FIXED POINT RESULT UNDER CIRIC TYPE MAPPING WITH SOME APPLICATION

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Abstract

The authors prove the existence and uniqueness of fixed points of mappings satisfying Ciric generalized and extended Banach contraction in the frame of quasi-partial metric spaces. The authors apply a solution for an integral equation as an application for analyzing the results. The results extend to generalize and include different types of known results. The results of this paper are theoretical and analytical in nature. Find and the existence and uniqueness of a fixed point for a quasi-contraction in partial order space and single-valued Ciric type contraction mapping to u- distance are proved. Attempt to design innovative fixed-point solutions in this research using the frame of quasi-partial metric space, and to extend these results by involving single-valued Ciric-type contraction mapping to u- distance.

Keywords: Fixed point theory, quasi-partial metric space, Banach contraction principle, Ciri'c type single-valued mapping, u-distance.

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1. INTRODUCTION

In fixed point theory, the Banach contraction principle is the remarkable result that was introduced by Banach [4] in 1922. Over the years, this theory was generalized by various researchers on different metric spaces. And they used the contraction principle for references to their theorems. Quasi-partial metric space was introduced by Karapinar *et. al.* in 2012. He proved the existence of a fixed point for self-mapping in quasi-partial metric space. By a generalization of the Banach contraction principle on a complete metric space, Ciric [5] (2011) proved the fixed-point theorem.

In 2012, Agarwal et al. [2] and Aydi et al. (2011) [1] obtained the fixed-point theorems and applications to nonlinear integral equations, Coupled fixed point results in cone metric spaces for w-compatible mappings. Akewe, H. and Olaoluwa, H., [3] (2020) established multistep-type construction of fixed points for multivalued-quasi-contractive maps in modular function spaces. Caristi [6] (1976) satisfied inwardness conditions and proved fixed point theorems for mappings. Ciric et al. [7, 8] (2010, 2011) introduced the Banach fixed point principle for nonlinear contractions in probabilistic metric spaces and generalized contractive mappings in ordered metric spaces. Cvetkovic c et al. [9](2011) generalized common fixed-point theorems for four mappings on cone metric type. Deepankar Dey et al. [10] (2021) obtained uniformity on generalized topological spaces.

In 2012, Hussain, N. et al. [11] established Suzuki-type fixed point results in metric type spaces. Jungck et al. [12, 13] (2006, 2009) presented fixed point theorems for occasionally weakly and weakly compatible mappings. R. Kannan [14] (1969) proved some results on fixed points. Okeke and Francis [15, 16] (2020, 2021) obtained some fixed-point theorems for a general class of mappings and Geraghty-type mappings applied to solving nonlinear Volterra-Fredholm integral equations in modular G-metric spaces. Parvaneh et al. [17] (2013), as an application presented to a system of integral equations. In 2001, T. Suzuki [18] was generalized distance and existence theorems in complete metric spaces. Ume S. [19] (2008) established extensions of minimization theorems and fixed point theorems on a quasi-metric space, he also proved existence theorems for generalized distance on complete metric spaces [20] (2010). Again in (2011) and (2013), he obtained fixed-point theorems for nonlinear contractions in Menger spaces [21] as well as Common fixed-point theorems for nonlinear contractions in a Menger space [22]. Again, Ume S. [23] (2015) proved fixed point theorems for ciric type mapping and application to integral equations.

Let $\boldsymbol{Y} : S \to S$ be a quasi-contractive mapping and $\boldsymbol{Y}$ be a complete metric space. Then $\exists\, b \in [0, 1]$ in such a way that, every $s^1, t^1 \in S$

$d(\boldsymbol{Y} s^1, \boldsymbol{Y} t^1) \leq b.max\, \{d(s^1, t^1), d(s^1, \boldsymbol{Y} s^1), d(t^1, \boldsymbol{Y} t^1)\}$.

Which holds $w \in S$ is a unique fixed point of $\boldsymbol{Y}$.

$lim_{m+1 \to \infty} \boldsymbol{Y}^{m+1} s^1 = w\ \forall s^1 \in S$.

$d(\boldsymbol{Y}^{m+1} s^1, w) \leq \left[\frac{b^{m+1}}{1-b}\right] d(s^1, \boldsymbol{Y} s^1)$ for $s^1 \in S$.

2. PRELIMINARIES

Now, we take a look at a few lemmas and definitions that are pertinent to our primary findings. We denote q_p= quasi partial metric space, N= natural quantity, R and R^+= all real and positive quantities.

