THE STABILITY OF SHEAR FLOW OF VISCOUS ELECTRICALLY CONDUCTING FLUID IN THE PRESENCE OF VELOCITY AND MAGNETIC SHEARS

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Abstract
The linear stability of shear flow of an incompressible, viscous electrically conducting fluid permeated by sheared magnetic field is investigated. An unbounded two layer model consisting of different viscosity and magnetic diffusivity fluids with different velocity shear and magnetic shears is examined for the two dimensional disturbances. An analytical study using the short wavelength approximation shows that the configuration is always unstable for different diffusivities and for different shears. When the magnetic field does not vanish on the interface it may be stabilizing or destabilizing the system depending on the values of certain parameters.

Key words: Stability, shear flow,electrically conducting fluid,unbounded two layer model magnetic shear, shortwavelength approximation
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1. Introduction

When a fluid is driven away from thermal and mechanical equilibrium, it will undergo a sequence of instabilities each of which leads, to a change in spatial or temporal structure. Transition from laminar to turbulent flow begins with the instability of flow. The stability of shear flows in the presence of applied magnetic field is important in geophysics and astrophysics. This study is based on the previous investigations by Drazin& Reid[4], Hooper & Boyd[6],Yih[11]. It was shown by Yih[11] that instability can occur when two co-flowing fluids have different viscosities. Hooper & Boyd [6] considered the stability analysis of two unbounded linear viscous shear flows with different shears, showed that the configuration is unstable when the fluids are of different viscosities and shears. In the presence of discontinuity in the electrical conductivity, a sheared magnetic field can give rise to a new instability. For two superimposed fluids of different electrical conductivities it was shown by Sneyd[9], Davidson & Lindsay[3], Bhattaacharya & Gupta[2] configuration is unstable when there is continuous or discontinuous variation in electrical conductivity.

Eun-jin Kim[5] studied the effect of flow shear and magnetic shear in the three-dimensional reduced magneto hydrodynamic turbulence. It was analytically shown that near the resonance surface, transport quenching by flow shear was weakened by magnetic shear as the latter does not involve with shearing process. Jyoti etal.[7] studied the influence of the magnetic field shear on the $E \times B$ (and/or gravitational) and the current convective instabilities occurring in the high-latitude ionosphere. It was shown that magnetic shear reduces the growth rate of these instabilities.Soumaya Hadj Salah etal.[8] studied the effect of heat transfer on shear flows around an obstacle which is useful in

determining the influence of water on buildings and port infrastructures. They obtained results supported by numerical simulations and concluded that the doubling of the fluid inlet temperature significantly modifies all the dynamic characteristics of the shear flow. These results is useful to exploit the flow of hot water discharged by power plants.TimourRadko[10]studied a systematic stability analysis of unsteady shear flows representing large-scale, low-frequency internal waves in the ocean. The analysis was based on the unbounded time-dependent Couette model. Linear analysis suggested that time-dependent spatially uniform shears were unstable regardless of the Richardson number. Alexandru & Hao Jia [1] proved asymptotic stability of the couette flow for the two-dimensional Euler equations in the domain . They proved that a small and smooth perturbation of the couette flow resulted in the velocity field which converges strongly to a nearby shear flow.
In this study, we considered the combined effect of velocity as well as magnetic shears on the stability of the interface formed by two unbounded shear flows of different viscosity as well as electrical conductivities and have determined the criteria for the growth rate, by using a regular perturbation analysis.

2. Mathematical Formulation

We consider the two co-flowing viscous unbounded electrically conducting shear flows separated by the interface at y = 0 in the presence of sheared magnetic field as shown in figure (1). We assume both the fluids are incompressible. The governing equations in each fluid are

$$\frac{\partial \vec{q}}{\partial t'}+(\vec{q}.\nabla)\vec{q} = -\frac{1}{\rho}\nabla P+\frac{1}{\rho\,\mu_0}(\vec{B}.\nabla)\vec{B}+\nu\nabla^2\vec{q} \quad (1)$$

$$\nabla.\vec{q}=0 \quad (2)$$

$$\frac{\partial \vec{B}}{\partial t'}+(\vec{q}.\nabla)\vec{B} = (\vec{B}.\nabla)\vec{q}+\lambda\nabla^2\vec{B} \quad (3)$$

$$\nabla.\vec{B} = 0 \quad (4)$$

where $P=p+\frac{\mu_m H^2}{2}$ is the total pressure. μ_0 is the magnetic permeability of the fluid. ρ, ν and λ are the density, kinematic viscosity and magnetic diffusivity of the fluid, $\vec{q}$ is the fluid velocity, $\vec{B}$ is the magnetic field.

