $\mathcal{D}$ in the complex plane, such that the function $(1 - |z|^2)||f'(z)|$ is bounded. $\mathcal{B}$ is a Banach space, with the norm defined by $\| f \|_{\mathcal{B}} = |f(0)| + \sup_{z \in \mathcal{D}} (1 - |z|^2)|f'(z)|$. This is known as the Bloch norm and the Bloch space's constituents are known as Bloch functions. We define the set

$$\mathcal{B} = \left\{ f \in H(D); \sup_{|z|<1} \{(1 - |z|^2)\left|f'(z)\right| < \infty\} \right\}.$$

The seminorm $\| f \|_{\mathcal{B}} = \sup_{|z|<1} \{(1 - |z|^2)|f'(z)|\}$ and the norm $\| f \|_{Bloch} = |f(0)| + \| f \|_{\mathcal{B}}$. The Bloch space is the set $\mathcal{B}$ with the norm $\| \cdot \|_{Bloch}$ [4].

2 Dynamical system induced by tensor product of composition and multiplication operators on tensor product of Bloch space of cross-section

Let X be a Hausdorff topological space. A vector fibration over X is a pair $\left(X, (F_x)\right)_{x \in X}$, where each F_x is a vector space over the field K (where $K = \mathbb{R}$ (or) $\mathbb{C}$). A cross-section over X is then any element of the Cartesian product $\prod_{x \in X} F_x$. The Cartesian product $\prod_{x \in X} F_x$ is made a vector space in the usual way and a vector space of cross-section over X is by definition any vector subspace of $\prod_{x \in X} F_x$.

If f and b are cross-section and Bloch function respectively over X. Then we will designate by $b[f]$ the positive valued function on X that takes x turns it into $b_x[f(y)]$. $L(x)$ denotes a vector space of cross-section over X. Now consider the Bloch spaces of cross sections over X with regard to the system of Bloch function $\mathcal{B}$ on X. $\mathcal{LB}_0(X) = \{f \otimes g \in L(y); b[f \otimes g]$ is holomorphic function and vanishes at infinity on X for each $b \in \mathcal{B}\}$ and $\mathcal{LB}_b(X) = \{f \otimes g \in L(y);\ b[f \otimes g]$ is a bounded function on X for each $b \in \mathcal{B}\}$. Then $\mathcal{LB}_0(X)$ and $\mathcal{LB}_b(X)$ are vector spaces and $\mathcal{LB}_0(X) \subseteq \mathcal{LB}_b(X)$.

Now for $b \in \mathcal{B}$ and $f \in L(X)$, if we put $\| f \otimes g \|_b = \sup_{|z|<1} \left\{(1 - |z|^2)\left|b'_y[f(z) \otimes g(z)]\right|, y \in Y, z \in \mathcal{D}\right\}$, then $\| \cdot \|_b$ can be regarded as a seminorm on either $L\mathcal{B}_0(X)$ or $L\mathcal{B}_b(X)$ and the family of seminorms $\{\| \cdot \|_b; b \in \mathcal{B}\}$ defines a Hausdorff locally convex topology on each of these spaces. This topology is denoted by τ_b and the vector space endowed with τ_b are referred to as Bloch space of cross section Because $\mathcal{B}$ is directed set of Bloch functions, τ_b has a basis of closed absolutely convex neighbourhood of the type [10].

$$\mathcal{N}_{b,\ell} = \{f \otimes g; f, g \in \mathcal{LB}_0(X,Y) \ni \| f \otimes g \|_b \leq 1\}. [10]$$

Theorem 2.1 *Let $\tau: X \to X$ and $\vartheta_t: X \to C$ be a continuous functions. Then $C_\tau f \otimes M_{\vartheta_t} g$ is bounded for every $t \in \mathbb{R}$, $f \otimes g \in \mathcal{LB}_0(X) \otimes \mathcal{LB}_0(X)$.*