Definition2.1: [19]

A function $q_p: S \times S \to R^+$ satisfying the condition

(*i*) $0 \leq q_p(s^1, s^1) = q_p(t^1, t^1) = q_p(s^1, t^1) \Longleftrightarrow s^1 = t^1$

(*ii*) $q_p(s^1, s^1) \leq q_p(t^1, s^1)$

(*iii*) $q_p(s^1, s^1) \leq q_p(s^1, t^1)$

(*iv*) $q_p(s^1, u) + q_p(t^1, t^1) \leq q_p(s^1, t^1) + q_p(t^1, u)$ for all $s^1, t^1, u \in V$

Then (S, q_p) known as quasi-partial metric space (QPMS) in such a way that S is a non-void set

Definition2.2: [23]

Let (S, d), p^* are metric space and u-distance on S. Thus, a sequence $\{s_m\} \in S$ known as p^* Cauchy if $\exists$ a function $\phi : S \times S \times R^+ \times R^+ \to R^+$ which satisfies all the conditions of u-distance except first, and there exist asequence $\{\mathfrak{z}_m\} \in S$ such that

$lim_{m\to\infty} sup\{\phi(ӡ_m, ӡ_m, p^*(ӡ_m, s_n^1), p^*(ӡ_m, s_n^1)) : n \geq m\} = 0$ or

$lim_{m\to\infty} sup\{\phi(ӡ_m, ӡ_m, p^*(s_n^1, ӡ_m), p^*(s_n^1, ӡ_m)) : n \geq m\} = 0.$

Lemma 2.3: [23]

Suppose a quasi-partial metric space (S, q_p) and a u-distance on S are on S be p^*. Let

$\{s_n^1\}$ of S which satisfy

$lim_{m\to\infty} sup\{ P^*(s_m^1, s_n^1)) : n \geq m\} = 0$ or

$lim_{m\to\infty} sup\{ P^*(s_n^1, s_m^1) : n \geq m\} = 0$ then

(i) It's a p^* Cauchy sequence.

(ii) $\{s_m^1\}$ be a Cauchy sequence if it's a p^* Cauchy sequence.

Lemma 2.4: [23]

Let a metric space is (S, d) and a u-distance p^* on S.

(i)Whether two sequences $\{s_m^1\}$ together with $\{t_m^1\}$ satisfied

$lim_{m\to\infty} p^*(ӡ, s_m^1)=0$ and $lim_{m\to\infty} p^*(ӡ, t_m^1)=0.$ And

$lim_{m\to\infty} p^*(s_m^1, ӡ) =0$ and $lim_{m\to\infty} p^*(t_m^1, ӡ) = 0$ for few $ӡ \in S$, then

$lim_{m\to\infty} p^*(s_m^1, t_m^1) = 0.$

(ii) $s^1= t^1$,if $p^*(ӡ, s^1)=0= p^*(ӡ, t^1)$ as well as $p^*(s^1, ӡ)=0= p^*(t^1, ӡ)$

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Lemma 2.5: [23]

Suppose a partial quasi metric space be (S, q_p) and a u-distance on S be p^*. $\{a_m\}$ and $\{b_m\}$be two sequences of S in such a way that

$lim_{m\to\infty} sup\{ p^* (a_m, a_n)) : n \geq m\} = 0$

along with $lim_{m\to\infty} sup\{ p^* (a_n, a_m): n \geq m\} = 0$

Then ∃ two sub sequences $\{a_{k_m}\}$ of $\{a_m\}$ and $\{b_{k_m}\}$ of $\{b_m\}$such that

$lim_{m\to\infty} q_p(\{a_{k_m}, b_{k_m}) = 0.$

Lemma 2.6:[23]

Let (S, q_P) and p^* are a quasi-partial metric space and a u- distance on S. There are two mappings $Y: S \to S$ including $\phi: R^+ \to R^+$ satisfy the conditions:

(a) $p^*(Y s^1, Y t^1) = \phi(max\{p^*(s^1, t^1), p^*(s^1, Y s^1), p^*(t^1, Y t^1),$

$p^*(t^1, s^1), p^*(Y s^1, s^1), p^*(Y t^1, t^1)\} \quad \forall s^1, t^1 \in S$ **(2.1)**

(b) A increasing function is ϕ such that $\phi^n(t^1) <$ (b) $t^1 > 0;$

(c) A increasing as well as bijective function is I-ϕ and I is identity mapping on R^+.

(d) $\sum_{m=1}^{\infty} \phi^m(t^1) < \infty$ for every $t^1 \in (0, \infty)$,

ϕ^n = n times ϕ composition was repeated with itself.

Then **(i)** for $s^1, t^1 \in S$ and $n, j, i \in N$

$d(Y s^1, Y t^1) \leq$ b (max $\{d(s^1, t^1), d(s^1, Y s^1), d(t^1, Y t^1)\}$

$max\{p^*(Y^j s^1, Y^i s^1), p^*(Y^j s^1, Y^i t^1), p^*(Y^j t^1, Y^i s^1),$

$p^*(Y^j t^1, Y^i t^1)\} \leq \phi(\zeta(O(s^1, t^1, n))$ where $j, i \leq m$

(ii) For $s^1, t^1 \in S$, $\zeta(O(s^1, t^1, \infty) \leq (1-\phi)^{-1}[a(s^1, t^1)]$,

where $a(s^1, t^1) = p^*(s^1, s^1) + p^*(t^1, t^1) + p^*(s^1, t^1) + p^*(t^1, s^1) + p^*(s^1, Y s^1) + p^*(Y s^1, s^1) + p^*(t^1 Y t^1) + p^*(Y t^1, t^1)$

(iii) For each $s^1 \in S$, $\{Y^{m+1}s^1\}$ a Cauchy sequence.

