A New Method for Balancing Chemical Reaction Equations

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Abstract

The main objective of this study is to propose a new method for balancing chemical reaction equations using knowledge of the inverse matrix obtained by performing elementary row operations. The proposed new method has also been applied in the example. From this method, it is shown that balancing chemical reaction equations using the new method gives the same solution as the Gauss-Jordan elimination.

Keywords Balancing Chemical Reaction, Elementary Row Operations, Gauss-Jordan Elimination, Inverse Matrix, System of Linear Equations.
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1. Introduction

Mathematics is important and can be applied to human daily life all the time, making humansthink logically, systematically, methodically, and with plans. In addition, nearby events that are directly or indirectly related to us can all be linked to mathematics. Because mathematics is a subject that can be integrated with many other fields of study as well. And we can apply mathematics in real life.

Chemistry is a branch of basic science that studies the structure and composition of matter. Changes and mechanisms of reactions that cause changes in matter. One main topic is chemical reaction equations. Everything in the universe is the result of infinite chemical reactions among different arrangements of elements. Chemical reactions can be described in simple terms what happens when certain reactants which may be components of a compound.Its molecules mix with other reactants or decompose to form products, which may be compounds with different molecules or elements. These reactions are often represented as chemical equations with the reactants on the left and the products on the right, and an arrow indicating the direction of the reaction. One common example is $2H_2 + O_2 \rightarrow 2H_2O$, which combines hydrogen gas (H_2) and oxygen gas (O_2) to create water (H_2O), which can be found all over the world and makes up 71% of the Earth's surface, but it is not a reaction. All of them have a one-to-one ratio like the formation of carbonic acid ($CO_2 + H_2O \rightarrow H_2CO_3$). Some reactions may use more reactants to produce product compounds. But some reactions may require only a small amount of reactants to produce the desired final product. From the past until the present, it can be seen that there are many scientists who study matter in order to use it for various purposes. There are many ways to balance chemical equations. The algebraic method is one of the methods that can be used to balance chemical reaction equations as well.

In the context of balance chemical reaction equations, many similar studies can be found in 2015, Gabriel and Onwukadescribed a procedure employing Gaussian elimination method in matrix algebra to balance chemical equations from easy to relatively complex chemical reactions. They found that 2 atom of sodium, 6 atoms of oxygen, 4 atoms of Hydrogen, and 1 atom of sulfur each on both the reactants and products made the chemical equation balance. This result satisfied the law of conservation of matter. There was no contradiction to the existing way of balancing chemical equations.

In 2018, Kafi and Abdillah presented a small sample of the wide variety of real-world problems regarding their study of linear systems. They showed that the problem in balancing chemical reaction can be described by homogeneous linear systems. The solution of the systems was obtained by performing elementary row operations. They also presented a computational calculation to show that mathematical software such as MATLAB can be used to simplify completion of the systems, instead of manually using row operations.

A formal and systematic method for balancing chemical reaction equations was also presented by Hamid (2019). A chemical reaction which possesses atoms with fractional oxidation numbers that have unique coefficients was studied. The chemical equations were balanced by representing the chemical reaction into systems of linear equations. Gauss elimination was used to solve the mathematical problem.

Later, in 2020, Johar discussed the equalization of chemical reactions using a system of linear equations with the Gaussian and Gauss-Jordan elimination. He found that it was possible to handle any chemical equation with given reactants and products.

In addition, Pandichelvi and Saranya (2022) applied the analysis of how the system of linear Diophantine equations in balancing chemical equations assimilated by the reactions of various chemical combinations. Their products were scrutinized.

Algebra is a branch of science that addresses mathematical problems. One of them is matrices. Applications of matrices can be used to solve various types of problems, such as solving systems of linear equations. Both systems of linear equations for real numbers and systems of linear equations for complex numbers, however, are the systems of linear equations that will be discussed in this study. System of linear equations of real numbers. Several commonly used methods for solving systems of linear equations are: Gaussian and Gauss-Jordan elimination methods.

Therefore, in this research, we present a new method for balancing chemical reaction equations usingknowledge about systems of linear equations and the inverse matrix. The proposed new method has also been applied in the example. We also examine the solution obtained using the Gauss-Jordan elimination.

2. Methodology

If there are a finite number of 2 or more linear equations with$x_1, x_2, \ldots, x_n$ variables, these linear equations are called that a system of linear equations and $s_1, s_2, \ldots, s_n$ is called the solution of the system if$x_1 = s_1, x_2 = s_2, \ldots, x_n = s_n$ is the solution of every equation in the system.

