EXPLORING MATHEMATICAL EDUCATION RESEARCH: A DISCUSSION ON LESTER'S INSIGHTFUL PAPER

#1 Mr.GOVINDARAPU RAJU, *Assistant Professor*
#2 Mr.BANGIMATAM SANDEEP KUMAR, *Assistant Professor*
Department of Mathematics,
SREE CHAITANYA INSTITUTE OF TECHNOLOGICAL SCIENCES, KARIMNAGAR, TS.

ABSTRACT:
Based on Lester's (2005) work, this comment examines the structure for academics interested in math education. In response to three of Lester's points, the author (a) expresses concern that current political forces in the United States may rigidly define scientific research in education; (b) provides a brief outline of Lester's framework and model for theory-based research in mathematics education; and (c) examines the importance of mathematical context in MER, which the MER community should discuss in response to the political forces that (inappropriately) shape our field.
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1. INTRODUCTION

The study undertaken by Lester holds tremendous importance within the field of mathematics education research (MER) due to its offering of a conceptual framework and a vision that drives MER researchers towards their work's aims and essential principles. My answer to the paper is divided into three pieces. In the first piece, I address Lester's concern about the dominant political forces in the United States that demand for tight constraints on scientific investigation within academic institutions. In addition, I provide a justification for the emergence of these forces that is not based on political ideology. In the second half, he presents his framework and model for theory-based research in mathematics education, as well as a recap of Lester's presentation. This section of the report might be read as a call to the MER community to face current political realities that unintentionally define the realm of scientific research in education. As suggested by Lester's findings, this section analyzes the argument for mandating a larger emphasis on philosophy and theory in mathematics teaching graduate degree programs. The significance of mathematical basis in MER is discussed in the paper's conclusion and third sections; this topic is not addressed in the body of the book. Despite the lack of explicit debate, I derived key findings from Lester's work that served as the foundation of my thesis: mathematical frameworks are required for theory-driven research in mathematics education. The proof-versus-argumentation phenomena will be used to demonstrate the implications of this argument. This argument contends that graduate programs that specialize in mathematics instruction are required to improve the quality of the mathematics component

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2. RIGID DEFINITION OF SCIENTIFIC RESEARCH IN EDUCATION

Lester's paper commences with an analysis of prevailing political dynamics in the United States, with the intention of delineating scientific research in education as a domain that predominantly utilizes randomized experiments and controlled clinical trials—what is scientifically-based research (SBR) as defined by the No Child Left Behind Act. The essential purpose of educational research, as per this specific definition, is to establish "what works," and SRB methodologies are indispensable in this pursuit. According to Lester, the experimental procedures of the SRB approach were extensively employed in MER until the 1970s. However, their lack of sufficient efficacy in resolving the question of "what works" led to its general rejection. Notwithstanding its criticality in resolving particular queries of research nature, it is obvious that the experimental research design approach was retained. Conversely, the idea that this particular technique was required for every MER investigation was rejected. Concerns regarding the intricacies of human thoughts and actions, particularly as they pertain to mathematics education and learning, have pushed the MER community to conclude that new research methodologies must be implemented and potentially even established. An illustration of this is the modern "teaching experiment" methodology, which was established in reply to apprehensions surrounding the manner in which mathematical information is learned in real-world classroom situations (Steffe & Thompson, 2000).

This leads the clear issue as to why advocates of SBR methodologies seem to have overlooked the arguments for their absence from MER, notwithstanding their insistence on their universal relevance to educational research irrespective of the topic being researched. Could this be linked to the paucity of faith among the general public and politicians in the caliber of educational research, specifically MER? According to evidence offered by Feuer, Towne, and Shavelson (2002), educational experts perceive the bulk of research in the field to be of low quality. In a number of different nations, objections over the applicability of educational research have been consistent, according to studies that extend beyond the United States (Levine, 2004). This difficulty is exacerbated, according to Feuer et al., by the absence of theory in educational research, which is precisely Lester's area of competence.