(iv) Every s^1, t^1 belongs to S and n belongs to N,

$p^*(Y^{m+1}s^1, Y^{m+1}t^1) \leq \phi^m(1-\phi)^{-1}(a(s^1, t^1))$

For each $s^1, t^1 \in S$, $\lim_{m+1\to\infty} p^*(Y^{m+1}s^1, Y^{m+1}t^1) = 0$ **(2.2)**

3. 3.U-DISTANCE

The notion of u-distance introduced by Ume [23] in 2010 by generalizing of τ distance. Which is as follows:

Let (X, d) be a metric space. A function $p^*: S \times S \to R_+$ is known as a u- distance on S If $\exists$ a function $\phi : S \times S \times R^+ \times R^+ \to R^+$ then the following condition holds for $s^1, t^1, \mathfrak{z} \in S$

(u_1) $p^*(s^1, \mathfrak{z}) \leq p^*(s^1, t^1) + p^*(t^1, \mathfrak{z})$;

(u_2) For each $s^1, t^1 \in R^+$ and for any $s^1 \in S$, $\epsilon > 0$, $\phi(s^1, t^1, 0, 0) = 0$ and $\phi(s^1, t^1, x, y) \geq min(x, y)$, then $\exists\, \delta > 0$ such that $|x - x_0| < \delta$; and $|y - y_o| < \delta\, x$, where $x_o, y, y_o \in R^+$

And $s^1 \in S$ which shows $|\phi(s^1, t^1, x, y) - \phi(s^1, t^1, 0, 0)| < \varepsilon$;

$(u3)$ $lim_{m\to\infty} s_m^1 = s^1$ and $lim_{m+1\to\infty} sup\, \{\phi(w_m, \mathfrak{z}_m, p^*(w_m, s_n^1), p^*(\mathfrak{z}_m, s_n^1)) : n \geq m\}$

Which shows $p^*(t^1, s^1) \leq lim_{m\to\infty} inf\, p^*(t^1, s_m^1)$ $\forall t^1 \in S$;

$(u4)$) $lim_{m\to\infty} sup\, \{p^*(s_m^1, w_n) : n \geq m\} = 0$, $lim_{m\to\infty} sup\, \{p^*(t_m^1, \mathfrak{z}_n) : n \geq m\} = 0$,

$lim_{m\to\infty} \phi\, \{(s_m^1, w_m, x_m, y_m)\} = 0$, $lim_{m\to\infty} \phi\, \{(t_m^1, w_m, x_m, y_m)\} = 0$,

Which implies that $lim_{m\to\infty} \phi(w_m, \mathfrak{z}_m, s_m^1, y_m) = 0$

(*u5*) $\lim_{m\to\infty} \sup \{ \phi (w_m, \mathfrak{z}_m, p^* (w_m, s_n^1), p^* (\mathfrak{z}_m, s_n^1)\}=0$,

$\lim_{m\to\infty} \sup \{ \phi (w_m, \mathfrak{z}_m, p^* (w_m, t_n^1), p^* (\mathfrak{z}_m, t_n^1)\}=0$

Which shows *that* $\lim_{m\to\infty} d (s_m^1, t_m^1)=0$ or

$\lim_{m\to\infty}\{ \phi (e_m, f_m, p^* (s_m^1, e_m), p^* (s_m^1, f_m)\}=0$, $\lim_{m\to\infty}\{ \phi (e_m, f_m, p^* (t_m^1, e_m), p^* (t_m^1, f_m)\}=0$

Imply $\lim_{m\to\infty} d (s_m^1, t_m^1)=0$.

Example 3.1[23]

Let the usual metric be $S = R$. Then p^*: $S\times S \longrightarrow R_+$; where. Then a u-distance on S is which is p^*, which is not a τ distance.

Example 3.2[23]

Let S be a normed space. Then p^*: $S \times S \to R_+$ defined by $p^* (s^1, t^1) = ||s^1||$ for each $s^1, t^1 \in S$. Then a u-distance on S is p^* which is not a distance known as τ distance.

