In Nano Topological Spaces, A New Concept of Generalized Nano Closed Sets

Dr.K.Dass
Assistant Profrosser,
Department of Mathematics,
The M.D.T Hindu College,Tirunelveli.
E-Mail Id:dassmdt@gmail.com

R.Uma Mageshwari
Research Scholar
Department of Mathematics,
The M.D.T Hindu College,Tirunelveli.
E-Mail Id: uma.mphil06@gmail.com
(Affiliated to Manonmaniam Sundaranar University, Tirunelveli – 627 012)

Abstract

This article contains, a new form of sets termed asn-D-open sets and n-D-closed sets on NanoTopological spaces as well as a few of the properties of the new types are studied.

Keywords: n-D- closed sets,n-D-open sets,n-D-closure.
DOI Number: 10.48047/NQ.2022.20.13.NQ88496 Neuroquantology 2022; 20(13): 4083-4092

Introduction

Closed sets in generalized form were first described by Levine [9] in 1970. This idea was first suggested as a generalization of closed sets in Spaces, which allowed for the introduction of new findings in general topology.

Regarding the subset H of a non-empty finite universe, which is described in the form oflower and upper approximations of H LellisThivagar[8] developed Nano spaces.

NO(nano- open) sets refer to the main components of nano spaces. He also provided definitions for NC(nano -closed) sets, NI(nano interiors), and NCL(nano- closures). Additionally, he investigated NSO(nano semi-Open), NO(nano-open), along with NPO(nano Pre-Open) sets, which are weaker variations of NO(nano open sets).

In this research, we establish new kinds of sets inNano -Topological spaces (NTS) termed n-D-open sets andn-D-closed and investigate the links between other nano sets.

I.Initiations:

Definition 1.1[8]

Let X be a non empty finite set of all objects termed as the relationship of universe, and let R be an equivalence relation on X termed as the relationship of indiscernibility. Then, X is partitioned into several disjoint equivalence classes. That elements in the same equivalence class are considered to be indistinguishable from each another. The approximation space is said to be the pair (X,R).Let $Z \subseteq X$.Then

i. LO-approximation of Z,as for R is the Set of objects, which may be classified as Z as for R in certain cases and it is denoted as $(LO)_R(Z)$.
(i.e) $(LO)_R(Z) = \bigcup_{z \in Z} \{R(z):R(z) \subseteq Z\}$,where R (z)denotes the equivalence class deterrmined by $z \in Z$.

ii. UP-approximation of Z as for R is the Set of all objects ,which may can be categorized as Z as for R and it is denoted by$(UP)_R(Z)$.

(i.e) $(UP)_R(Z)=. \bigcup_{z\in Z} \{R(z):R(z)\cap Z\neq \phi\}$

iii. BO-approximationof Zas for R is the Set of all objects ,which can be neither categorized as Znor as not Z as for R and it is denoted as $BO_R(Z)$.

(i.e) $BO_R(Z) = (UP)_R(Z) - (LO)_R(Z)$.

Property 1.2[8]

If (X,R) is an approximation space and Z,H$\subseteq$ X,then

i. $(LO)_R(Z) \subseteq Z \subseteq (UP)_R(Z)$.
ii. $(LO)_R(\phi) = (UP)_R(\phi) = \phi$.
iii. $(LO)_R (X) = (UP)_R(X) = X$.
iv. $(UP)_R(Z\cup H) = UP_R(Z)\cap UP_R(H)$.
v. $(UP)_R(Z\cap H) \subseteq (UP)_R(Z)\cup (UP)_R(H)$.
vi. $(LO)_R(Z\cup H) = (LO)_R(Z) \cup (LO)_R(H)$.
vii. $(LO)_R(Z\cap H) = (LO)_R(Z) \cap (LO)_R(H)$.
viii. $(LO)_R(Z) \subseteq (LO)_R(H)$ and $(UP)_R(Z) \subseteq (UP)_R(H)$,whenever $Z\subseteq H$.
ix. $(UP)_R(Z^c) = [(LO)_R(Z)]^c$and $(LO)_R(H^c) = [(UP)_R(Z)]^c$
x. $(LO)_R[(LO)_R(Z)] = (UP)_R[(LO)_R(Z)] = (LO)_R(Z)$.
xi. $(UP)_R(UP)_R(X)] =(LO)_R [(Up)_R(Z)] = (UP)_R(Z)$

Definition 1.3[8]

Let X be a non empty finite set of all objects called the relationship of universe, and let R be an relation of equivalenceon Xand $\tau_R(Z) =\{\phi, (LO)_R(Z), (UP)_R(Z),(BO)_R (Z),U\}$ .Then by property 1.1 $\tau_R(Z)$ satisfies the axioms

i. X and Ø arebelongs to $\tau_R(Z)$.
ii. The ($\cup$) of the components of (X, $\tau_R(Z)$)is to be found in the $\tau_R(Z)$.
iii. The ($\cap$) of the finite components of $\tau_R(Z)$ is to be found in $\tau_R(Z)$.