Proof. To prove that $C_\tau f \otimes M_{\vartheta_t} g$ is bounded.
It is enough to prove that $C_\tau f \otimes M_{\vartheta_t} g$ is continuous at the origin.
We claim that $(C_\tau \otimes M_{\vartheta_t})(\mathcal{N}_{b,l}) \subseteq \mathcal{N}_{b,l}$.
For, $f \otimes g \in \mathcal{N}_{b,l}$ we have,

$$\begin{aligned}
\left\|C_\tau f \otimes M_{\vartheta_t} g\right\|_b &= \sup_{|z|<1}\left\{(1-|z|^2)\left|b_y'\left[C_\tau f(z) \otimes M_{\vartheta_t} g(z)\right]\right| \text{foreveryt} \in \mathbb{R}\text{andz} \in \mathcal{D}\right\} \\
&= \sup_{|z|<1}\left\{(1-|z|^2)\left|b_y'[(f \circ \tau)(z) \otimes \vartheta_t(z) g(z)]\right| \text{ for every t} \in \mathbb{R} \text{ and z} \in \mathcal{D}\right\} \\
&= \sup_{|z|<1}\left\{(1-|z|^2)|b_y'[f(\tau(z)) \otimes e^{th(z)} g(z)]|\right\} \\
&\leq \sup_{|z|<1}\left\{(1-|z|^2)|b_y'[f(z) \otimes e^{|t||h(z)|} g(z)]|\right\} \\
&= \sup_{|z|<1}\left\{(1-|z|^2)|b_y'[f(z) \otimes g(z)]| \text{ast} \to 0\right\} \\
&= \| f \otimes g \|_b \leq 1
\end{aligned}$$

Therefore $\left\|C_\tau f \otimes m_{\vartheta_t} g\right\|_b \leq 1$.
Therefore $C_\tau f \otimes M_{\vartheta_t} g$ is continuous at the origin.
Therefore $C_\tau f \otimes M_{\vartheta_t} g$ is bounded.

Theorem 2.2 *Let $B(E)$ be the Banach algebra of all bounded linear operators on E. Let $L^\infty(X, B(E))$ be the space of bounded functions. Let $g_\alpha(\tau_{t_\alpha})$ converges to $g(\tau_t)$ in $L^\infty(X, B(E))$ and let f_α be a sequence converging to f in $\mathcal{LB}_0(X,E)$. Then the product $f_\alpha g_\alpha(\tau_{t_\alpha})$ converges to $f g(\tau_t)$ in $\mathcal{LB}_0(X,E) \otimes L\mathcal{B}_0(X,E)$.*

Proof. Let $g_\alpha(\tau_{t_\alpha}$) converges to $g(\tau_t)$ in $L^\infty(X, B(E))$ and let f_α be a sequence converging to f in $\mathcal{LB}_0(X,E)$.
To prove that $f_\alpha g_\alpha(\tau_{t_\alpha})$ converges to $f g(\tau_t)$ in $\mathcal{LB}_0(X,E) \otimes \mathcal{LB}_0(X,E)$. Now,

$$\begin{aligned}
\| f_n g_n(\tau_{t_n}) - f g(\tau_t) \|_B &= \sup_{|z|<1}\{(1-|z|^2)|[f_n g_n(\tau_{t_n}) - f g(\tau_t)]'(z)|; \quad z \in \mathcal{D}\} \\
&= \sup_{|z|<1}\{(1-|z|^2)|[f_n g_n(\tau_{t_n})]'(z) - \left[f g(\tau_t)\right]'(z)\right|; z \in \mathcal{D}\} \\
&= \sup_{|z|<1}\left\{(1-|z|^2)\left|f_n(z)[g_n(\tau_{t_n})]'(z)\right) + g_n\left(\tau_{t_n}(z)\right) f_n'(z)\right. \\
&\quad -f(z)[g(\tau_t)]\text{^}'(z)) - g(\tau_t (z)) f\text{^}'(z)|; z \in \mathcal{D}\} \\
&= \sup_{|z|<1}\{(1-|z|^2)|f_n(z)[g_n(\tau_{t_n})]'(z) \\
&\quad -f_n(z)[g(\tau_t)]'(z) + f_n(z)[g(\tau_t)]'(z) \\
&\quad +g_n\left(\tau_{t_n}(z)\right) f_n'(z) - f_n'(z) g\left(\tau_t(z)\right) + f_n'(z) g\left(\tau_t(z)\right) \\
&\quad -f(z)\left[g(\tau_t)\right]'(z) - g\left(\tau_t(z)\right) f'(z)\right|; z \in \mathcal{D}\} \\
&= \sup_{|z|<1}\{(1-|z|^2)|f_n(z)||[g_n(\tau_{t_n})]'(z) - [g(\tau_t))]'(z)|\}
\end{aligned}$$