4. MAIN RESULT OF FIXED POINT FOR Ψ GENERALIZED SINGLE-VALUED AND P- CONTRACTIVE CIRIC TYPE MAPPING

Here we present the main theorems which are proved by generalizing and improving the conditions given by Ume, JS. (2015) in the frame of quasi-partial-metricspace. 3405

Theorem 4.1: Suppose there is any complete quasi partial metric space and u-distance be (S, q_p) and p^*.
Let a Ciric type mapping is $\boldsymbol{Y} : S \to S$,
Which is ψ generalized single-valued and p-contractive, satisfies the conditions (b), (c), and (d) of Lemma 2.6, Then

(i) $\lim_{m+1\to\infty} \boldsymbol{Y}^{m+1}s^1 = z$ *for each* $s^1 \in S$.

(ii) $p^* (\boldsymbol{Y}^{m+1} s^1, \mathfrak{z}) \le \sum_{k=m}^{n+1} \phi^{k+1}((1 - \phi)^{-1}(a (s^1)) \ \forall s^1 \in S$

Where $a(s^1) = [4p (s^1, s^1) + 2p^*(s^1, \boldsymbol{Y} s^1) + 2p^* (\boldsymbol{Y} s^1, s^1)$

(iii) In S, $\mathfrak{z}$ is a unique fixed point of $\boldsymbol{Y}$, and $p^*(\mathfrak{z}, \mathfrak{z}) = 0$.

Proof:

Proof (i): Let $s^1, t^1 \in S$ and $s_{m+1}^1 = \boldsymbol{Y}^{m+1}s^1$ and $t_{m+1}^1 = \boldsymbol{Y}^{m+1}t^1 \quad \forall m + 1 \in N$

then we have to prove, $\{ s^1_{m+1}\}$ is a Cauchy sequence.

Let $s^1 \in S$ be an arbitrary point and define $s^1_{m+1} = \boldsymbol{Y}^{m+1}s^1$ for every $m+1 \in N$

By the hypothesis of equation (2.1) and (b) of Lemma 3.3, we have

$p^*(s^1_{m+1}, s^1_{m+2}) = p^*(\boldsymbol{Y}\ s^1_m , \boldsymbol{Y}\ s^1_{m+1})$

$\leq \phi\ (max\ \{p^*(s^1_m , s^1_{m+1}), p^*(s^1_m , s^1_{m+1}), p^*(s^1_{m+1} , s^1_{m+2}),$

$p^*(s^1_{m+1} , s^1_m), p^*(s^1_{m+1} , s^1_m), p^*(s^1_{m+2} s^1_{m+1})\}),$ **(4.1)**

Similarly , $p^*(s^1_{m+2}, s^1_{m+1}) = p^*(\boldsymbol{Y}\ s^1_{m+1}, \boldsymbol{Y}\ s^1_m)$

$\leq \phi\ (\max\ \{p^*(s^1_{m+1} , s^1_m), p^*(s^1_{m+1} , s^1_{m+2}), p^*(s^1_m , s^1_{m+1}),$

$p^*(s^1_m , s^1_{m+1}), p^*(s^1_{m+2} , s^1_{m+1}), p^*(s^1_{m+1} , s^1_m)\}),$ **(4.2)**

And

$p^*(s^1_m, s^1_{m+1}) = p^*(\boldsymbol{Y}\ s^1_{m-1} , \boldsymbol{Y}\ s^1_m)$

$\leq \phi\ (max\ \{p^*(s^1_{m-1} , s^1_m), p^*(s^1_{m-1} , s^1_m), p^*(s^1_m , s^1_{m+1}), p^*(s^1_m , s^1_{m-1}),$

$p^*(s^1_m , s^1_{m-1}), p^*(s^1_{m+1} , s^1_m)\}),$ **(4.3)**

And $p^*(s^1_{m+1}, s^1_m) = p^*(\boldsymbol{Y}\ s^1_m , \boldsymbol{Y}\ s^1_{m-1})$

$\leq \phi(\max\ \{p^*(s^1_m , s^1_{m-1}), p^*(s^1_m , s^1_{m+1}), p^*(s^1_{m-1} , s^1_m), p^*(s^1_{m-1} , s^1_m),$

$p^*(s^1_{m+1} , s^1_m), p^*(s^1_m , s^1_{m-1})\}),$ **(4.4)** 3406

And $p^*(s^1_m, s^1_{m+2}) = p^*(\boldsymbol{Y}\ s^1_{m-1} , \boldsymbol{Y}\ s^1_{m+1})$

$\leq \phi\ (\max\ \{p^*(s^1_{m-1} , s^1_{m+1}), p^*(s^1_{m-1} , s^1_m), p^*(s^1_{m+1} , s^1_{m+2}),$

$p^*(s^1_{m+1} , s^1_{m-1}), p^*(s^1_m , s^1_{m-1}), p^*(s^1_{m+2} , s^1_{m+1})\}),$ **(4.5)**

And $p^*(s^1_{m+2}, s^1_m) = p^*(\boldsymbol{Y}\ s^1_{m+1} , \boldsymbol{Y}\ s^1_{m-1})$