$\tau_R(Z)$form of topologycalled as the topology within Nano on X as for Z.We call (X,$\tau_R(Z)$) as the N-T opological spaces(NTS).The components of the N- topology are known as (N- open)sets in (X,$\tau_R(Z)$) and (N- open $)^c$ of the N- topology are known as N-closed sets in (X, $\tau_R(Z)$).

Remark 1.4[8]

If (X,$\tau_R(Z)$) is the nano topology on Z with respect to X,then the setB $=\{(LO)_R(H),(UP)_R(H),(BO)_R(H)\}$is the basis for $\tau_R(Z)$.

Definition 1.5[8]

Consider (X, $\tau_R(Z)$) , a N-topological space with respect to Z,where $Z\subseteq X$ and $H\subseteq X$.

i. The N-interior of the set H is represented by the notation N-int(H).**N-int(H)**, and is considered as the ($\cup$) of all N-open subsets present in H. The biggest N- open subset of H is N-int(H).

ii. The ($\cap$) of all N-closed sets containing H is known as the N-closure of the set H, and it is represented by the N-cl(H). The smallest N-closed set, N-cl(H) contains H.

Definition 1.6[7]

Let (X,$\tau_R(Z)$) be a N-topological spaceand H $\subseteq$X.Then H is said to be

i. **N- Pre-open** if $H\subseteq$ N-cl(N-int(H)).
ii. **N- Semi-open** if $H\subseteq$ N-int(N-cl(H)).
iii. **N- α-open**if $H\subseteq$ N-int(N-cl(N-int(H))).
iv. **N- regular open** if H= N-int(N-cl(H)).
v. **N- π –open** if it is finite ($\cup$) of all N-regular open sets.

Definition 1.7

Consider (X,$\tau_R(Z)$) be a N- topological spaces.A subset H is said to be

i. **Nano generalized closed set (N-g-closed)**[3] if N-cl(H)$\subseteq$V no matter when $H\subseteq V$ and here V is N-**o**pen in X. 4084
ii. **Nano Generalized pre-closed set(N-gp-closed)**[4] if N-pcl(H) $\subseteq$V no matter when $H\subseteq V$ and here V is N-open in X.
iii. **Nano π generalized pre -closed (N-πgp-closed)**[5] if N-pcl(H) $\subseteq$V no matter when, $H\subseteq V$ and V is N-π -open in X.
iv. **Nano Generalized $\wedge$ closed set(N-$\hat{g}$-closed)[6]** if N-cl(H) $\subseteq$V no matter when $H\subseteq V$ and V is N- semi open in X.

v. **Nano Generalized pre-regular closed set (N-gpr-closed)**[11] if N-pcl(H) $\subseteq$V no matter when, H$\subseteq$V and V is N-regular open in X.

vi. **Nano Generalized semi pre-closed set (N-gsp-closed)** [12] if N-spcl(H)$\subseteq$ V no matter when H$\subseteq$V and V is N-open in X.

Remark 1.8[14]

Regarding a subset H of a collection of N-spaces (X, τ_R(Z)),

i)N-cl(X–H) = X–N-int(H).