Consider a system of linear equations which consists of m equations and n variables$(x_1, x_2, \ldots, x_n)$ can be written as follows:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m \end{aligned} \qquad (1)$$

where a_{ij} and b_i $(1 \leq i \leq m, 1 \leq j \leq n)$ are real number constants. The form of the system of linear equations in equation (1) can be written as

$$AX = B \qquad (2)$$

where

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}_{m\times n}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{n\times 1}, \quad B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}_{m\times 1}. \qquad (3)$$

Matrix A is the coefficient matrix of the system of linear equations

and a_{ij} is the element in the row i and column j of matrix A.

Let $$[A \mid B] = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} & b_m \end{bmatrix}_{m\times(n+1)}.$$

$[A \mid B]$ is called the augmented matrix of the system.

Two systems of linear equations are equivalent if one system of equations is derived from the other system by the following actions.

E1. Switch any two rows, that is, switch between ith row and jth row $(R_i \leftrightarrow R_j)$.

E2. Multiply every element in ith row by a constant$c$ which is not zero $(R_i \to cR_i)$.

E3. Replace$i$th row with the sum of that row and a multipleof a non-zero constant$c$to$j$th row $(R_i \to R_i + cR_j; i \neq j)$.

We call the operations in E1, E2, and E3 that the elementary row operations. And the rank value of a matrix$A$, denoted byrank$A$, is the number of every non-zero element row of the echelon matrix.

Consider a system of linear equations $AX = B$ where$A$ is a $m \times n$ dimensional matrix. One of the following statements must be true (Hogben (2007)).

1. If rank $[A \mid B] =$ rank $A =$ the number of variables in the system , then the system of linear equations has aunique solution.

2. If rank $[A \mid B] =$ rank $A <$ the number of variables in the system, then the system of linear equations has an infinitely many solutions.

3. If rank $[A \mid B] >$ rank A , then the system of linear equations has no solution.

We perform elementary row operations on the augmented matrix to obtain a row echelon matrix. Then use reverse substitution to get a system of equations. It will be a system of equations that is easy to find the solution. This method is called Gaussian elimination. From the Gaussian elimination,we will perform elementary row operations on the additive matrix until we get a reduced row echelon matrix which can read the solution value of the system of equations immediately for convenience without having to substitute the values back. Sometimes the solution of some variables may depend on other variables. Therefore, values must be assigned to some variables. This method is called Gauss-Jordan elimination.

In this research, we propose a new method for balancing chemical reaction equations using the inverse matrix obtained by performing elementary row operations. The method for finding A^{-1} of an n-dimensional square matrix$A$ can be summarized as follows. Starting from the matrix $[A|I_n]$ and performing elementary row operations on the matrix $[A|I_n]$ until $[A|I_n]$ changes to $[I_n|C]$, that is $[A|I_n] \sim [I_n|C]$ and will get $C = A^{-1}$. If A is an $n \times n$dimensional matrix from which the inverse matrix can be found, then for each matrix B which has dimension $n \times 1$, the system of equations $AX = B$ will have exactly one solution which is $X = A^{-1}B$.

3. Results

3.1 New method for balancing chemical reaction equations using inverse matrix.

Example: Balance the following chemical reaction equation
$KI + MnO_2 + H_2SO_4 \rightarrow I_2 + KHSO_4 + MnSO_4 + H_2O$.
Solution: This chemical reaction consists of 6 elements: Potassium (K), Iodine (I), Manganese (Mn), Sulfur (S), Hydrogen (H), and Oxygen (O). A balanced equation can be obtained by determining the values ofunknown coefficient$(x_1, x_2, x_3, x_4, x_5, x_6, x_7)$. Therefore
$x_1KI + x_2MnO_2 + x_3H_2SO_4 \rightarrow x_4I_2 + x_5KHSO_4 + x_6MnSO_4 + x_7H_2O$.
We will get a system of linear equations consisting of 6 equations and 7 variables as follows

Potassium(K): $x_1 = x_5$
Iodine(I): $x_1 = 2x_4$
Manganese(Mn): $x_2 = x_6$
Sulfur(S): $x_3 = x_5 + x_6$
Hydrogen(H):$2x_3 = x_5 + 2x_7$
Oxygen(O):$2x_2 + 4x_3 = 4x_5 + 4x_6 + x_7$.