$\leq \phi\ (\max\ \{p^*(s^1_{m+1} , s^1_{m-1}), p^*(s^1_{m+1} , s^1_{m+2}), p^*(s^1_{m-1} , s^1_m),$

$p^*(s^1_{m-1} , s^1_{m+1}), p^*(s^1_{m+2} , s^1_{m+1}), p^*(s^1_m , s^1_{m-1})\}),$ **(4.6)**

$\leq \phi\ (\max\ \{p^*(s^1_m , s^1_m), p^*(s^1_m , s^1_{m+1}), p^*(s^1_m , s^1_{m+1})\})$ **(4.7)**

Substitute the value of (2) - (7) in (1) and by hypotheses (a), (b), and (c) of Lemma 2.6, we have

$p^*(s^1_{m+1}, s^1_{m+2}) \leq \phi\ (max\ \{p^*(s^1_j , s^1_i) : m \leq j, i \leq m + 2\}) \leq \varphi^2\ (\max\ \{p^*(s^1_j , s^1_i) : m-1 \leq j, i \leq m + 2\})$

Proceeding in this manner

$\leq \phi^m\ (\max\ \{p^*(s^1_j , s^1_i) : 1 \leq j, i \leq m + 2\})$

$\leq \phi^m\ (\zeta\ (O\ (s^1, s^1, \infty))$

$\leq \phi^m ((1-\phi)^{-1} a (s^1))$ **(4.8)**

Where $(a(s^1)) = 4p^*(s^1, s^1) + 2[p^*(s^1, \boldsymbol{Y} s^1) + p^*(\boldsymbol{Y} s^1, s^1)]$

Now if m + 1 < n + 1, then by (8)

$p^*(s^1_{m+1}, s^1_{n+1}) \leq p^*(s^1_{m+1}, s^1_{m+2}) + p^*(s^1_{m+2}, s^1_{m+3}) + \ldots\ldots\ldots\ldots + p^*(s^1_n, s^1_{n+1})$

$= \sum_{k=m}^{n-1} p*(s^1_{k+1}, s^1_{k+2})$

$\leq \sum_{k=m}^{n-1} \phi^k ((1-\phi)^{-1} a(s^1))$

So $p^*(s^1_{m+1}, s^1_{n+1}) \leq \sum_{k=m}^{n-1} \phi^{k+1}((1-\phi)^{-1} a(s^1))$ **(4.9)**

By (d) of Lemma 2.6 and (4.9),

We get $\lim_{m\to\infty} \sup \{p^*(s^1_{m+1}, s^1_{n+1})) : n+1 \geq m+1\} = 0$ **(4.10)**

By Lemma 2.4 and (4.10),

s^1_{m+1} is a Cauchy Sequence, $\{\boldsymbol{Y}^{m+1} s^1\}$ is a Cauchy Sequence.

So S is complete and s^1_{m+1} converges to some $\mathfrak{z} \in S$.

Proof (ii):

Due to equation (9), (d) of Lemma 2.6, Lemma 2.4, Definition 2.2, and (u3), we have

$$p^*(s^1_{m+1}, \mathfrak{z}) \leq \lim_{n\to\infty} \inf p^*(s^1_{m+1}, s^1_{n+1}) \leq \sum_{k=m}^{n-1} \phi^{k+1}((1-\phi)^{-1} a(s^1))$$

Which proves (ii) part of theorem.

Proof (iii):

Since $\boldsymbol{Y}$ has a unique fixed-point w ∈ S and (iv) of Lemma 2.6 such that

$\lim_{m+1\to\infty} \boldsymbol{Y}^{m+1} t = \boldsymbol{Y} \mathfrak{z}$ **(4.11)**

from equation (2.2) and (4.8)

$\lim_{m+1\to\infty} \sup \{\sup [p^*(\boldsymbol{Y}^{m+1} s^1, \boldsymbol{Y}^{n+1} t^1) : n+1 > m+1]\}$

$\leq \lim_{m+1\to\infty} \sup \{\sup [p^*(\boldsymbol{Y}^{m+1} s^1, \boldsymbol{Y}^{n+1} s^1) + p^*(\boldsymbol{Y}^{n+1} s^1, \boldsymbol{Y}^{n+1} t^1) : n+1 > m+1]\}$

$\leq \lim_{m+1\to\infty} \sup \{\sup [p^*(\boldsymbol{Y}^{m+1} s^1, \boldsymbol{Y}^{n+1} s^1) : n+1 > m+1]\} +$

$\lim_{m+1\to\infty} \sup \{\sup p^*(\boldsymbol{Y}^{n} s^1, \boldsymbol{Y}^{n} t^1) : n > m]\} = 0$ **(4.12)**

So, we obtain,

$\lim_{m+1\to\infty} \sup \{\sup [p^*(\boldsymbol{Y}^{m+1} s^1, \boldsymbol{Y}^{n+1} t^1) : n+1 > m+1]\} = 0$ **(4.13)**

From equation (4.13) and lemma (2.5)

∃ two sequences $\{s^1_{k_{m+1}}\}$, $\{t^1_{k_{m+1}}\}$ both are subsequences of $\{s^1_{m+1}\}$ and $\{t^1_{m+1}\}$

in such a way that

$lim_{m+1\to\infty} q_p(s^1_{m+1}, t^1_{m+1})=0$ **(4.14)**

Since $\boldsymbol{Y}$ has a unique fixed-point z in S and from equation (10) and lemma 2.5;

$q_p(\mathfrak{z}, \boldsymbol{Y}\mathfrak{z}) = 0$

Therefore $\mathfrak{z}$ is a fixed point of $\boldsymbol{Y}$.