In the unperturbed state, $\vec{q}=(U(y)=\sigma y',0)$, $\vec{B}=(B_0+s\,y',0)$. $P=P_0$, where σ, s, B_0, P_0 are constants, σ and s are respectively the shear intensity of the mean flow and the magnetic shear intensity. We now consider a two-dimensional perturbation given by $\vec{q}=(\sigma y'+u',v')$, $P=P_0+P'$, $\vec{B}=(B_0+s\,y'+b_x',b_y')$.

Inthe unperturbed state, balance of normal stress and continuity of the tangential component of the magnetic field require

$$(B_0)_1=(B_0)_2,\ (P_0)_1=(P_0)_2. \quad (5)$$

Whereas the continuity of the tangential component of the electric field and velocity field require

$$\lambda_1 s_1=\lambda_2 s_2,\quad \mu_1\sigma_1=\mu_2\sigma_2 \quad (6)$$

where s_1, s_2 are magnetic shears, σ_1, σ_2 are velocity shears, μ_1, μ_2 are viscous diffusivities and λ_1, λ_2 are magnetic diffusivities in fluid 1 and 2 respectively.

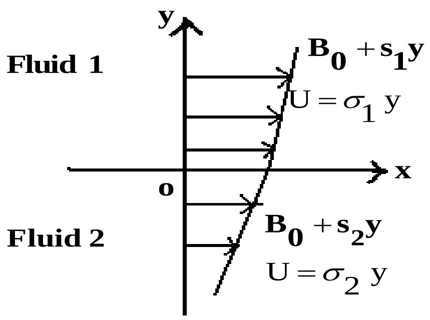

Fig 1. A sketch of the physical problem.

Let $y'=\eta'(x',t')$ be the equation of the interface between the two fluids in the perturbed state. The kinematic boundary condition at the interface, after linearizing and assuming normal modes as before, gives

$v' = -i\alpha' c' \eta'$ at $y' = 0$. (7)

We now impose the requirement of continuity of the tangential and normal components of velocity, shear and normal stress, tangential and normal components of the magnetic field and tangential component of the electrical field at the perturbed interface. We linearize and assume normal modes each of the dependent variables with (x', t') -dependence in the form $\exp\left[i\alpha'(x' - c't')\right]$,

We now introduce a stream function $\psi(y')$ and $\phi(y')$ for the magnetic field such that

$$u' = \frac{d\psi}{dy'},\; v' = -\frac{d\psi}{dx'},$$

$$b'_x = \frac{d\phi}{dy'}, b'_y = -\frac{d\phi}{dx'} \quad (8)$$

The linearized equations by eliminating pressure reduces to

$$i\alpha'(\sigma y' - c')\left(\frac{d^2}{dy'^2} - \alpha'^2\right)\psi = v\left(\frac{d^2}{dy'^2} - \alpha'^2\right)^2 \psi$$

$$+ \frac{i\alpha'}{\rho\mu_0}\left(B_0 + s y'\right)\left(\frac{d^2}{dy'^2} - \alpha'^2\right)\phi \quad (9)$$

$$i\alpha'(\sigma y' - c')\phi = i\alpha'\left(B_0 + s y'\right)\psi$$

$$+\lambda\left(\frac{d^2}{dy'^2} - \alpha'^2\right)\phi \quad (10)$$

The requirement that the perturbations vanish as $y' \to \pm\infty$, together constitute the eigenvalue problem governing linear stability. The length scale L and time scale T are defined as

$$L = \left(\frac{\lambda_2^2 \mu_0 \rho_2}{s_2^2}\right)^{\frac{1}{4}},\; T = \frac{\left(\mu_0 \rho_2\right)^{\frac{1}{2}}}{s_2}$$