For this new proposed method, we will find the solution by arranging one of the variables in terms of the remaining variables. In this example, we choose to write the variable x_7 in terms of the remaining 6 variables $(x_1, x_2, x_3, x_4, x_5, x_6)$. So we can rewrite the system of linear equations in the following form:

$x_1 + 0x_2 + 0x_3 - 0x_4 - x_5 - 0x_6 = 0x_7$
$x_1 + 0x_2 + 0x_3 - 2x_4 - 0x_5 - 0x_6 = 0x_7$
$0x_1 + x_2 + 0x_3 - 0x_4 - 0x_5 - x_6 = 0x_7$
$0x_1 + 0x_2 + x_3 - 0x_4 - x_5 - x_6 = 0x_7$
$0x_1 + 0x_2 + 2x_3 - 0x_4 - x_5 - 0x_6 = 2x_7$
$0x_1 + 2x_2 + 4x_3 - 0x_4 - 4x_5 - 4x_6 = x_7$

Let

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & -2 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & 2 & 0 & -1 & 0 \\ 0 & 2 & 4 & 0 & -4 & -4 \end{bmatrix} \text{ and } B = x_7 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 2 \\ 1 \end{bmatrix}.$$

Then

$$\left[A|I\right]=\begin{bmatrix}1&0&0&0&-1&0&1&0&0&0&0&0\\1&0&0&-2&0&0&0&1&0&0&0&0\\0&1&0&0&0&-1&0&0&1&0&0&0\\0&0&1&0&-1&-1&0&0&0&1&0&0\\0&0&2&0&-1&0&0&0&0&0&1&0\\0&2&4&0&-4&-4&0&0&0&0&0&1\end{bmatrix}$$

$$\underset{\sim}{R_2\to R_2-R_1}\begin{bmatrix}1&0&0&0&-1&0&1&0&0&0&0&0\\0&0&0&-2&1&0&-1&1&0&0&0&0\\0&1&0&0&0&-1&0&0&1&0&0&0\\0&0&1&0&-1&-1&0&0&0&1&0&0\\0&0&2&0&-1&0&0&0&0&0&1&0\\0&2&4&0&-4&-4&0&0&0&0&0&1\end{bmatrix}$$

$$\underset{\sim}{R_2\leftrightarrow R_3}\begin{bmatrix}1&0&0&0&-1&0&1&0&0&0&0&0\\0&1&0&0&0&-1&0&0&1&0&0&0\\0&0&0&-2&1&0&-1&1&0&0&0&0\\0&0&1&0&-1&-1&0&0&0&1&0&0\\0&0&2&0&-1&0&0&0&0&0&1&0\\0&2&4&0&-4&-4&0&0&0&0&0&1\end{bmatrix}$$

$$\underset{\sim}{R_6\to R_6-2R_2}\begin{bmatrix}1&0&0&0&-1&0&1&0&0&0&0&0\\0&1&0&0&0&-1&0&0&1&0&0&0\\0&0&0&-2&1&0&-1&1&0&0&0&0\\0&0&1&0&-1&-1&0&0&0&1&0&0\\0&0&2&0&-1&0&0&0&0&0&1&0\\0&0&4&0&-4&-2&0&0&-2&0&0&1\end{bmatrix}$$

$$\underset{\sim}{R_3\leftrightarrow R_4}\begin{bmatrix}1&0&0&0&-1&0&1&0&0&0&0&0\\0&1&0&0&0&-1&0&0&1&0&0&0\\0&0&1&0&-1&-1&0&0&0&1&0&0\\0&0&0&-2&1&0&-1&1&0&0&0&0\\0&0&2&0&-1&0&0&0&0&0&1&0\\0&0&4&0&-4&-2&0&0&-2&0&0&1\end{bmatrix}$$

$$\begin{array}{c} R_5 \to R_5 - 2R_3 \\ \sim \\ R_6 \to R_6 - 4R_3 \end{array} \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -2 & 1 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & -2 & -4 & 0 & 1 \end{bmatrix}$$

$$\begin{array}{c} R_4 \to -\frac{R_4}{2} \\ \sim \end{array} \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{2} & 0 & \frac{1}{2} & -\frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & -2 & -4 & 0 & 1 \end{bmatrix}$$

$$\begin{array}{c} R_1 \to R_1 + R_5 \\ R_3 \to R_3 + R_5 \\ \sim \\ R_4 \to R_4 + \frac{1}{2}R_5 \end{array} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 2 & 1 & 0 & 0 & -2 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & \frac{1}{2} & -\frac{1}{2} & 0 & -1 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & -2 & -4 & 0 & 1 \end{bmatrix}$$