UNIQUENESS

Suppose $\mathfrak{z} = \boldsymbol{Y}\mathfrak{z}$ and $v = \boldsymbol{Y} v$

Then by hypothesis, we get

$p^*(\mathfrak{z}, v) = p^*(v, \mathfrak{z}) = p^*(\boldsymbol{Y}\mathfrak{z}, \boldsymbol{Y} v) \le \psi(max\{p^*(v, \mathfrak{z}), p^*(v, v), p^*(\mathfrak{z}, \mathfrak{z}), p^*(\mathfrak{z}, v)\})$ **(4.15)**

$p^*(\mathfrak{z}, \mathfrak{z})=p^*(\boldsymbol{Y}\mathfrak{z}, \boldsymbol{Y}\mathfrak{z}) = p^*(v,v)=p^*(\boldsymbol{Y} v, \boldsymbol{Y} v)\le\psi(max\{p^*(v, \mathfrak{z}), p^*(v, v), p^*(\mathfrak{z}, \mathfrak{z}), p^*(\mathfrak{z}, v)\})$

From hypothesis and equation (4.15)

$max\{p^*(v, \mathfrak{z}), p^*(v, v), p^*(\mathfrak{z}, \mathfrak{z}), p^*(\mathfrak{z}, v)\} = 0$ **(4.16)**

From (4.16) and Lemma 2.3, we obtained $v = \mathfrak{z}$. So $\mathfrak{z}$ is the unique fixed point of $\boldsymbol{Y}$ in S.

From the above theorem, we have following corollary:

Corollary 4.2: A complete quasi partial metric space be (S, d_q) and p is au-distance on S. Let $T: S \to S$ be a mapping, satisfy the following assertations

(i) $p^*(\boldsymbol{Y} s^1, \boldsymbol{Y} t^1) \le j\,(\max[p^*(s^1, t^1), p^*(s^1, \boldsymbol{Y} s^1), p^*(t^1, \boldsymbol{Y} t^1), p^*(t^1, \boldsymbol{Y} s^1),$

$p^*(t^1, s^1), p^*(\boldsymbol{Y} s^1, s^1), p^*(\boldsymbol{Y} t^1, t^1), p^*(\boldsymbol{Y} t^1, s^1), p^*(\boldsymbol{Y} s^1, t^1)$

For some $j \in (0, 1)$ and for all $(s^1, t^1) \in S$

(ii) $\forall\, s^1 \in S$ with $\lim_{m+1\to\infty} \boldsymbol{Y}^{m+1}s^1=c_{s^1}\in S$,such that $\lim_{m+1\to\infty} \boldsymbol{Y}^{m+1}s^1 = \boldsymbol{Y} c_{s^1}$

Then, $\mathfrak{z}$ is a unique fixed point of $\boldsymbol{Y}$ and $p^*(\mathfrak{z}, \mathfrak{z}) = 0$.

5. EXISTENCE OF A SOLUTION FOR AN INTEGRAL EQUATION

Let the set of all continuous functions defined on [0, 1] is $\phi : R^+ \longrightarrow R^+$, satisfyconditions (b), (c), and (d) of Lemma 2.6.

(i) A non-decreasing is ψ and $\psi(y) < y$ for all $y > 0$;

(ii) I-ψ is a non-decreasing and bijective function, where I is identity mappingon R^+;

(iii) $\sum_{k=0}^{n} \psi^m(y) < \infty$ for each $y \in (0, \infty)$.

Where ψ^m is n times repeated composition of ψ with itself.

Let d, p^*: S ×S→ R^+ and ϕ: S×S×R^+× R^+ →R^+ be mappings which is defined as

$$d(s^1, t^1)= sup_{y\in[0,1]} \mid s^1(y) - t^1(y) \mid ; p(s^1, t^1)= sup_{y\in[0,1]} \mid s^1(y) \mid , \text{ and } \phi(s^1, t^1, x, y) = s^1$$

∀ s^1, t^1 ∈S and x, y ∈ R^+. Then (S, q_p) is a quasi-partial metric space and h is a u-distance on the set S.

