Even Vertex Analytic Mean Labeling On Some Special Graphs

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ABSTRACT

A graph labeling is an assignment of integers to the vertices or edges, or both, subject to certain conditions. A graph G with p vertices and q edges is said to have an even vertex analytic mean labeling if there exists an injective function $f: V(G) \to \{2,4,6,\ldots,2q\}$ such that the induced map $f^*: E(G) \to \{1,2,3,\ldots,N\}$ defined by

$$f^*(e=uv) = \begin{cases} \frac{|[f(u)]^2-[f(v)]^2|}{2} & if\ |[f(u)]^2-[f(v)]^2|\ is\ even \\ \frac{|[f(u)]^2-[f(v)]^2|+1}{2} & if\ |[f(u)]^2-[f(v)]^2|\ is\ odd \end{cases}$$

and the edge labels are distinct. A graph that admits an even vertex analytic mean labeling is called an even vertex Analytic Mean Graph.

Keywords: Graph Labeling, Even vertex Analytic Mean Graph, Even vertex Analytic Mean Labeling.

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1. INTRODUCTION

Throughout this paper, by a graph we mean a finite, undirected, simple graph. Let $G(V,E)$ be a graph with p vertices and q edges. For notations and terminology we follow [2]. The concept of mean labeling was introduced by Somasundaram and Ponraj [4] and analytic mean labeling was introduced by Tharmaraj and Sarasija[5]. Motivated the results in [5]& [6] we introduced a new mean labeling called odd vertex analytic mean labeling. We proved that Octopus graph O_n , $K_{1,n}$ ʘ

$K_{1,n}$,Double $Z_n - path$, Crown graph $CR_n(G)$, Super Ladder $SL(G)$, Friendship design graph $DS^r(G)$.

Definition1 . 1 An Octopus graph O_n , $(n \geq 2)$ can be constructed from a fan graph F_n , $(n \geq 2)$ by joining a star graph $K_{1,n}$ with sharing a common vertex, where n is any positive integer. $i.e., O_n = F_n + K_{1,n}$.

Definition1 . 2 The Corona $G_1 \odot G_2$ of two graphs G_1 and G_2 is defined as the graph G obtained by taking one copy of G_1 (which has P_1 vertices) and P_1 copies of G_2 and then joining the i^{th} vertex of G_1 to every vertices in the i^{th} copy of G_2 .

Definition1 . 3 Double $Z_n - path$ graph is obtained by connecting two paths P_n of vertices $u_i's \ and \ v_i's$ by appending $u_i v_{i+1} \ and \ v_i u_{i+1} \ for \ 1 \leq i \leq n-1$.

Definition1 . 4 Crown graph $CR_n(G)$ is obtained by appending a pendent vertex to each vertex of a regular cycle.

Definition1 . 5 Super Ladder graph $SL(G)$ is a graph obtained from a ladder extending the pendent vertices w, x, y, z with u_1, v_1, u_n, v_n respectively on both sides of upper and lower part of the ladder. Then the ladder graph is said to be Super ladder graph.

Definition1 . 6 Design graphconsisting of r copies of $DS(G)$ concatenated at exactly one vertex is said to be a friendship design graph and is denoted by $DS^r(G)$.

2. MAIN RESULTS

Theorem2.1

An Octopus graph O_n is an even vertex analytic mean graph.

Proof:

Let $V = \{u_i / 1 \leq i \leq 2n+1\}$ be the vertex set and

$E = \{u_1 u_i / 1 \leq i \leq 2n+1\} \cup \{u_i u_{i+1} / 2 \leq i \leq n\}$ be the edge set of an Octopus graph O_n .

Here $|V(G)| = 2n+1 \ \ and \ \ |E(G)| = 3n-1$

Define a function $f : V \to \{2,4,6,...2q\}$ by

$f(u_i) = 2i$, $1 \leq i \leq 2n+1$

Then the edges are labeled as follows

For $2 \leq i \leq 2n+1$

$$f^*(u_1 u_i) = \frac{\left|[f(u_1)]^2 - [f(u_i)]^2\right|}{2}$$
$$= \frac{\left|4 - 4i^2\right|}{2}$$
$$= 2i^2 - 2$$

For $2 \le i \le n+1$

$$f^*(u_i u_{i+1}) = \frac{\left|[f(u_i)]^2 - [f(u_{i+1})]^2\right|}{2}$$
$$= \frac{\left|[(2i)]^2 - [(2i+2)]^2\right|}{2}$$
$$= 4i + 2$$

Then the edge labels are distinct.