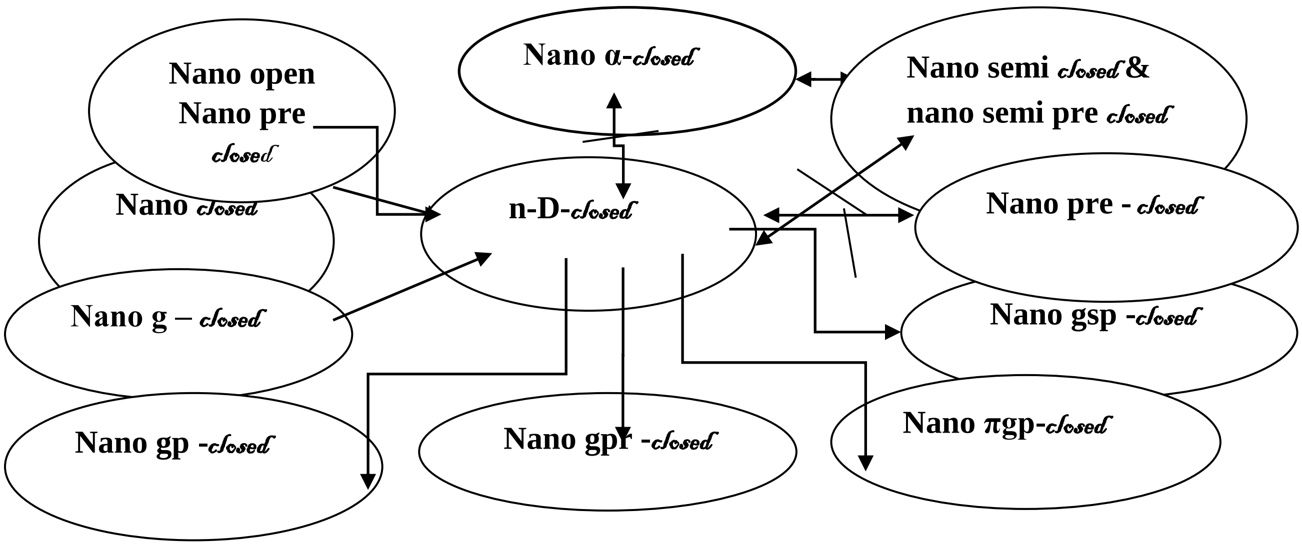
ii)N-int(X–H) = X–N-cl(H).

Lemma 1.9[8]

A subset H is N-$\hat{g}$ open iff if F$\subseteq$N-int(H) no matter when F is N- semi closed and F$\subseteq$H.

Remark 1.10

From the following diagram and can infer the following from our current findings. A$\rightarrow$B(A B) represents A implies B but not conversely (A and B are independent of each other)



II. n-D-closed sets, Fundamental Properties

Definition 2.1

A subset H is said to beNano-D-closed set (n-D-closed set)if N-pcl(H)$\subseteq$N-int(V),whenever H$\subseteq$Vand Vis N-$\hat{g}$ open in (X, τ_R(Z)).

Theorem 2.2

Each Nano closed is also n-D-closed

Proof

Let Hbe a nano closed set.

Let H$\subseteq$ V and V be a nano $\hat{g}$ open in X.

By lemma1.8,H$\subseteq$N-int(V).

Also,N-cl(H) = H (SinceH is nano closed)

Thus N-cl(H)$\subseteq$N-int(V).

Therefore,N-pcl(H) $\subseteq$N-int(V).

Thus, H is a n-D-closed set.

Remark 2.3

The reverse of the theorem 2.2 does not have to be correct.

Let X={p,q,r,s} with X/R={{r},{s},{p,q}}.

Let H={p,r}.

Then τ_R(Z) = {ϕ,{r},{p,q},{p,q,r},U}.

Thus {p,q} is n-D-closed set but not nano closed.

Theorem 2.4

Each nano pre- closed and nano open set of (X, τ_R(Z)) is n-D-closed set.

Proof:

Let H be nano open and nano pre closed set in (X, τ_R(Z)).

Let H$\subseteq$V and V is nano $\hat{g}$ open.

As of H is nano pre closed,N-pcl(H) = H and H is nano open, N-int (H) =H,That is,N-pcl(H) $\subseteq$N-int (V).

Thus H is n-D- closed set.

Remark 2.5

The reverse statement of the theorem 2.4 is not correct.
Let X ={p,s,u,y} with X/R={{p},{u},{s,y}.
Let H={p,s}.
Then τ_R(Z)= {ϕ,{p},{s,y},{p,s,y},X}.
Here {p} is nano open and nano pre- closed but not n-D-closed set.
Thus {u} is n-D-closed set but not nano open and nano pre-closed.

Theorem 2.6

Each Nano generalized (N-g-closed)closed set is an n-D-closed set.

Proof:

Let H be a nano generalized closed set.
Let H$\subseteq$ V and V be a nano $\hat{g}$ open.
As of His nano generalized closed set,N-cl(H)$\subseteq$V and V is nano open.
As of every nano closed set is nano pre -closed set,N-pcl(H)$\subseteq$N-cl(H).
That is,N-pcl(H)$\subseteq$N-cl(H)$\subseteq V$ = N-int(V)(Since V is nano open)
Therefore, N-pcl(H)$\subseteq$N-int(V)
Thus H is n-D-closed set.