3. THE ROLE OF THEORY

Lester's work might potentially be viewed as a challenge to the mathematics education research community (MER) to stimulate greater investigation in the subject, as this was one of the purposes of the SRB movement—to address the perspective that initially sparked the movement. "Promoting and reinforcing a scientific culture," which is "a set of norms and practices and an ethos of honesty, openness, and continuous reflection, including how research quality is evaluated" (Feuer et al., 2002, p. 4), is important, among other things, to develop enhanced research. Lester raises attention to a fundamental flaw in the existing scientific culture at MER, which is an inadequate focus on philosophy and theory. "The function of theory and the philosophical foundations of our research" have been overlooked, according to the author (p. 457, emphasis added). Three key reasons are recognized by Lester as contributing to this insufficiency. The present initiative by the government is a comparatively recent development.

It is quite plausible that the prioritizing of "what works" research by funding bodies has lowered the tendency of scientists to formulate hypotheses. The last two issues are as follows: (a) the general perplexity among academics regarding the definition of theoretical or conceptual approaches to research, and (b) the prevalent belief among scientists that they do not possess the qualifications necessary to investigate philosophical and theoretical matters. These two obstacles, in contrast to the initial concern, originate internally within the MER community and are directly attributable to the dearth of "practical and philosophical foundations for future research" courses and experiences that are customary in graduate programs across the United States (p. 461). Due to the fact that these are internal issues, Lester contends that the MER community is most qualified to handle them. Lester accomplishes this objective effectively by requesting that the MER community "grow more proficient at fostering a predilection [among graduate students] for meticulously conceived frameworks to direct our research." As an illustration, he asserts that "the researcher can only speculate or offer no explanation in the absence of a [research] framework" (p. 461). Similar thoughts have been expressed by others: "One of the most crucial factors in establishing the theoretical underpinnings of mathematics education is without a doubt the training of researchers in the field" (Batanero, Godino, Steiner, & Wenzelburger, 1992, p.2).

Lester's appeal for the progression of theory-driven research in mathematics education is substantiated by (a) an elucidation of the function that theory assumes in the realm of education research and (b) an examination of how the philosophical standpoint of an individual influences the character of the research pursued. Lester proposes a theoretical framework that permits the assessment of educational research and MER with respect to the initial condition. Stokes' (1997) "dynamic" paradigm for scientific and technological research, which crosses "considerations of utility and the pursuit of fundamental understanding" (p. 465), serves as the cornerstone for Lester's methodology. Lester's method is beneficial since it emphasizes the requirement of merging MER theory and practice. Existing knowledge (pertaining to fundamental issues) and present products (including educational policies and curricula) operate as inputs for "use-inspired basic research" (i.e., inquiries that unearth improved products and knowledge), as illustrated in this diagram.

Lester investigates the probable applicability of these technologies to a critical research subject in mathematics education with reference to the second argument. The individual gives light on Churchman's (1971) taxonomy of inquiry systems, which includes the systems proposed by Leibniz-Lockean, Kantian, Hegelian, and Singerian. Regarding the usefulness of "traditional" vs "reform" curriculum in producing mathematical proficiency, attitudes among educators are split. In this debate, Lester does not suggest that the application of Churchman's paradigm might, in theory, give a solution to this or any other educational challenge. Conversely, he argues that

Churchman's framework permits academicians to contemplate key concerns relevant to their field of study. Lester's examination of these two domains gives convincing reasons, highlighting the requirement for mathematics education doctorate programs to more fully integrate philosophy and theory into their curricula. This is the outcome of the interaction of three elements. Researchers must create a sound philosophical foundation prior to diving into fundamental concerns such as the ethics and applicability of research findings, the defense of research arguments, and the nature of conclusions and evidence. Furthermore, in line with the initial point, proper theoretical preparedness is important in order to effectively navigate such scenarios. When utilizing the Kantian inquiry system, for instance, it is imperative to grasp the procedure for formulating diverse studies from various theoretical vantage points, along with the reasoning behind why each of these designs may require unique data collection and lead to substantially differing interpretations of the study's results. Consequently, a full grasp of the varied theories and their consequences for the field of mathematics education and learning is important. Moreover, to cultivate an interest in philosophical and theoretical thought, beginner researchers are required to address tough challenges, such as philosophical and theoretical conundrums. In contrast to the frequently supplied descriptive courses, which comprise of exercises aimed to aid students in learning the multiple concepts and inquiry processes necessary for MER, problem-solving programs offer major advantages to graduate students. Nevertheless, the purpose of teaching our graduate students in "Singerian" inquiry—as articulated by Lester—is more significant than mere familiarity with a particular theory or inquiry system.