Hence, the Octopus graph O_n is an even vertex analytic mean graph.

Example 4354

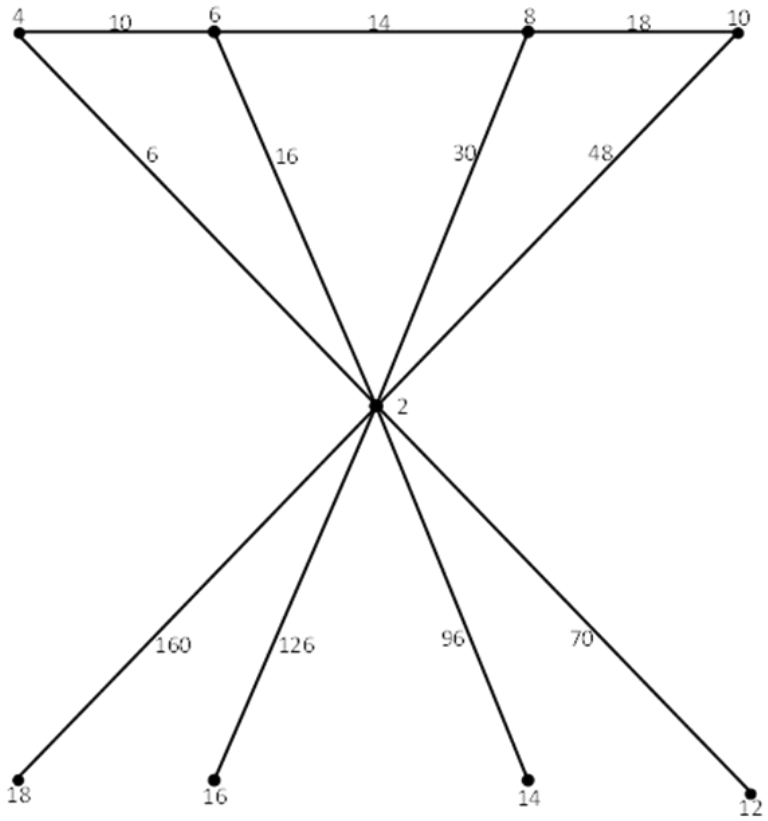

Figure 1.Octopus graph O_4

Theorem 2.2

$K_{1,n}$ ʘ $K_{1,n}$ is an even vertex analytic mean graph.

Proof:

Let $V=\{u_i\ /1\le i\le n\}\cup\{a,b\}$ be the vertex set and $E=\{au_i/1\le i\le n\}\cup\{bu_i\ /1\le i\le n\}$ be the edge set of $K_{1,n}$ ʘ $K_{1,n}$.

Here $|V(G)|=n+2 \quad and \quad |E(G)|=2n$

Define a function $f:V\to\{1,3,5,...2q-1\}$ by

$f(a)=2$

$f(b)=4$

$f(u_i)=2i+4,\ 1\le i\le n$

Then the edges are labeled as follows

$$f^*(au_i)=\frac{\left|[f(a)]^2-[f(u_i)]^2\right|}{2}$$
$$=\frac{\left|[(2)]^2-[(2i+4)]^2\right|}{2}$$
$$=2i^2+8i+6$$

$$f^*(bu_i)=\frac{\left|[f(b)]^2-[f(u_i)]^2\right|}{2}$$
$$=\frac{\left|[(4)]^2-[(2i+4)]^2\right|}{2}$$
$$=2i^2+8i$$

Then the edge labels are distinct.

Hence, $K_{1,n}$ ʘ $K_{1,n}$ is an even vertex analytic mean graph.

