SOLVING DIFFERENTIAL EQUATIONS IN THE RICCATI MATRIX WITH AN ADVANCED VARIATIONAL ITERATION METHOD

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ABSTRACT:
The convergence of the iterative procedure’s approximate solution is investigated. By applying the variational iteration method (VIM) to the general solution form of iterative approximate solutions, a novel variational iteration scheme is created with a faster convergence rate to an approximate solution after a limited number of iterations. The updated strategy accelerated the process of obtaining the exact solution by employing the Lagrange multiplier to build an extra correction function.

Keywords: Riccati matrix differential equations, variational iteration, he’s method, approximate solution

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1. INTRODUCTION
The ideas that underpin the Riccati equation, which has found increasing applicability in finance, engineering, and economics. Numerous researchers have made efforts to address the difficulties using traditional ways. Analytical approaches, on the other hand, were used to solve the nonlinear Riccati using Ansonian decomposition techniques (see reference). Tan and Abbasbandy used the Homotopy Analysis Method (HAM) to solve the quadratic Riccati equation once more. His thorough investigation of the variational iteration method (see references, and) established him as a pioneer.
The use of VIM to solve nonlinear problems has proved its usefulness and simplicity. Real-world concerns can be expressed in systems of differential equations, integral equations, algebraic equations, and differential equations developed from control problems using mathematical modeling. Because of the inherent complexity of analyzing the solutions to these models, numerical and approximative methods appear to be viable alternatives for overcoming these barriers. A large body of scholarly literature compares variational iteration methods (VIM) to other approximative and numerical techniques. When compared to the alternatives, VIM routinely demonstrates greater performance in terms of both speed and accuracy.

Nonetheless, the Riccati Matrix Differential Equation (RMDE), which is well recognized, is used in a wide range of applications. A variety of approaches can be used to obtain the analytical solution of RMDEs with constant coefficients. Watch out for T. Nguyen and others. The method proposed by Nguyen T.et al. is shown to be trustworthy and numerically effective. Recent developments in VIM implementation have been recognized, as stated in references. The results of matrix equations have been validated analytically and theoretically, and RMDE has been widely used as a control model for decades.

It is recommended that the reader refer to the following queries for further areas of implementation. VIM, an improved iteration of the generic Lagrange multiplier approach, has demonstrated the capacity to treat a wide range of nonlinear situations effectively and precisely.

Our research makes an innovative contribution by developing a variational iteration method that solves nonlinear terms in a differentiable manner with regard to the dependent variable and its derivatives. An improved version of the VIM software designed primarily to generate precise approximation numerical solutions for RMDE problems.

Solution of RMDE by VIM
The Riccati equation system is expected to be defined as follows: As described below, the recently proposed
The use of VIM to apply a correction function to the RMDE allows for the development of a sequence of iterations. \( z_0 = 0, 1, 2, \ldots \)

where \( \lambda \) is the Lagrange multiplier.

2. FORMULATION OF NEW VIM FOR SOLVING RMDES

Let’s start with the linear and nonlinear operators represented by \( F \) and \( N \). The new linear operator is defined as follows:

\[
\phi_0(t) + \phi_1(t) - \phi_0(t) + \phi_1(t) = 0
\]

where \( \phi_0(t) = \frac{\phi(t)}{a(t)} + f(t) \) with the components of the nonlinear operator \( \lambda \) necessary for the determination of the sequence for the RMDE as:

\[
\Lambda = \frac{\phi_0(t)}{a(t)} + f(t) - t - 1
\]

by decomposing the nonlinear operator \( \Lambda \) into two parts of linear and nonlinear respectively given by \( \Phi_0 \) and \( \Phi_1 \), where

\[
\phi_0(t) = \frac{\phi(t)}{a(t)} + f(t)
\]

The subsequent estimated generated sequence of solutions by the method with respect to the defined operators as in (5) nonlinear RMDE is defined by:

\[
\Phi_0(t) + \Phi_1(t) = 0
\]

where \( \Phi_1 \) is to be evaluated from the sequence. The correction functional is expressed as:

\[
\psi_0(t) = \psi_0(t) + \int_0^t (\psi_0(s) + \psi_1(s)) \, ds
\]

where \( \psi_0(t) \) is assumed as a modified variation with \( \delta \psi_0 \) = 0.

3. CONVERGENCE CRITERIA

Let's integrate by parts on \( \psi_1 \), we have:

\[
\psi_0(t) = \psi_0(t) + \int_0^t f(t) \, ds + \int_0^t f(t) \, ds
\]

By setting \( \delta \psi_0(t) = 0 \) and \( \delta \psi_1(t) = 0 \), The Euler-Lagrange result to the following differential equation: \( 1 + x(t) = 0 \)

With boundary condition: \( 1 + x(t) = 0 \).

The solution of (18) and (19) is \( x(t) = -1 \).

The iteration scheme is reduced to a sequence after the substitution \( x(t) = \psi_0 \) to (2)

\[
\psi_0(t) = \psi_0(t) + \int_0^t (\psi_0(s) - \psi_0(t)) \, ds
\]
Illustration of the new algorithm
To demonstrate how well the new method works, a simple example is used.

Consider the scalar RDE $y'(t) - t^2 = 1$ for $1.0 \leq t \leq 1$
with exact solution $y(t) = \frac{t^2}{2} + C$.

The new scheme of VIM for (27) using iteration defined in (20) with the initialisation point $y(0) = 0$:

\[
\begin{align*}
\eta_0 &= 0, \\
\eta_1 &= 0, \\
\eta_2 &= 0, \\
\eta_3 &= 0, \\
\eta_4 &= 0, \\
\eta_5 &= 0, \\
\eta_6 &= 0, \\
\eta_7 &= 0, \\
\eta_8 &= 0, \\
\eta_9 &= 0, \\
\eta_{10} &= 0.
\end{align*}
\]

Using the same iteration technique for $y_1(t), y_2(t), ..., y_{10}(t)$, The table below shows the exact solutions with the approximate solution errors.

<table>
<thead>
<tr>
<th>Steps of Iterations</th>
<th>Exact solution $y(t)$</th>
<th>Absolute errors at $y_0$</th>
<th>Absolute errors at $y_{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>1.00037371</td>
<td>2.11e-16</td>
<td>2.1105e-16</td>
</tr>
<tr>
<td>0.2</td>
<td>1.002419025</td>
<td>3.91e-13</td>
<td>1.6575e-15</td>
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<td>3.3987e-11</td>
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<tr>
<td>0.5</td>
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<td>2.5009e-8</td>
<td>2.9365e-9</td>
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<tr>
<td>0.6</td>
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<td>2.906e-7</td>
<td>3.4122e-7</td>
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<tr>
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<td>1.082727491</td>
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<tr>
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<tr>
<td>0.9</td>
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<td>9.3482e-4</td>
</tr>
<tr>
<td>1</td>
<td>1.2106</td>
<td>1.7338e-4</td>
<td>4.2658e-5</td>
</tr>
</tbody>
</table>
Fig 1: This is the ninth edition, which contains valid answers but obvious errors.

Fig 2: The ninth version has both exact corrections and complete errors.

4. CONCLUSION
The solution of the Riccati matrix differential equations was discovered using the upgraded VIM, which resulted in faster convergence to the exact answer and reduced error as the number of iterations approached zero.

REFERENCES