Remark 2.7

The converse of the preceding theorem does not have to be true.
Let X={a,b,c,d} with X/R={{c},{d},{a,b}}.
Let H={a, c}.
Here {a,b} is n-D-closed set but not Nano generalized closed set.

Theorem 2.8

Each n-D-closed set is Nano generalized pre regular (N-gpr-closed)closed set.

Proof:

Let H be n-D-closed set.
Let H$\subseteq$V and V be a nano regular open.
As of each nano regular open is nano open and each nano open is nano $\hat{g}$ open.
ThereforeN-pcl(H) $\subseteq$N-int (V)$\subseteq$V
Therefore,N-pcl(H) $\subseteq$V.
Thus H is nano generalized pre regular closed set.

Remark 2.9

The converse of the preceding theorem does not have to be true.
Let X={p,q,u,w} with X/R={{u},{w},{p,q}}.
Let H={p,u}$\subseteq$X
Here {p,u} is N-gpr-closed set but not Nano-closed set.

Theorem 2.10

Each n-D-closed set is Nano generalized pre closed set (N-gp-closed set).

Proof:

Let H be n-D-closed set.
Let H$\subseteq$ V and V be a nano open.
As of each nano open is nano $\hat{g}$ open,we get N-pcl(H)$\subseteq$N-int(V) $\subseteq$V.
Thus H is Nano gp-closed.

Remark 2.11

The converse of the preceding theorem does not have to be true.
Let X={p,q,u,w} with X/R={{u},{w},{p,q}}.
Let H={p,u},
Here {p} is N-gp-closed set but not n-D-closed set.

Theorem 2.12

Each n-D-closed set is Nano generalized semi pre closed set (N-gsp-closed set).

Proof:

Let H be n-D-closed set.
Let H$\subseteq$V and V be a nano open.
As of each nano open is nano $\hat{g}$ open,we get N-pcl(H)$\subseteq$N-int(V) $\subseteq$V.
Therefore,N-spcl(H)$\subseteq$N-pcl(H)$\subseteq$V, we get H is N-gsp – closed set.

Remark 2.13

The converse of the preceding theorem does not have to be true.
Let X={a,b,c,d} with X/R={{c},{d},{a,b}}.
Let H={a,c},
Here H={b,c} is N-gsp-closed set but not n-D-closed set.

Theorem2.14

Each n-D-closed set is Nano πgp- closed set.

Proof:

Let H be n-D-closed set.
Let H $\subseteq$ V and V be a nano π-open.

As of each nano π-open is nano $\hat{g}$ open,we get N-pcl(H) $\subseteq$ N-int(V) $\subseteq$ V.
Thus H is Nano πgp- closed set.

Remark 2.15

The converse of the preceding theorem does not have to be true.
Let X={a,b,c,d} with X/R={{a},{c},{b,d}}.
Let H={a,b}
Here H={b}is nano πgp - closed set but not n-D-closed.

Remark 2.16

N- pre-closedness and n-D-closedness are separate concepts. This is verified by the example below.

Example 2.17

Let X={a,b,c,d} with X/R={{c},{d},{a,b}}.
Let H={a,c}.
Then $\tau_R(Z)$ ={X, ϕ ,{c},{a,b},{a,b,c}}
and the set H ={b}is Nano pre- closed but not n-D-closed set.

Remark 2.18

N-α-closedness and n-D-Closedness are separate concepts. This is verified by the example below.

Example2.19

Let X={a,b,c,d} with X/R= {{c},{d},{a,b}}.
Let H={a,c}
Then $\tau_R(Z)$ ={X, ϕ ,{c},{a,b},{a,b,c}}
The set H={a,d}is n-D-closed but not N- α-closed set.
Then the set H={b} is N-α-closed set but not n-D-closed.

Remark 2.20

Nano semi-closed and Nano semi-pre-closed sets have no influence on n-D-closed sets.
The example that follows demonstrates it.

Example 2.21

Let X={a,b,c,d}with X/R={{a},{b},{c,d}}.
Let H={a,d}.
Therefore, $\tau_R(Z)$ ={X, ϕ ,{c},{c,d},{a,c,d}}
The set H= {a} is both Nano semi closed and Nano semi pre closed but not n-D-closed set.
Let X={a,b,c,d}with X/R={{c},{d},{a,b}}.
Let H={a,c}.
Therefore, $\tau_R(Z)$= {ϕ,{c},{a,b},{a,b,c},U}.
The set H ={b,d}is n-D-closed but not both nano semi closed and nano semi pre -closed.