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Example

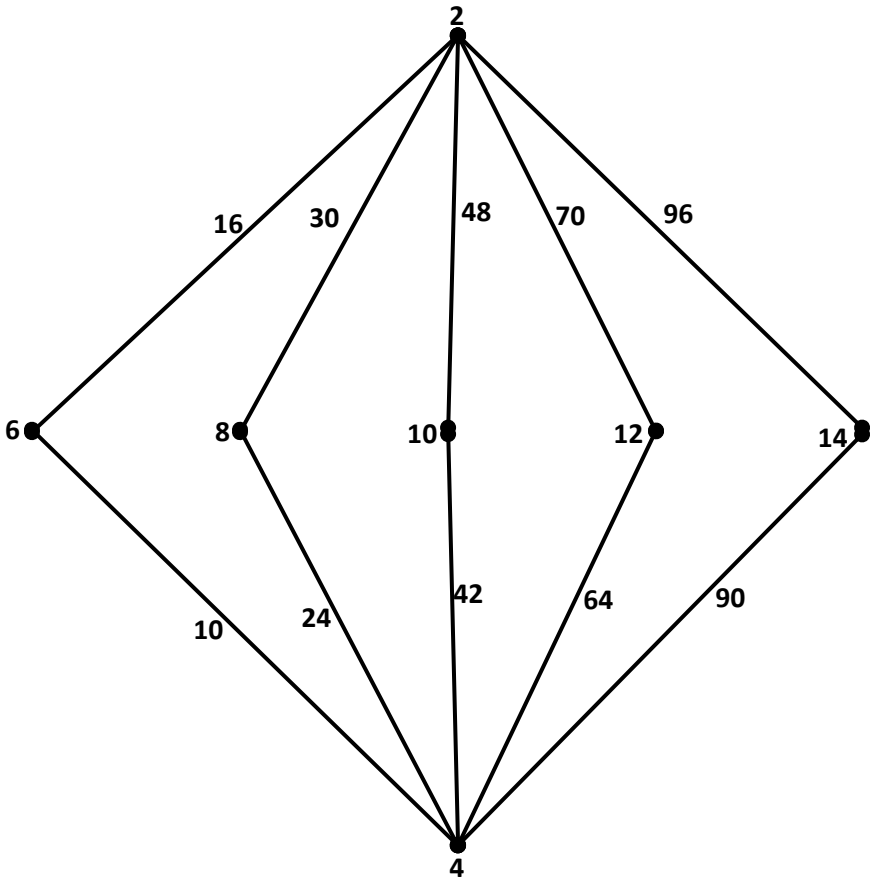

Figure 2. $K_{1,5} \odot K_{1,5}$

Theorem 2.3

Double $Z_n - path\ DZ_n$ is an even vertex analytic mean graph.

Proof:

Let $V = \{u_i\ /1 \le i \le n\} \cup \{v_i\ /1 \le i \le n\}$ be the vertex set and

$E = \{u_i\, u_{i+1} /1 \le i \le n-1\} \cup \{v_i\, v_{i+1} /1 \le i \le n-1\} \cup \{u_i\, v_{i+1} /1 \le i \le n-1\} \cup \{v_i\, u_{i+1} /1 \le i \le n-1\}$
be the edge set of Double $Z_n - path$.

Here $|V(G)| = 2n \quad and \quad |E(G)| = 4n-4$

Define a function $f : V \to \{2,4,6,\ldots 2q\}$ by

$f(u_i) = 2i\ ,\ 1 \le i \le n$

$f(v_i) = 2n + 2i\ ,\ 1 \le i \le n$

Then the edges are labeled as follows

For $1 \le i \le n-1$

$$f^*(u_i u_{i+1}) = \frac{\left|[f(u_i)]^2 - [f(u_{i+1})]^2\right|}{2}$$
$$= \frac{\left|[(2i)]^2 - [(2i+2)]^2\right|}{2}$$
$$= 4i+2$$

For $1 \le i \le n-1$

$$f^*(v_i v_{i+1}) = \frac{\left|[f(v_i)]^2 - [f(v_{i+1})]^2\right|}{2}$$
$$= \frac{\left|[(2n+2i)]^2 - [(2n+2i+2)]^2\right|}{2}$$
$$= 4n+4i+2$$

For $1 \le i \le n-1$

$$f^*(u_i v_{i+1}) = \frac{\left|[f(u_i)]^2 - [f(v_{i+1})]^2\right|}{2}$$
$$= \frac{\left|[(2i)]^2 - [(2n+2i+2)]^2\right|}{2}$$
$$= 2n^2+4n+4ni+4i+2$$

For $1 \le i \le n-1$

$$f^*(v_i u_{i+1}) = \frac{\left|[f(v_i)]^2 - [f(u_{i+1})]^2\right|}{2}$$
$$= \frac{\left|[(2n+2i)]^2 - [(2i+2)]^2\right|}{2}$$
$$= 2n^2+4ni-4i-2$$

Then the edge labels are distinct.