Implementing this strategy involves consistent review of the underlying assumptions of inquiry systems. Irrespective of their seeming simplicity, essential notions must be questioned in order to generate fresh views on the subject at hand. Consequently, an appraisal of the ethical considerations and moral principles that occur throughout the process of ideation unavoidably arises. Page 463

Given the aforementioned characteristics, I am certain that Lester's corpus of work would serve as an outstanding framework for a sequence of graduate-level courses that attempt to engage students' philosophical and conceptual thinking regarding educational research. Conversely, the paper's narrative fails to address the role of mathematical context in theory-based research relevant to mathematics education.

The Role of Mathematical Context

It is evident that Lester's work does not comprehensively cover all facets of the mathematics education preparation that is necessary for scholars to conduct theory-based research. Nevertheless, the narrative of the study neglects to explicitly address the considerable influence of the disciplinary setting on conceptual and structural issues.

upon which we conducted our investigation. However, I successfully discerned fundamental elements of Lester's work that formed the basis of my thesis, which argues that investigations into mathematics education led by theory must be grounded in a mathematical framework. Based on this line of reasoning, it is my conviction that Lester's request for graduate students to participate in theory-based research within the MER community ought to be accompanied by an effort to foster the growth of robust mathematical foundations among this cohort.

An initial element of Lester's paradigm is its equitable emphasis on fundamental comprehension and practical factors. The study viewpoint determines the pertinent aspects and (supposed) interrelationships of the phenomena represented by these abstractions. Lester defines "research framework" as "a foundational structure of concepts (i.e., interrelationships and abstractions) that serves as the structural foundation for an investigation into a specific phenomenon." The aforementioned interrelationships and abstractions subsequently furnish validation and support for every component of the study. Page 458

Lester concludes the third and final segment by mentioning the context:... Due to [a specific type of conceptual framework or framework]:

The study conducted by Lester, which adheres to three guiding principles, may serve as a conceptual framework for researchers attempting to comprehend the essence and intent of MER. Upon examining these components collectively, Lester's work can be perceived as a framework comprising three guiding principles that assist scholars in comprehending the essence and objective of MER.

Understanding the fundamental problems in mathematics education and learning and applying this knowledge to the evaluation of existing products and the creation of new ones pertaining to "mathematics" are the objectives of MER (see Harel, in press). Four concepts are defined as outcomes resulting from the implementation of these instructional objectives:

Ultimately, the objective of mathematics education is to assist students in cultivating cognitive processes and comprehension that resemble those employed by contemporary mathematicians. Collectively, these four criteria substantiate the assertion that mathematics education theory-driven research must be grounded in the present mathematical environment. The present argument shall be examined within the framework of a separate phenomenon, namely "argumentation" as opposed to "proof."

Considerable endeavors are underway to improve the prevailing atmosphere in mathematics courses, encompassing, among other strategies, the cultivation of persuasive language, debate, and discussion. The application of the primary guiding principle requires knowledge and application, or fundamental usage and comprehension considerations. This subject matter was selected for two reasons: (a) academics across disciplines are striving to comprehend argumentation and integrate it into the syllabi at every grade level; and (b) a solid mathematical foundation seems superfluous given that this research is predominately grounded in socio cultural, socio constructivist, and situative theoretic perspectives. This undertaking is unquestionably crucial, if not crucial. Nevertheless, "mathematical reasoning" and "argumentation" are not interchangeable terms. Failing to differentiate between the two could potentially lead to an overemphasis on developing argumentation skills, if not mathematical reasoning altogether. In order to be pertinent to mathematical discourse, a study paradigm that satisfies the four criteria stated above must incorporate a comprehension of the fundamental differences between mathematical reasoning and argumentation. Moreover, it is imperative that this inquiry underscore the critical significance of possessing an extensive comprehension of mathematics.

When employing mathematical deduction, it is critical to differentiate between the status and substance of a given assertion (Duval, 2002). Status is determined by the sequence of knowledge and deduction, as opposed to content (including hypotheses and conclusions). Consequently, the veracity of one statement differs from that of another in mathematics.

would almost surely enhance the quality of mathematics instruction.