III. n-D-closed sets Properties

Definition 3.1

The intersection($\cap$) of each nano $\hat{g}$ open subsets of (X, $\tau_R(Z)$) holding A is called N $\hat{g}$ -kernel of A and it is represented by **N $\hat{g}$ - ker(A).**

Theorem 3.2

If a subset H of (X, $\tau_R(Z)$) is n-D-closed,then N-pcl(H) $\subseteq$ N $\hat{g}$ –ker(H).

Proof:

Assume H is n-D-Closed set.
Then N-pcl(H) $\subseteq$ N-int (V),H $\subseteq$ V and V is N $\hat{g}$ open set.
Let x $\in$ N-pcl(H)
Assume x $\notin$ N $\hat{g}$ - ker(H).
Then there exists a N $\hat{g}$ open V containing H such that x $\notin$ V.
SinceVis N $\hat{g}$ open set containing H, x $\notin$ N-pcl(H), which is contrary.

Example 3.3

Let X ={a,b,c,d}with X/R ={{c},{d},{a,b}}.
Let H = {a,c}.
Then $\tau_R(Z)$= { ϕ ,{c},{a,b},{a,b,c},Û}.
Let H={a}.Then N $\hat{g}$ - ker({a}) = {a}.

Remark 3.4

The converse of the preceding theorem does not have to be true.
Let H={b}.

Then N- $\hat{g}$ ker({b}) = {b}
Therefore,N-pcl(H) = {b} $\subseteq$ N- ker(H).
But H ={b} is not n-D-closed set.

Theorem3.5
If H and K are n-D-closed sets,then H $\cap$ K is n-D-closed set.
Proof:

Let H $\cap$ K $\subseteq$ V and V is nano $\hat{g}$ open.
As of H and K are n-D-closed sets, N-pcl(H) $\subseteq$ N-int (V)and N-pcl(K) $\subseteq$ N-int(V),
V is nano $\hat{g}$ open.
Also,N-pcl(H $\cap$ K) $\subseteq$ N-pcl(H) $\cap$ N-pcl(K) $\subseteq$ N-int (V).
Therefore,H $\cap$ K is a n-D-closed set.

Theorem 3.6
If H and K are n-D-closed sets,then H $\cup$ K is n-D-closed set.
Proof:

Let H $\cup$ Y $\subseteq$ V and V is nano $\hat{g}$ open.
As of H and K are N-D-closed sets, N-pcl(H) $\subseteq$ N-int (V) and N-pcl(K) $\subseteq$ N-int(V),
V is nano $\hat{g}$ open.
N-pcl(H $\cup$ K) = N-pcl(H) $\cup$ N-pcl(K) $\subseteq$ N-int (V).
Therefore,H $\cup$ K is a n-D-closed set.

Theorem 3.7
A set H is n-D-closed set $\Rightarrow$ N-pcl(H) –H contains no non-empty nano $\hat{g}$ closed set.
Proof:

Let F be a non-emptynano $\hat{g}$ closed set. such that F $\subseteq$ N-pcl(H) – H.
Then F $\subseteq$ N-pcl(H) andH $\subseteq$ X– F.
Since X–F is nano $\hat{g}$ open set and H is n-D-closed,N-pcl(A) $\subseteq$ N-int (X – F) = X– N-cl(F).
Thus N-cl(F) $\subseteq$ X– N-pcl(H).
Thus F $\subseteq$ X–N-pcl(H).
Therefore,F $\subseteq$ N-pcl(H) $\cap$ {X–N-pcl(H)}= ϕ.

Remark 3.8
The converse of the theorem 3.7 is not true.

Let X = {a,b,c,d}with X/R ={{a},{b},{c,d}}.
Let H= {a,d}.
Then τ_R(Z)={ϕ,{a},{c,d},{a,c,d},X}
Consider H ={a,d}.
N-pcl(H)-H ={b}.
$\Rightarrow$ N-pcl(H)- Hcontains no non- empty nano $\hat{g}$ closed sets.
But H= {a,d} is not n-D-closed set.

Theorem 3.9
If H is n-D-closed set and H $\subseteq$ K $\subseteq$ N-pcl(H),then K is n-D-closed set.
Proof:

Let K $\subseteq$ V, where V is nano $\hat{g}$ open.
Then H $\subseteq$ K $\Rightarrow$ H $\subseteq$ V.
As of H is N-D-closed,N-pcl(H) $\subseteq$ N-int(V).
Also, K $\subseteq$ N-pcl(H) $\Rightarrow$ N-pcl(K) $\subseteq$ N-pcl(H).
Therefore,N-pcl(H) $\subseteq$ N-int (V) and so K is n-D-closed set.