$$+\sup_{|z|<1}\{(1-|z|^2)|[g(\tau_t)]'(z)||f_n(z)-f(z)|\}$$
$$+\sup_{|z|<1}\left\{(1-|z|^2)|f_n'(z)|\left|g_n\left(\tau_{t_n}(z)\right)-g\left(\tau_t(z)\right)\right|\right\}$$
$$+\sup_{|z|<1}\{(1-|z|^2)|g(\tau_t(z))||f_n'(z)-f'(z)|\}$$
$$=|f_n(z)| \parallel g_n(\tau_{t_n})-g(\tau_t)\parallel_B + \parallel g(\tau_t)\parallel_B |f_n(z)-f(z)|$$
$$+\parallel f_n \parallel_B |g_n(\tau_{t_n}(z))-g(\tau_t(z))| + |g(\tau_t(z))| \parallel f_n - f \parallel\parallel_B \to 0$$

as $\|g_n(\tau_{t_n})-g(\tau_t)\|_{\mathcal{B}} \to 0$, $|f_n(z)-f(z)| \to 0$, $|g_n(\tau_{t_n}(z))-g(\tau_t(z))| \to 0$ and $\|f_n-f\|_{\mathcal{B}} \to 0$. Therefore $\|f_n g_n(\tau_{t_n})-fg(\tau_t)\|_B \to 0$. Therefore $f_\alpha g_\alpha(\tau_{t_\alpha})$ converges to $fg(\tau_t)$ in $\mathcal{LB}_0(X,E)\otimes\mathcal{LB}_0(X,E)$.

Theorem 2.3 *Let $\nabla:\mathbb{R}\times\mathcal{LB}_0(\mathbb{R})\otimes\mathcal{LB}_0(\mathbb{R})\to L(\mathbb{R})\otimes L(\mathbb{R})$ be the function defined by $\nabla(t,f\otimes g)=(C_{\tau_t}\otimes M_{\vartheta_t})(f\otimes g)$ for all $t\in\mathbb{R}$ and $f\otimes g\in\mathcal{LB}_0(\mathbb{R})\otimes\mathcal{LB}_0(\mathbb{R})$. Then ∇ is a linear dynamical system on $\mathcal{LB}_0(\mathbb{R})\otimes\mathcal{LB}_0(\mathbb{R})$.*

Proof. Since $C_{\tau_t}\otimes M_{\vartheta_t}$ is a tensor product on $\mathcal{LB}_0(\mathbb{R})\otimes\mathcal{LB}_0(\mathbb{R})$ for every $t\in\mathbb{R}$ and $f\otimes g\in$ 4730
$\mathcal{LB}_0(\mathbb{R})\otimes\mathcal{LB}_0(\mathbb{R})$. It can be easily seen that $\nabla(o,f\otimes g)=f\otimes g$ and $\nabla(s+t,f\otimes g)=\nabla(s,\nabla(t,f\otimes g))$.

In order to show that $\nabla(t,f\otimes g)$ is a dynamical system on $\mathcal{LB}_0(\mathbb{R})\otimes\mathcal{LB}_0(\mathbb{R})$. It is enough to prove that ∇ is continuous. Let $g_\alpha\to g$ and let $(t_\alpha,f_\alpha\otimes g_\alpha)$ be a set in $\mathbb{R}\times\mathcal{LB}_0(\mathbb{R})\otimes\mathcal{LB}_0(\mathbb{R})$ such that $(t_\alpha,f_\alpha\otimes g_\alpha)\to(t,f\otimes g)$. We shall prove that, $\nabla(t_\alpha,f_\alpha\otimes g_\alpha)\to\nabla(t,f\otimes g)$.

$$\parallel \nabla(t_\alpha,f_\alpha\otimes g_\alpha)-\nabla(t,f\otimes g)\parallel_B = \parallel (C_{\tau_{t_\alpha}}\otimes M_{\vartheta_{t_\alpha}})(f_\alpha\otimes g_\alpha)-(C_{\tau_t}\otimes M_{\vartheta_t})(f\otimes g)\parallel_B$$
$$=\sup_{|z|<1}\{(1-|z|^2)|[c_{\tau_{t_\alpha}}f_\alpha\otimes M_{\vartheta_{t_\alpha}}g_\alpha - C_{\tau_t}f\otimes M_{\vartheta_t}g]'(z)|\}$$
$$=\sup_{|z|<1}[(1-|z|^2)|\{(C_{\tau_{t_\alpha}}f_\alpha)'(z)\otimes(M_{\vartheta_{t_\alpha}}g_\alpha)'(z)\}-\{(C_{\tau_t})'(z)\otimes(M_{\vartheta_t}g)'(z)\}|]$$
$$=\sup_{|z|<1}[(1-|z|^2)|\{(f_\alpha\circ\tau_{t_\alpha})'(z)-(f\circ\tau_t)'(z)\}\otimes\{(\vartheta_{t_\alpha}g_\alpha)'(z)-(\vartheta_t g)'(z)\}|]$$
$$=\sup_{|z|<1}[(1-|z|^2)|\{f_\alpha'(\tau_{t_\alpha})-f'(\tau_t)\}(z)\otimes\{\vartheta_{t_\alpha}(z)[g_\alpha'(z)-g'(z)]+\vartheta_{t_\alpha}'(z)[g_\alpha(z) - g(z)]\}|]$$
$$=\sup_{|z|<1}[(1-|z|^2)]|(f_\alpha(\tau_{t_\alpha})-f(\tau_t))'(z)\otimes\{\vartheta_{t_\alpha}(z)(g_\alpha-g)'(z)+\vartheta_{t_\alpha}'(z)[(g_\alpha - g)(z)]\}|] = \parallel f_\alpha(\tau_{t_\alpha})-f(\tau_t)\parallel_B \otimes |\vartheta_{t_\alpha}(z)| \parallel g_\alpha - g\parallel_B + \parallel \vartheta_{t_\alpha}\parallel_B |(g_\alpha-g)(z)| \to 0$$