Hence, Double $Z_n - path$ is an even vertex analytic mean graph.

Example

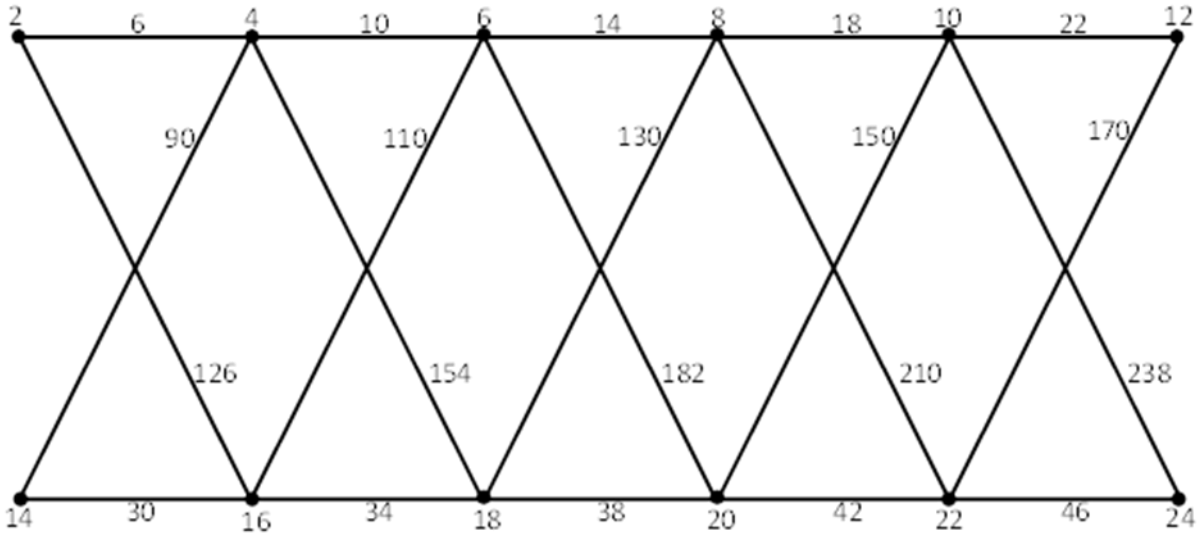

Figure 3. DZ_6 **- graph**

Theorem 2.4

Crown graph $CR_n(G)$ is an even vertex analytic mean graph.

Proof:

Let $V=\{u_i\ /1\le i\le n\}\cup\{v_i\ /1\le i\le n\}$ be the vertex set and

$E=\{u_i u_{i+1}/1\le i\le n-1\}\cup\{u_1 u_n\}\cup\{u_i v_i/1\le i\le n\}$ be the edge set of Crown graph.

Here $|V(G)|=2n \quad and \quad |E(G)|=2n$

Define a function $f:V\to\{2,4,6,\ldots 2q\}$ by

$f(u_i)=4i-2,\ 1\le i\le n$

$f(v_i)=4i\ ,\ 1\le i\le n$

Then the edges are labeled as follows

For $1\le i\le n-1$

$$f^*(u_i u_{i+1})=\frac{\left|[f(u_i)]^2-[f(u_{i+1})]^2\right|}{2}$$

$$=\frac{\left|[(4i-2)]^2-[(4i+2)]^2\right|}{2}$$

$$=16i$$

$$f^*(u_1 u_n) = \frac{\left|\left[f(u_1)\right]^2 - \left[f(u_n)\right]^2\right|}{2}$$
$$= \frac{\left|\left[(2)\right]^2 - \left[(4n-2)\right]^2\right|}{2}$$
$$= 8n^2 - 8n$$

For $1 \le i \le n$

$$f^*(u_i v_i) = \frac{\left|\left[f(u_i)\right]^2 - \left[f(v_i)\right]^2\right|}{2}$$
$$= \frac{\left|\left[(4i-2)\right]^2 - \left[(4i)\right]^2\right|}{2}$$
$$= 8i - 2$$

Then the edge labels are distinct.