Here we prove the solution of integral equation of the existence theorem by the theorem

$$s^1(y) = r(s^1, y)+ \int_0^1 O(y, x)f (x, s^1(x)dx \qquad \textbf{(5.1)}$$

Where s^1∈ S, r: S×R→R, O: [1,0]×[o,1]→R and f : [1,0] ×R →R are given mappings.

In support of above application, we shall prove a theorem.

Theorem 5.1: Let the following assertation holds:

(I_1) There is a continuous mapping $r : S \times R \to R$ such that

$|r (s^1, y) \leq \frac{1}{2}\psi(|s^1(y)|)$ for all $s^1 \in S$ and y ∈ R.

(I_2) $O : [0, 1] \times [0, 1] \to R$ is a continuous mapping in such a way that

$O (y, x) \leq \frac{1}{2}$ for all y, x ∈ [0, 1].

(I_3) f: $[0, 1] \times R \to R$ is a continuous mapping in such a way that

$|f(x, s^1(x)) | \leq \psi s^1(x)$ ∀$s^1 \in S$ and x ∈ [0, 1].

(I_4) for each s^1 ∈ S with $\lim_{m\to\infty} \boldsymbol{Y}^m s^1 = c_{s^1}$ ∈ S, there exist t^1∈ S such that $\lim_{m\to\infty} \boldsymbol{Y}^m t^1 = \boldsymbol{Y} c_{s^1}$

Then equation (4.1) has a solution $s^1 \in S$.

Proof: Let a mapping $\boldsymbol{Y} : S \to S$ defined by

$(\boldsymbol{Y} s^1) (y) = r(s^1, y) + \int_0^1 O(y, x)f (x, s^1(x)dx \quad \forall s^1 \in S$ and y ∈ [0, 1].

From (I_1), (I_2) *and* (I_3)

$| (\boldsymbol{Y} s^1)(y)| = |r(s^1, y) + O (y, x) f(x, s^1(x)dx|$

$| (\boldsymbol{Y} s^1)(y)| = |r(s^1, y) +\int_0^1 O(y, x) f(x, s^1(x)dx|$

$\leq |r (s^1, y) | + |\int_0^1 O (y, x) f(x, s^1(x)dx|$

$\leq |r\ (s^1, y)| + |\int_0^1 O\ (y, x)|\ |f(x, s^1(x)\ dx|$

$\leq |r\ (s^1, y)| + \frac{1}{2} |\int_0^1 \psi(|s^1(x)|dx$

$\leq \frac{1}{2}\ (\psi|(s^1(y)|) + \frac{1}{2} |\int_0^1 \psi(\, sup_{y\in[0,1]} |\ s^1(y)\ |\ dx|$

$\leq \frac{1}{2}\ (\psi|(s^1(y)|) + \frac{1}{2} |\int_0^1 \psi(\, sup_{y\in[0,1]} |\ s^1(y)|)\ dx|$

$\leq \frac{1}{2}\ (\psi|(s^1(y)|) + \frac{1}{2}\ \psi(\, sup_{y\in[0,1]} |\ s^1(y)\ |\ )$

$\forall\ s^1 \in S$ and $x \in [0, 1]$. Then

$p\ (\boldsymbol{Y}\ s^1, \boldsymbol{Y}\ \ t^1) = sup_{y\in[0,1]}\ |\ (\boldsymbol{Y}\ s^1)(y)|\ \leq sup_{y\in[0,1]}\ \{\frac{1}{2}(\psi|\ s^1(y)\ |) + \frac{1}{2}\psi(sup_{y\in[0,1]} |\ s^1(y)\ |)\}$

$\leq \frac{1}{2}(\psi|\ s^1(y)\ |) + \frac{1}{2}\psi(sup_{y\in[0,1]}\ |\ s^1(y)\ |)$

$\leq \psi(sup_{y\in[0,1]}\ |\ s^1(y)\ |)$

$\leq \psi\ (max\ \{sup_{y\in[0,1]}\ |\ s^1(y)|, sup_{y\in[0,1]}\ |\ t^1(y)\ |, sup_{y\in[0,1]}\ |\ \boldsymbol{Y}\ s^1\ (y)\ |,$
$sup_{y\in[0,1]}\ |\ (\boldsymbol{Y}\ \ t^1\ )(y)\ |\})$

$= \psi(max\{p^*(s^1, t^1), p^*(s^1, \boldsymbol{Y}\ s^1), p^*(t^1, \boldsymbol{Y}\ \ t^1), p(t^1, s^1), p^*(\ \boldsymbol{Y}\ s^1, s^1), p^*(\ \boldsymbol{Y}\ t^1, t^1)\})$

$\forall\ s^1, t^1 \in S.$
There for each and every assumption of theorem 3.5 is satisfied. Hence the mapping ***Y*** has a fixed point. This point is the solution of the integral equation (5.1).
Now if $s^1, t^1 \in (1, \infty) \cap Q$, then

$$q_p(Gs^1, Gt^1) = \max\ \{(t^1 - \tfrac{4}{9}) - (\ s^1 - \tfrac{4}{9}), 0\} + (s^1 - \tfrac{4}{9})$$