$$\begin{array}{c} R_6 \to \frac{R_6}{2} \\ \sim \end{array} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 2 & 1 & 0 & 0 & -2 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & \frac{1}{2} & -\frac{1}{2} & 0 & -1 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & -2 & 0 & \frac{1}{2} \end{bmatrix}$$

$$\begin{matrix} R_1 \to R_1 - 2R_6 \\ R_2 \to R_2 + R_6 \\ R_3 \to R_3 - R_6 \\ \sim \\ R_4 \to R_4 - R_6 \\ R_5 \to R_5 - 2R_6 \end{matrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 2 & 2 & 1 & -1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & -\frac{1}{2} \\ 0 & 0 & 0 & 1 & 0 & 0 & \frac{1}{2} & -\frac{1}{2} & 1 & 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 2 & 2 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & -2 & 0 & \frac{1}{2} \end{bmatrix}.$$

Thus we get $A^{-1} = \begin{bmatrix} 1 & 0 & 2 & 2 & 1 & -1 \\ 0 & 0 & 0 & -2 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 1 & 1 & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & 1 & 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 2 & 2 & 1 & -1 \\ 0 & 0 & -1 & -2 & 0 & \frac{1}{2} \end{bmatrix}.$

And $A^{-1} \cdot B = x_7 \begin{bmatrix} 1 & 0 & 2 & 2 & 1 & -1 \\ 0 & 0 & 0 & -2 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 1 & 1 & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & 1 & 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 2 & 2 & 1 & -1 \\ 0 & 0 & -1 & -2 & 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 2 \\ 1 \end{bmatrix}$

$$= x_7 \begin{bmatrix} 1 \\ \frac{1}{2} \\ \frac{3}{2} \\ \frac{1}{2} \\ 1 \\ \frac{1}{2} \end{bmatrix}.$$

If we let x_7 is the maximum value of the denominator of each answer above, then

$x_7 = max$ {1,2,2,2,1,2}=2.

Let $$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = x_7 \begin{bmatrix} 1 \\ \frac{1}{2} \\ \frac{3}{2} \\ \frac{1}{2} \\ 1 \\ \frac{1}{2} \end{bmatrix} = 2 \begin{bmatrix} 1 \\ \frac{1}{2} \\ \frac{3}{2} \\ \frac{1}{2} \\ 1 \\ \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \\ 1 \\ 2 \\ 1 \end{bmatrix}.$$

Thus, we get the solution for this system of linear equations:

$$x_1 = 2, x_2 = 1, x_3 = 3, x_4 = 1, x_5 = 2, x_6 = 1\text{และ} x_7 = 2.$$

Finally, we will obtain a balanced chemical reaction equation

$$2KI + MnO_2 + 3H_2SO_4 \rightarrow I_2 + 2KHSO_4 + MnSO_4 + 2H_2O. \qquad \square$$

Note that this new proposed method is used to solve the system of linear equations for n equations and $n + 1$ variables.

3.2 Gauss-Jordan Elimination.

Next, the solution for the system of linear equations can be found using the Gauss-Jordan elimination method to check the above answer as well.

From the example above, we have the system of linear equations

$$
\begin{aligned}
x_1 \qquad\qquad - x_5 \qquad\qquad &= 0 \\
x_1 \qquad - 2x_4 \qquad\qquad\qquad &= 0 \\
x_2 \qquad\qquad\qquad - x_6 \qquad &= 0 \\
x_3 \qquad - x_5 - x_6 \qquad &= 0 \\
2x_3 \qquad - x_5 \qquad - 2x_7 &= 0 \\
2x_2 + 4x_3 \qquad - 4x_5 - 4x_6 - x_7 &= 0
\end{aligned}
$$

That is

$$
\left[A\middle|B\right]=\begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 2 & 0 & -1 & 0 & -2 & 0 \\ 0 & 2 & 4 & 0 & -4 & -4 & -1 & 0 \end{bmatrix}
$$

$$
\underset{\sim}{R_2 \to R_2 - R_1}\begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 2 & 0 & -1 & 0 & -2 & 0 \\ 0 & 2 & 4 & 0 & -4 & -4 & -1 & 0 \end{bmatrix}
$$

$$
\underset{\sim}{R_2 \leftrightarrow R_3}\begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 2 & 0 & -1 & 0 & -2 & 0 \\ 0 & 2 & 4 & 0 & -4 & -4 & -1 & 0 \end{bmatrix}
$$

$$\underset{\sim}{R_6 \to R_6 - 2R_2} \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 2 & 0 & -1 & 0 & -2 & 0 \\ 0 & 0 & 4 & 0 & -4 & -2 & -1 & 0 \end{bmatrix}$$