Hence, Crown graph $CR_n(G)$ is an even vertex analytic mean graph.

Example

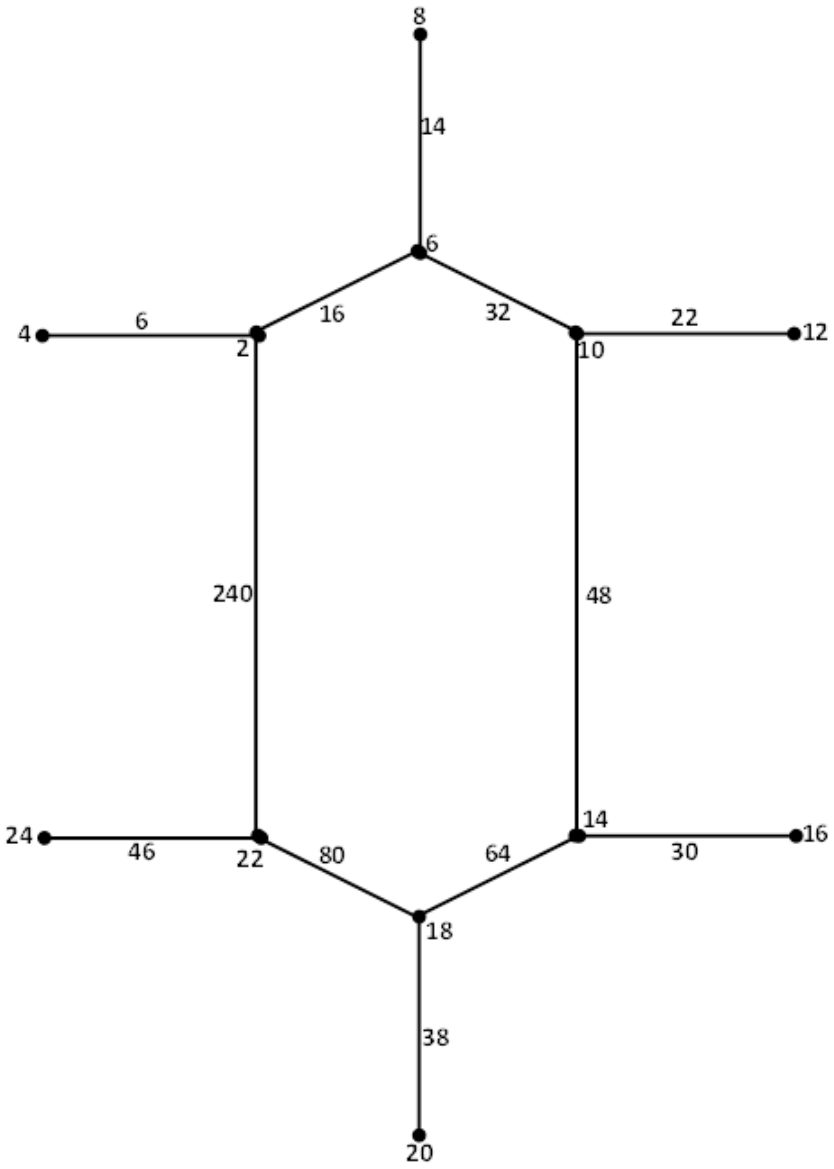

Figure 4.Crown graph

Theorem 2.5

Super Ladder $SL(G)$ is an even vertex analytic mean graph.