as $g_\alpha\to g$ and $f_\alpha(\tau_{t_\alpha})\to f(\tau_t)$. Therefore $\nabla(t,f\otimes g)$ is a dynamical system on $\mathcal{LB}_0(\mathbb{R})\otimes\mathcal{LB}_0(\mathbb{R})$.

Next we have to prove that ∇ is linear. That is to prove that

$$\nabla[t,\alpha(f\otimes g)+\beta(i\otimes j)]=\alpha\nabla(t,f\otimes g)+\beta\nabla(t,i\otimes j)$$

for all $t \in \mathbb{R}, \alpha, \beta \in C$, $f \otimes g$ and $i \otimes j \in \mathcal{LB}_0(\mathbb{R}) \otimes \mathcal{LB}_0(\mathbb{R})$. Therefore $\nabla(t, f \otimes g)$ is a linear dynamical system on $\mathcal{LB}_0(\mathbb{R}) \otimes \mathcal{LB}_0(\mathbb{R})$.

Theorem 2.4 *Let $\tau: X \to X$ and $\vartheta_t: X \to C$ be a continuous functions. Then $M_{\vartheta_t} f \otimes C_\tau g$ is bounded for every $t \in \mathbb{R}$, $f \otimes g \in \mathcal{LB}_0 \otimes \mathcal{LB}$.*

Proof. To prove that $M_{\vartheta_t} f \otimes C_\tau g$ is bounded. It is enough to prove that $M_{\vartheta_t} f \otimes C_\tau g$ is continuous at the origin.
We claim that $(M_{\vartheta_t} \otimes C_\tau)(\mathcal{N}_{b,l}) \subseteq \mathcal{N}_{b,l}$. For, $f \otimes g \in \mathcal{N}_{b,l}$ we have

$$\begin{aligned} \| M_{\vartheta_t} f \otimes C_\tau g \|_b &= \sup_{|z|<1} \left\{(1-|z|^2)\left|b_y^{'}\left[M_{\vartheta_t} f(z) \otimes C_\tau g(z)\right]\right| \text{ for every } t \in \mathbb{R} \text{ and } z \in \mathcal{D}\right\} \\ &= \sup_{|z|<1} \{1-|z|^2)\left|b_y^{'}[f(z) \otimes g(z)]\right| \text{as} t \to 0\} \\ &= \| f \otimes g \|_b \leq 1 \end{aligned}$$

Therefore $\left\|M_{\vartheta_t} f \otimes C_\tau g\right\| \leq 1$.
Therefore $M_{\vartheta_t} f \otimes C_\tau g$ is continuous at the origin.
Therefore $M_{\vartheta_t} f \otimes C_\tau g$ is bounded.

Theorem 2.5 *Let $\nabla: \mathbb{R} \times \mathcal{LB}_0(\mathbb{R}) \otimes \mathcal{LB}_0(\mathbb{R}) \to L(\mathbb{R}) \otimes L(\mathbb{R})$ be the function defined by $\nabla(t, f \otimes g) = M_{\vartheta_t} \otimes C_{\tau_t}(f \otimes g)$ for all $t \in \mathbb{R}$ and $f \otimes g \in \mathcal{LB}_0(\mathbb{R}) \otimes \mathcal{LB}_0(\mathbb{R})$. Then ∇ is a linear dynamical system on $\mathcal{LB}_0(\mathbb{R}) \otimes \mathcal{LB}_0(\mathbb{R})$.*