$$\underset{\sim}{R_3 \leftrightarrow R_4} \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & -2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & -1 & 0 & -2 & 0 \\ 0 & 0 & 4 & 0 & -4 & -2 & -1 & 0 \end{bmatrix}$$

$$\begin{matrix} R_5 \to R_5 - 2R_3 \\ \sim \\ R_6 \to R_6 - 4R_3 \end{matrix} \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & -2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & -1 & 0 \end{bmatrix}$$

$$\underset{\sim}{R_4 \to -\frac{R_4}{2}} \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & -1 & 0 \end{bmatrix}$$

$$\begin{matrix} R_1 \to R_1 + R_5 \\ R_3 \to R_3 + R_5 \\ \sim \\ R_4 \to R_4 + \frac{1}{2}R_5 \end{matrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 2 & -2 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & -1 & 0 \end{bmatrix}$$

$$R_6 \to \frac{R_6}{2} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 2 & -2 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -\frac{1}{2} & 0 \end{bmatrix}$$

$$\begin{matrix} R_1 \to R_1 - 2R_6 \\ R_2 \to R_2 + R_6 \\ R_3 \to R_3 - R_6 \\ \sim \\ R_4 \to R_4 - R_6 \\ R_5 \to R_5 - 2R_6 \end{matrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -\frac{3}{2} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -\frac{1}{2} & 0 \end{bmatrix}$$

From the final matrix, which is in the form of a reduced row echelon matrix, we get rank $[A|B]=6$, rank $A=6$ and the number of variables(n)in the system is 7 , that is, rank $[A|B]=$ rank $A<n$. Therefore, the system of linear equations has infinitely many solutions.And it can be seen that

$$x_1 - x_7 = 0$$

$$x_2 - \frac{1}{2}x_7 = 0$$

$$x_3 - \frac{3}{2}x_7 = 0$$

$$x_4 - \frac{1}{2}x_7 = 0$$

$$x_5 - x_7 = 0$$

$$x_6 - \frac{1}{2}x_7 = 0$$

or

$$x_1 = x_7$$

$$x_2 = \frac{1}{2}x_7$$

$$x_3 = \frac{3}{2}x_7$$

$$x_4 = \frac{1}{2}x_7$$

$$x_5 = x_7$$

$$x_6 = \frac{1}{2}x_7$$

where x_7 can be any real number constant. This causes the desired chemical reaction to have a positive integer number of elements and be in at least fraction form. Therefore, when setting x_7 = 2, we will get

$$x_1 = 2$$

$$x_2 = 1$$

$$x_3 = 3$$

$$x_4 = 1$$

$$x_5 = 2$$

$$x_6 = 1$$

Hence, from the problem, we substitute the values $x_1 = 2,\ x_2 = 1,\ x_3 = 3,\ x_4 = 1,\ x_5 = 2,\ x_6 = 1$ and $x_7 = 2$, we will get a balanced chemical reaction equation. That is

$$2KI + MnO_2 + 3H_2SO_4 \rightarrow I_2 + 2KHSO_4 + MnSO_4 + 2H_2O. \qquad \square$$

4. Conclusion

Linear algebra is a branch of mathematics that can be applied to solve problems in many fields, such as economics, engineering, computer, social, statistics, physics, biology, chemistry, etc., which need to calculate and find solutions by solving systems of linear equations. In this research, we present an application of linear algebra in mathematics and chemistry using the matrices involving variables for the problem of balancing chemical reaction equations. Using matrices helps in writing large systems of linear equations in a concise, easy-to-understand way.

Study and finding solutions of systems of linear equations are considered in the study of linear algebra. Therefore, here we have integrated creativity, analytical and synthetic thinking. We propose a new method for balancing chemical reaction equations using the inverse matrix. The new method is presented with exampletothe systemof $n$linear equations and $n+1$ variables for solving thechemical equation balancing problem. The solution is also examined using the Gauss-Jordan elimination. The result shows that the new proposed methodgives the same solution as the Gauss-Jordan elimination. In our example, we obtain

$2KI + MnO_2 + 3H_2SO_4 \rightarrow I_2 + 2KHSO_4 + MnSO_4 + 2H_2O$,

that is, 2 atom of potassium(K), 2atom of iodine(I), 1atom of manganese(Mn), 3atom of sulfur(S), 6atom of hydrogen(H), and 14atom ofoxygen(O)each on both the reactants and products makes the chemical equation balance.

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