Proof:

Let $V=\{u_i\ /1\le i\le n\}\cup\{v_i\ /1\le i\le n\}\cup\{x,y,z,w\}$ be the vertex set of Super Ladder and

$E=\bigcup_{i=1}^{3}E_i$ *where* $E_1=\{u_iu_{i+1}\ ,\ v_iv_{i+1}/1\le i\le n-1\}$

$$E_2=\{u_iv_i\ /1\le i\le n\}$$
$$E_3=\{u_1w,v_1x,u_ny,v_nz\}$$

Here $|V(G)|=2n+4$ *and* $|E(G)|=3n+2$

Define a function $f:V\to\{2,4,6,\ldots 2q\}$ by

$f(u_i)=2i\ ,\ 1\le i\le n$
$f(v_i)=2n+2i\ ,\ 1\le i\le n$
$f(w)=4n+2$
$f(x)=4n+4$
$f(y)=4n+6$
$f(z)=4n+8$

Then the edges are labeled as follows

For $1\le i\le n-1$

$$f^*(u_i\,u_{i+1})=\frac{\left|[f(u_i)]^2-[f(u_{i+1})]^2\right|}{2}$$
$$=\frac{\left|[(2i)]^2-[(2i+2)]^2\right|}{2}$$
$$=4i+2$$

For $1\le i\le n-1$

$$f^*(v_i\,v_{i+1})=\frac{\left|[f(v_i)]^2-[f(v_{i+1})]^2\right|}{2}$$
$$=\frac{\left|[(2n+2i)]^2-[(2n+2i+2)]^2\right|}{2}$$
$$=4n+4i+2$$

For $1\le i\le n$

$$f^*(u_i\,v_i)=\frac{\left|[f(u_i)]^2-[f(v_i)]^2\right|}{2}$$
$$=\frac{\left|[(2i^2)]^2-[(2n+2i)]^2\right|}{2}$$
$$=2n^2+4ni$$

$$f^*(u_1 w)=\frac{\left|[f(u_1)]^2-[f(w)]^2\right|}{2}$$
$$=\frac{\left|[(2)]^2-[(4n+2)]^2\right|}{2}$$
$$=8n^2+8n$$

$$f^*(v_1 x)=\frac{\left|[f(v_1)]^2-[f(x)]^2\right|}{2}$$
$$=\frac{\left|[(2n+2)]^2-[(4n+4)]^2\right|}{2}$$
$$=6n^2+12n+6$$

$$f^*(u_n y)=\frac{\left|[f(u_n)]^2-[f(y)]^2\right|}{2}$$
$$=\frac{\left|[(2n)]^2-[(4n+6)]^2\right|}{2}$$
$$=6n^2+24n+18$$

$$f^*(v_n z)=\frac{\left|[f(v_n)]^2-[f(z)]^2\right|}{2}$$
$$=\frac{\left|[(4n)]^2-[(4n+8)]^2\right|}{2}$$
$$=32n+32$$

Then the edge labels are distinct.

Hence, Super Ladder $SL(G)$ is an even vertex analytic mean graph.

Example

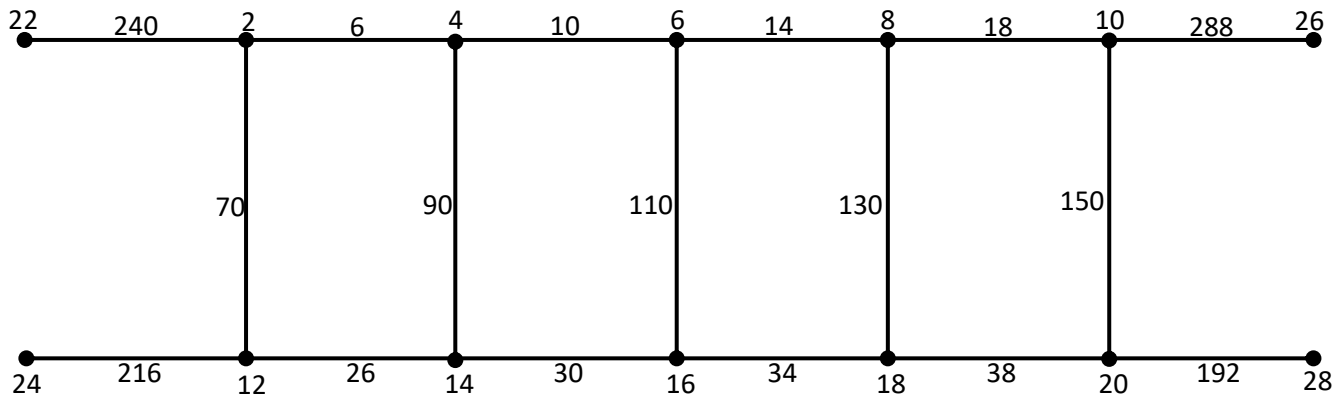

Figure 5.Super Ladder Graph

Theorem2.6

Friendship design graph $DS^r(G)$ is an even vertex analytic mean graph.