Theorem 3.10

If a subset H of (X, τ_R(Z)) is N $\hat{g}$ open and n- D-closed,then His Nano pre closed.
Proof:

Since H is Nano $\hat{g}$ open,we get N-pcl(H) $\subseteq$ N-int(H) $\subseteq$ H.
But H $\subseteq$ N-pcl(H).
Thus His nano pre- closed.

Theorem 3.11
A nano regular of (X, τ_R(Z)) is Nano gpr - closed iff H is n-D- closed in (X, τ_R(Z)).
Proof:
Necessity

Let H $\subseteq$ V and V is a nano $\hat{g}$ open in (X, τ_R(Z).
Since His nano regular open and nanogpr -closed,H is nano pre closed.
As of every nano regular open is nanoopen,we get H is nano open and nano pre-closed.
ThusH is n-D-closed.

Sufficiency
Assume H $\subseteq$ V and V be a nano regular open in (X, τ_R(Z))

Since every nano regular open is nano open,which is nano $\hat{g}$ open,we get N-pcl(H) $\subseteq$ N-int(V) $\subseteq$ V.
Thus His N-gpr-closed set.

Theorem 3.12
Let H be n-D-closed in (X, τ_R(Z)),then H is nano pre closed iff N-pcl(H) – H is nano $\hat{g}$ closed.
Proof:

Necessity
Let H be a nano pre closed.
Then N-pcl(H) = H.
Thus N-pcl(H) – H = ϕ ,which is nano $\hat{g}$ closed.

Sufficiency
Assume N-pcl(H) – H is nano $\hat{g}$ closed.
As of H is n-D-closed and by theorem 3.4, N-pcl(H) – H= ϕ .
Then N-pcl(H)= H.
Thus H is nano pre closed.

Theorem 3.13
A Nano open set H of (X, τ_R(Z))) is N-gp -closed iff H is n- D-closed.
Proof:
Let H be nano open and N-g-closed.
Let H $\subseteq$ V andV be nano $\hat{g}$ open in Û.
Since H is nanoopen, we get H= N-int(H) $\subseteq$ N-int (V).
As of N-int(V) is nano open,we get N-pcl(H) $\subseteq$ N-int (V).
Thus H is n-D-Closed.

Sufficiency
This part is true bytheorem 2.10.

Theorem 3.14
If a subset H of (X, τ_R(Z))) is nano open and nano regular closed,then H is n-D-closed.
Proof:
Let H be nano open and nano regular closed.
Since every nano regular closed is nano pre closed,we get by theorem 2.4, H is N-D-closed.

Theorem 3.15
In a N-topological space(X, τ_R(Z)),for each x $\in$ X,{x} is nano $\hat{g}$ closed (or)its complement X–{x} is n-D – closed in(X, τ_R(Z)).
Proof:
Assume that {x}is not nano $\hat{g}$ closed in (X, τ_R(Z))
Then X–{x} is nano $\hat{g}$ open and hence the only nano $\hat{g}$ open containing X–{x} $\subseteq$ X.
Then N-pcl(X–{x}) $\subseteq$ X.
Thus X-{x} is n-D-closed in(X, τ_R(Z))).
.

Definition 3.15: n-D-Closure of H
Let H be a subset of a nano topological space (X, τ_R(Z)).
The n-D-closure of H is defined as the ($\cap$) of all n-D-closed sets that are containing H and is denoted by
n-D-Cl(H).

Lemma 3.16
If a subset H of (X, τ_R(Z)) is n-D -closed,then H = n-D-Cl(A).
Proof:
If H is n-D- closed,then H is the smallest n-D- closed set containing itself.
Thus n-D-Cl(H) = H.

Remark 3.17
The converse of the preceding theorem does not have to be true. It is clear from the below example.
Example 3.18
Let X={a,b,c,d}with X/R={{c},{d},{a,b}}.
Let H $\subseteq$ Z={a,c} $\subseteq$ X. ϕ
Then, τ_R(Z)= {,{c},{a,b},{a,b,c},X}.
Let H={a}
Then n-D-Cl(H)={a}.
H= n-D-Cl(H)={a}.
But H={a} is a not a n-D-Closed set.

Definition 3.19
Let(X, τ_R(Z)) be a N- Topological space where Z$\subseteq$X,H$\subseteq$X, x$\in$X.Then, x is said to be a **nano limit point of H** if every nano open set containing x contains a point of H different from x.