In order to accomplish these objectives, theory-based MER requires that MER inquiries be conducted within the research frameworks established by Lester.

The concepts and connections advocated for in the study framework must be described and illustrated using mathematical terminology, as stated in Principle 1.

The topic of what "mathematical context" is still unanswered." Addressing this point is beyond the scope of this short article, but the stance I provide here is predicated on a specific definition of For special interpretations of these two terms, see Harel (in press a, in press b). It is critical to emphasize that these phrases do not imply correct understanding. When referring to what pupils know, the terms merely describe the knowledge—correct or incorrect, useful or impractical—that the students currently possess. However, the ultimate goal is for students to develop ways of understanding and thinking that are compatible with those that have been institutionalized in the mathematics discipline, those that the mathematics community as a whole accepts as correct and useful in solving mathematical and scientific problems. This goal is worthless unless it takes into account the reality that the process of learning entails the building of flawed, even erroneous, ways of understanding and deficient, if not incorrect, ways of thinking.

It is imperative to acknowledge that there are others who hold a different perspective regarding mathematics education (see, for instance, Millroy, 1992). Other field—can only be assessed by its position in logical value, not by epistemic worth (degree of confidence). Students tend to focus on content and have difficulty separating status from content. As a result, many propositions in mathematics appear trivial to students because they are judged in terms of epistemic values rather than logical values. For example, a decisive majority of students taking a geometry course for mathematics majors in their senior year had genuine difficulties understanding why it is necessary to substantiate the proposition "For any double cone, there is a plane that intersects it in an ellipse." Their robust perceptual proof scheme (Harel & Sowder, 1989) compelled them to make epistemic value judgments rather than logical value judgments. Similarly, due to attachment to content, students, particularly undergraduate mathematics majors, struggle with arguments by contradiction and contrapositive proofs when the conclusion of the statement to be demonstrated is self-evident. When a proposition a b is to be established and the students regard the assertion b as self-evident, they are prone to struggle with proofs that presume not b. Their main difficulty is distinguishing the content of b from its status.Another related feature of mathematical reasoning that is particularly significant and a source of difficulty for students is that the status of propositions changes during the construction of a proof: the conclusion of one deductive step may become a hypothesis of another. These are critical traits that must be present in every type of mathematical conversation, informal or formal (!).Outside of mathematics, however, they are not the primary concern: the strength of the arguments presented in support or opposition to a proposition is far more important.

To ascertain whether "argumentation" or "mathematical reasoning" is being advanced, it seems that an academic interested in social interaction in the mathematics classroom must explicitly or implicitly address critical questions such as whether mathematical reasoning originates from argumentative discourse and how, and whether a researcher conducting a teaching experiment or observing a classroom debate must possess a solid mathematical foundation.

The distinctions between "argumentation" and "proof" discussed above are critical and distinguishing features of mathematical reasoning in comparison to thinking in other fields. Despite this, students—even senior undergraduate mathematics majors—have difficulty understanding these features. This implies that graduate programs in mathematics teaching should prioritize the mathematical content component of their course requirements. Of course, in order to adhere to Lester's concept of "research framework," mathematics education researchers must understand much more than proof: they must understand the constructs of "argumentation," "social interaction," and "norms," as well as essential elements of various theoretical perspectives, such as sociocultural, cognitivist, socioconstructivist, and situative theoretic perspectives, in which these constructs reside. Furthermore, dealing with the learning and teaching of proof inevitably leads to questions about the epistemology and history of the concept, such as distinguishing between didactical obstacles—difficulties that result from narrow or faulty instruction—and epistemological obstacles—difficulties that are unavoidable because of the concept's meaning (see Brousseau, 1997). This is why graduate mathematics teaching programs must contain advanced mathematics courses as well as cognition, sociology, philosophy, and mathematics history.

Schoenfeld argues that mathematics is a critical concept in mathematics education as a distinct discipline due to its distinctive structures, historical context, and epistemological framework. Schoenfeld (2000) provided an opinion on the objective of the Mathematics Education Research Establishment (MER), which is consistent with the four-principle structure mentioned earlier: "Most importantly, the primary aim of mathematics education research is to comprehend the essence of mathematical cognition, instruction, and research."

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