Proof:

Let $V=\left\{u_i^{\,k}\ /1\le i\le n\ ,1\le k\le r\right\}\cup\left\{a\right\}\cup\left\{w_i\ /1\le i\le r\right\}$ be the vertex set and

$E=\left\{u_i^{\,k}\,u_{i+1}^{\,k}\ /\ 1\le i\le n-1,1\le k\le r\right\}\cup\left\{u_1^{\,k}a,\ u_n^{\,k}a\ /\ 1\le k\le r\right\}\cup\left\{aw_i\ /\ 1\le i\le n\right\}$ be the edge set of Friendship design graph $DS^r(G)$.

Here $|V(G)|=r(n+1)+1\ \ and\ \ |E(G)|=5r$

Define a function $f:V\to\{2,4,6,\ldots 2q\}$ by

$f(u_i^{\,k})=2n(k-1)+2i\ /\ 1\le i\le n\ ,\ 1\le k\le r$

$f(w_i)=2nr+2i\ ,\ 1\le i\le r$

$f(a)=2nr+2r+2$

Then the edges are labeled as follows

For $1\le i\le n-1,1\le k\le r$

$$f^*(u_i^{\,k}\,u_{i+1}^{\,k})=\frac{\left|\left[f(u_i^{\,k})\right]^2-\left[f(u_{i+1}^{\,k})\right]^2\right|}{2}$$

$$=\frac{\left|\left[2n(k-1)+2i\ \right]^2-\left[2n(k-1)+(2i+2)\right]^2\right|}{2}$$

$$=4nk-4n+4i+2$$

$$f^*(u_1^{\,k}\,a)=\frac{\left|\left[f(u_1^{\,k})\right]^2-\left[f(a)\right]^2\right|}{2}$$

$$=\frac{\left|\left[2n(k-1)+2\ \right]^2-\left[2nr+2r+2)\right]^2\right|}{2}$$

$$=4n^2k+4n-4nk+2n^2r^2+2r^2+4nr^2+4r+4nr-2n^2k^2-2n^2$$

$$f^*(u_n^{\,k}\,a)=\frac{\left|\left[f(u_n^{\,k})\right]^2-\left[f(a)\right]^2\right|}{2}$$

$$=\frac{\left|\left[2n(k-1)+2n\right]^2-\left[2nr+2r+2\right]^2\right|}{2}$$

$$=2n^2r^2+2r^2+4nr^2+4r+4nr-2n^2k^2+2$$

$$f^*(w_i\, a) = \frac{\left|\left[f(w_i)\right]^2 - \left[f(a)\right]^2\right|}{2}$$

$$= \frac{\left|\left[2nr + 2i\right]^2 - \left[2nr + 2r + 2\right]^2\right|}{2}$$

$$= 2n^2r^2 + 2r^2 + 4nr^2 + 4r + 4nr - 2n^2r^2 - 2i^2 - 4nri + 2$$

Then the edge labels are distinct.

Hence, the Friendship design graph $DS^r(G)$ is an even vertex analytic mean graph.