Definition 3.20
The set of all nano limit points of H is said to be **nano derived set of H.** It is denoted by **n-D[H].**

Definition 3.21
Let (X, τ_R(Z)).be a N-topolgical space where Z $\subseteq$X,H$\subseteq$X, x$\in$X.Then x is said to be a **nanopre limit point of H** if every nanopre-open set containing x contains a point of H different from x.

Definition 3.22
The set of all Nano pre limit points of H is said to be **nanopre derived set of H.**
It isdenoted by **n-D$_P$[H].**

Theorem 3.23
If n-D[H]$\subseteq$n-D$_P$[H] for each subset H of a space(X, τ_R(Z)) ,then the union of n-D-closed sets is n-D-closed *set.*
Proof:
Let H and K be two n- D-closed sets is n- D-closed sets.
Let V be a N$\hat{g}$ open.set such that H$\cup$K$\subseteq$ V.
Then N-pcl(H)$\subseteq$N-int (V) and N-pcl(K) $\subseteq$ N-int (V).
Since for each subset H of X,n-D$_P$[H]$\subseteq$n-D[H].
Therefore,N-cl(H)$\subseteq$N-pcl(H) and N-cl(K)$\subseteq$ N-pcl(K).
Hence N-cl(H$\cup$K)=N-cl(H)$\cup$N-cl(K) = N-pcl(H)$\cup$N-pcl(K)$\subseteq$N-int (V).
ButN-pcl(H$\cup$K) $\subseteq$N-cl(H$\cup$K)
Hence N-pcl(H$\cup$K)$\subseteq$N-int (V).
Thus H$\cup$K is not n-D-closed sets.

Theorem 3.24
A subset H of ((X, τ_R(Z)) is nano regular open iff H is nano open and n- D-closed.

Proof:
Necessity
Assume H is nano open and n-D-closed set.
Then His nano $\hat{g}$ open and n-D-closed.
By theorem 3.10,H is nano pre- closed.
Hence N-cl(nint(H))$\subseteq$H.
Since H is Nano open and H = N-int(H),we get N-cl(H) = H.
Hence N-int(H) = N-int(N-cl(H)).
Thus H is nano regular open.

Sufficiency
Assume H be complement of nano regular cosed.
Then H is nano open.
Let V be Nano $\hat{g}$ open and H$\subseteq$V.
Assume N-pcl(H) $\not\subset$N-int(H).
Then H$\cup$N-cl(Nint(H))$\not\subset$N-int(V)
$\Rightarrow$N-cl(N-int(H))$\not\subset$N-int(V).
Since H is nano open and H=N-int(H),we get N-cl(H)$\not\subset$N-int(V),which is a contrary..

IV. n-D- open sets

Definition 4.1
A subset H of a N- topological spaces ((X, τ_R(Z)) is called n-D -open sets if its complement X – H is n- D-closed in (X, τ_R(Z))).
Theorem 4.2
Let (X, τ_R(Z))).be a N-topological space.Then, the following statement are hold.
i. Each nano open set is n-D-open.
ii. Each Nano pre-open is n-D-open.
iii. Each Nano g-open is n-D-open.
iv. Each n-D-open is Nano gpr-open.
v. Each n-D-open is Nano gp – open.
vi. Each n-D-open is Nano gsp-open.
vii. Each n-D-open is Nano πgp-open
Proof:
Proof follows from the previoustheorem 2.2,2.3,2.6.2.8,2.10,2.12,2.14.
Theorem 4.3
If H and K are two n-D-open,then H$\cap$K is n-D -open.
Proof:
Let H and K be two n-D-open sets.
Then X–Hand X–K are n-D-closed sets.

By theorem 3.6, X–H $\cup$ X–K are n-D-closed sets in X.
That is,X–(H $\cap$ K) is n-D-closed set in X.
Therefore, H $\cap$ K is n-D-open in X.

Theorem 4.4

If H and K are two n-D-open subsets , then H $\cup$ K is n-D -open.

Proof:

Let H and K be two N-D-open sets.
Then X– H and X – K are n-D-closed sets.
By theorem 3.5,(X– H) $\cap$ (X – K) are n-D-closed sets in X.
That is,X – (H $\cup$ K) is n-D-closed set in X.
Therefore, H $\cup$ K is n-D-open in X.

Theorem 4.5

Every nano-clopen subset of (X, τ_R(Z)), is n-D-open.

Proof:

Consider Hbe any nano-clopen subset of (X, τ_R(Z))).
Let X–H $\subseteq$ G and Gis nano $\hat{g}$ open in X.
As of every nano closed is nano pre -closed,we get N-pcl(H) = X– H= N-int (H) $\subseteq$ N-int (V).
Hence X–H is n-D -closed and hence H is n-D-open.