$q_p(Gs^1, Gt^1) = \max\ \{(t^1 - s^1), 0\} + (s^1 - \frac{4}{9})$
Now we take three conditions i.e., $s^1 = t^1, s^1 > t^1, s^1 < t^1$.
(i) If $s^1 = t^1$ then $q_p\ (Gs^1, Gt^1) = k \max\ \{q_p\ (s^1, t^1), q_p\ (s^1, Gs^1), q_p\ (t^1, Gt^1), q_p\ (s^1, Gt^1), q_p(t^1, Gs^1)\}$
By fixed point theory $s^1 = Gs^1$ and $t^1 = Gt^1$

$q_p\ (Gs^1, Gs^1) = k\ \max\ \{q_p\ (s^1, t^1), q_p\ (s^1, s^1), q_p\ (t^1, t^1), q_p\ (s^1, t^1), q_p\ (t^1, s^1)\}$

$max\{(s^1 - s^1), 0\} + (s^1 - \frac{4}{9})\ \ = k\ \max\ \{q_p\ (s^1, s^1), q_p\ (s^1, s^1), q_p\ (s^1, s^1), q_p\ (s^1, s^1, q_p\ (s^1, s^1)\}$

$(s^1 - \frac{4}{9}) = k\ \{q_p\ (s^1, s^1)\} = \frac{4}{13} q_p\ (s^1, s^1)$

(ii)If $s^1 > t^1$ then $q_p(Gs^1, Gt^1) \le k \max\{q_p(s^1, t^1), q_p(s^1, Gs^1), q_p(t^1, Gt^1), q_p(s^1, Gt^1), q_p(t^1, Gs^1)\}$

By fixed point theory $s^1 = Gs^1$ and $t^1 = Gt^1$

.$q_p(s^1, t^1) \le k \max\{q_p(s^1, t^1), q_p(s^1, s^1), q_p(t^1, t^1), q_p(s^1, t^1), q_p(t^1, s^1)\}$

$q_p(s^1, t^1) \le k \max\{q_p(s^1, t^1)\} \le \frac{4}{13} q_p(s^1, t^1)$

(iii)If $s^1 < t^1$ then $q_p(Gs^1, Gt^1) \le k \max\{q_p(s^1, t^1), q_p(s^1, Gs^1), q_p(t^1, Gt^1), q_p(s^1, Gt^1), q_p(t^1, Gs^1)\}$

By fixed point theory $s^1 = Gs^1$ and $t^1 = Gt^1$

$q_p(s^1, t^1) \le k \max\{q_p(s^1, t^1), q_p(s^1, s^1), q_p(t^1, t^1), q_p(s^1, t^1), q_p(t^1, s^1)\}$

$q_p(s^1, t^1) \le k \max\{q_p(s^1, t^1)\} \le \frac{4}{13} q_p(s^1, t^1)$

In each case, it's a contradiction

Now, if $s^1, t^1 \in \overline{B(s_0^1, r)} \cap Q$ then

$q_p(Gs^1, Gt^1) = \max\{\frac{1}{10}t^1 - \frac{1}{10}s^1, 0\} + \frac{1}{10}s^1$

$q_p(Gs^1, Gt^1) = \max\{\frac{1}{10}[(t^1 - s^1, 0\} + s^1]\} = \max \frac{1}{10} q(s^1, t^1)$

$q_p(Gs^1, Gt^1) = \frac{1}{10} q_p(s^1, t^1) < \frac{4}{13} q_p(s^1, t^1)$

So $q_p(Gs^1, Gt^1) \le k \max q_p(s^1, t^1), q_p(s^1, Gs^1), q_p(t^1, Gt^1), q_p(s^1, Gt^1), q_p(t^1, Gs^1)$ is fulfilled. 3411

Furthermore, 0 is the fixed point of G and $q_p(s^1, t^1) = 0$

6. CONCLUSION

In the computational analysis, the algorithms is the computations that evaluate their running time of this algorithms. The running time play an important role in computer algorithm. In current paper, following the original task of Ume [23] and prove fixed point of Ciric type single valued and p-contractive map in quasi- partial metric space. Here we proven the existence and uniqueness of fixed points using ψ generalized single valued and p-contractive ciric type map with respect to u distance in Theorem 4.1 and Theorem 5.1 has been proved for the support of application portation. The noble idea of Banach was enforced by Ume for ciric type in complete metric space. Moreover, this result are no technical for modelling the algorithm complexity and, motived by this disadvantage, we proved the existence and uniqueness of a fixed point theorem in partial quasi- metric space. In the present paper, we focus our achievements on construct a fixed point on the frame of partial quasi- metric space that allowed a program verification mathematical tool. Current result is useful to find more applications in another field of fixed-point theory.

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