Proof. Since $M_{\vartheta_t} \otimes C_{\tau_t}$ is a tensor product on $L(\mathbb{R}) \otimes L(\mathbb{R})$ for every $t \in \mathbb{R}$ and $f \otimes g \in \mathcal{LB}_0(\mathbb{R}) \otimes \mathcal{LB}_0(\mathbb{R})$ we have $\nabla(o, f \otimes g) = f \otimes g$ and $\nabla(s+t, f \otimes g) = \nabla(s, \nabla(t, f \otimes g))$.
In order to show that $\nabla(t, f \otimes g)$ is a dynamical system on $\mathcal{LB}_0(\mathbb{R}) \otimes \mathcal{LB}_0(\mathbb{R})$. It is enough to prove that ∇ is continuous. Now,

$$\begin{aligned} &\| \nabla(t_\alpha, f_\alpha \otimes g_\alpha) - \nabla(t, f \otimes g) \|_B = \| \left(M_{\vartheta_{t_\alpha}} \otimes C_{\tau_{t_\alpha}}\right)(f_\alpha \otimes g_\alpha) - \left(M_{\vartheta_t} \otimes C_{\tau_t}\right)(f \otimes g) \|_B \\ &= \sup_{|z|<1} \{(1-|z|^2)|[(M_{\vartheta_{t_\alpha}} f_\alpha \otimes C_{\tau_{t_\alpha}} g_\alpha) - (M_{\vartheta_t} f \otimes C_{\tau_t} g)]^{'}(z)|\} \\ &= \sup_{|z|<1} [(1-|z|^2)|\{(M_{\vartheta_{t_\alpha}} f_\alpha)^{'}(z) \otimes (C_{\tau_{t_\alpha}} g_\alpha)^{'}(z)\} - \{(M_{\vartheta_t} f)^{'}(z) \otimes (C_{\tau_t} g)^{'}(z)\}|] \\ &= \sup_{|z|<1} [(1-|z|^2)|\{(\vartheta_{t_\alpha} f_\alpha)^{'}(z) \otimes (g_\alpha \circ \tau_{t_\alpha})^{'}(z)\} - \{(\vartheta_t f)^{'}(z) \otimes (g \circ \tau_t)^{'}(z)\}|] \\ &= \sup_{|z|<1} [(1-|z|^2)|\{(\vartheta_{t_\alpha} f_\alpha)^{'}(z) - (\vartheta_t f)^{'}(z)\} \otimes \{(g_\alpha \circ \tau_{t_\alpha})^{'}(z) - (g \circ \tau_t)^{'}(z)\}|] \\ &= \sup_{|z|<1} [(1-|z|^2)|[\vartheta_{t_\alpha}(z)\{f_\alpha^{'}(z) - f^{'}(z)\} + \vartheta_{t_\alpha}^{'}(z)\{f_\alpha(z) - f(z)\}] \otimes [g_\alpha^{'}(\tau_{t_\alpha} - g^{'}(\tau_t)](z)|] \\ &= |\vartheta_{t_\alpha}(z)|\{\| f_\alpha - f \|_B + \| \vartheta_{t_\alpha} \|_B |(f_\alpha - f)(z)|\} \otimes \| g_\alpha(\tau_{t_\alpha}) - g(\tau_t) \|_B \to 0 \end{aligned}$$

as $f_\alpha \to f$ and $g_\alpha(\tau_{t_\alpha}) \to g(\tau_t)$.
Therefore $\nabla(t, f \otimes g)$ is a dynamical system on $\mathcal{LB}_0(\mathbb{R}) \otimes \mathcal{LB}_0(\mathbb{R})$.

Next we have to prove that ∇ is linear. That is to prove that

$$\nabla[t, \alpha(f \otimes g) + \beta(i \otimes j)] = \alpha\nabla(t, f \otimes g) + \beta\nabla(t, i \otimes j)$$

for all $t \in \mathbb{R}, \alpha, \beta \in C f \otimes g$ and $i \otimes j \in \mathcal{LB}_0(\mathbb{R}) \otimes \mathcal{LB}_0(\mathbb{R})$.
Therefore $\nabla(t, f \otimes g)$ is a linear dynamical system on $\mathcal{LB}_0(\mathbb{R}) \otimes \mathcal{LB}_0(\mathbb{R})$.

Note 2.6 *All results in this section hold if we replace $\mathcal{LB}_b(X, E)$ as a substitute of $\mathcal{LB}_0(X, E)$.*

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