Theorem 4.6

Le (X, τ_R(Z)))be a N-topological spaces and H $\subseteq$ X.Then H is n- D-open $\Leftrightarrow$ N-cl(S) $\subseteq$ N-pint (H),whenever S $\subseteq$ H and S is nano $\hat{g}$ closed.

Proof:

Necessity

Let H be the n-D -open in (X, τ_R(Z))). Then S $\subseteq$ H and S is nano $\hat{g}$ closed.
Then X –H is n-D-closed and it is contained in the nano $\hat{g}$ open X – S.
Hence N-pcl(X–A) $\subseteq$ N-int(X-H)
That is, X–(N-pcl(H)) $\subseteq$ X–(N-cl(S)).
Hence N-cl(S) $\subseteq$ N-pint(H).

Sufficiency

If S is nano $\hat{g}$ closed set such that N-cl(S) $\subseteq$ N-pint(H) whenever S $\subseteq$ H.
It follows that X–H $\subseteq$ X–S and X–N-pint(H) $\subseteq$ X–(N-cl(S)).
Thus N-pcl(X–H) $\subseteq$ N-int(X-S).
Hence X–H is n- D-closed set and n-D -open set.

Theorem 4.7

If N-pint(H) $\subseteq$ K $\subseteq$ Hand H is n-D-open,then K is n-D-open.

Proof:

If N-pint(H) $\subseteq$ K $\subseteq$ H,then X– H $\subseteq$ X – K $\subseteq$ N-pint(H).
That is, X– H $\subseteq$ X –K $\subseteq$ N-pcl(X– H).
As of X – H is n-D-closed set and by theorem 3.5, X – K is n-D-closed.
Hence Kis Nano open.

Theorem 4.8

If H $\subseteq$ S is n-D-closed,then N-pcl(H) – His n-D-open.

Proof:

Let H be n-D-closed.
By theorem 3.7, N-pcl(H) – H contains no non – empty nano $\hat{g}$ closed.
Thus ϕ =S $\subseteq$ N-pcl(H) – H and ϕ =S be nano $\hat{g}$ closed.
Clearly N-cl(S) $\subseteq$ N-pint(N-pcl(H) – H.
Hence by theorem 4.5,N-pcl(H)-H is n-D-open.

References

[1] Adinatha C. Upadhya ,On quasi nano p-normal spaces ,International journal of recent Scientific Research-8(6)2017,17748-17751
[2] Antony Rex Rotrigo and K.Dass,A new type of generalized closed sets,International journal Mathematical Archieve-3(4),2012:1517-1523.
[3] K.Bhuvaneshwari and K.Mythili ,Nano Generalized closed sets,International Journal of Scientific and Research Publications ,4(5),2014,1-3.
[4] K.Bhuvaneshwari and K.Mythili ,nano Generalized Pre closed sets and nano Pre Generalized closed sets in nano Spaces,International journal of innovative Research in Science,Engineering and Technology,3(10),2014,16825-16829.
[5] Rajasekran ,Thangavel siva subramanian raja,On nano π gp-closed setsin Journal of New Theory.

[6] R.Lalitha and A.Francina shalini ,On nano Generalized ^ - closed sets and open sets in Nano spaces ,International journalof Applied Research,2017,3(8);368-371

[7] Lellis thivagar .M and Carmel Richard ,On nano forms of Weakly open sets International Journal of Mathematics and Statistical Invention ,Volume 1,issue 1,August 2013,pp(33-37).

[8] LellisThivagar and caramel Richard,Notes on nano Spaces (Communicated)

[9] Levine. N. Generalized closed sets in Topology ,Rend Cire,Math Palermo,1963,19(2);89-96.

[10]Njastad O, On some classes of nearly open sets, Pacific J. Math, 15(1965); 961-970.

[11]R.Parvathy and S.Praveena ,On nano generalized pre Regular closed sets in nano Spaces,IOSR journalof Mathematics (IOSR-JM),13(2)(2017)pp(56-60).

[12]Shalini S.B and Indira Rani.K ,On nano generalized – semi pre closed sets and nano semi pre- generalized closed set in nano Spaces ,International journal of Physics and mathematical sciences ,2015,vol5(3) pg 27-31.

[13]VeerakumarMKRS closed sets and LC-functions,IndianJ Math,2001,43(2); 231-247.

[14]M.Lellis and caramel ,Notes on Spaces.(Communicated)