Example

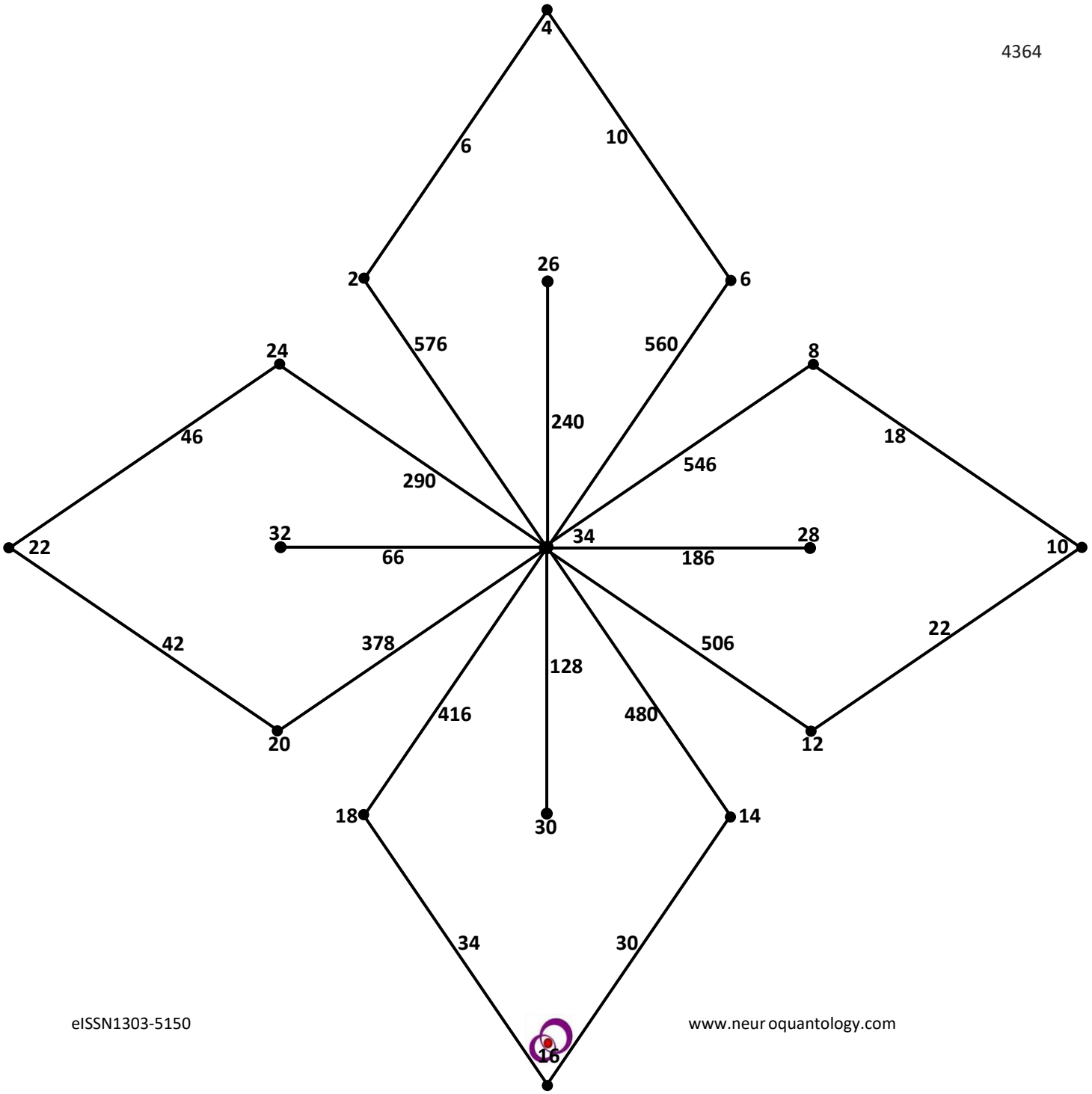

Figure 6.Friendship Design Graph $DS^4(G)$

REFERENCES

1. Gallian, A 2022, 'A Dynamic Survey of Graph Labeling', The Electronic Journal of Combinatorics, # DS6.
2. Harary, F 1972, Graph Theory, Addison-Wesley, Reading, Mass.
3. Kannan, M, Vikramaprasad, R &Gopi, R 2017 'Even Vertex Odd Mean Labeling of Some Graphs', Global journal of pure and applied mathematics , vol.13, no. 3, pp. 1019-1033.
4. Somasundaram, S &Ponraj, R 2003, 'Mean labelings of graphs',National Academy Science Letter, 26,210-213.
5. Tharmaraj, T &Sarasija,PB 2014, 'Analytic Mean Graphs', Int. Journal. of Math. Analysis, vol 8, no.12, pp.595-609.
6. Vasuki, R, Nagarajan, A &Arockiaraj, S 2013 'Even vertex Odd Mean Labeling of Graphs, SutJ.Math, 49(2), pp. 79-92.