and non-dimensional variables are defined as

$$\left(X, Y, 1/\alpha\right) = \left(\frac{s_2^2}{\lambda_2^2 \mu_0 \rho_2}\right)^{1/4}\left(x', y', 1/\alpha'\right)$$

$$\left(\varphi_1, \varphi_2\right) = \left(\frac{1}{\mu_0 \rho_2}\right)^{1/2}\left(\phi'_1, \phi'_2\right),$$

$$\left(\psi_1, \psi_2\right) = \left(\frac{1}{\mu_0 \rho_2}\right)^{1/2}\left(\psi'_1, \psi'_2\right).$$

$$C = \left(\frac{\mu_0 \rho_2}{\lambda_2^2 s_2^2}\right)^{1/4} c. \quad (11)$$

The rescaled coordinates and a rescaled phase speed are defined by

$$(x, y) = \alpha(X, Y),\; C_1 = \alpha C \quad (12)$$

Substituting the above into equations (8) and (9) and writing the equations separately for the two fluids, we have

$$\left(\frac{d^2}{dy^2} - 1\right)^2 \psi_1 = -\frac{i}{\alpha^2}\frac{m}{r P_2}\left\{r(\alpha M + \chi y)\varphi_1\right.$$

$$\left.+\left(C_1 - N_1 Q y\right)\left(\frac{d^2}{dy^2} - 1\right)\psi_1\right. \quad (13)$$

$$\left(\frac{d^2}{dy^2} - 1\right)^2 \psi_2 = -\frac{i}{\alpha^2}\frac{m}{r P_2}\left\{r(\alpha M + y)\varphi_2\right.$$

$$\left.+\left(C_1 - N_2 Q y\right)\left(\frac{d^2}{dy^2} - 1\right)\psi_2\right. \quad (14)$$

$$\left(\frac{d^2}{dy^2} - 1\right)\varphi_1 = -\frac{i}{\alpha^2}\chi\left\{(\alpha M + \chi y)\psi_1\right.$$

$$\left.+\left(C_1 - N_1 Q y\right)\varphi_1\right\} \quad (15)$$

At $y' = 0$, it follows that $\psi_1 = \psi_2$ and $\phi_1 = \phi_2$

$$\left(\frac{d^2}{dy^2} - 1\right)\varphi_2 = -\frac{i}{\alpha^2}\left\{(\alpha M + y)\psi_2\right.$$

$$\left.+\left(C_1 - N_2 Q y\right)\varphi_2\right\} \quad (16)$$

Here

$$m = \frac{\mu_2}{\mu_1} = \frac{\sigma_1}{\sigma_2} \quad , \quad r = \frac{\rho_2}{\rho_1}, \quad \chi = \frac{\lambda_2}{\lambda_1} = \frac{s_1}{s_2},$$

$$P_2 = \frac{\gamma_2}{\gamma_1}, \qquad N_1 = \frac{\sigma_1}{s_2}, \qquad N_2 = \frac{\sigma_2}{s_2},$$

$$M = \frac{B_0}{\left(\mu_0 \rho_2 \lambda_2^2 s_2^2\right)^{1/2}}, Q = \left(\mu_0 \rho_2\right)^{1/2}$$

$$S = \frac{T}{\rho_2 \lambda_2} \left(\frac{\mu_0 \rho_2}{\lambda_2^2 s_2^2} \right)^{1/4} \tag{17}$$

Where M and S are the magnetic and surface tension parameters. Further $\psi_1 \to 0$, $\varphi_1 \to 0$ as $y \to \infty$;

$\psi_2 \to 0$, $\varphi_2 \to 0$ as $y \to -\infty$. (18)

3. A Regular Perturbation Analysis for Short Wavelength

From the equations (13) – (16) and the boundary conditions it is evident that $1/\alpha^2$ can be used as an expansion parameter for carrying out a regular perturbation analysis for disturbances of short wavelength. Accordingly, we assume the expansions

$$\psi_1(y) = \sum_{n=0}^{\infty} \frac{a_n(y)}{\alpha^{2n}} e^{-y},$$

$$\psi_2(y) = \sum_{n=0}^{\infty} \frac{b_n(y)}{\alpha^{2n}} e^{y},$$

$$\varphi_1(y) = \sum_{n=0}^{\infty} \frac{g_n(y)}{\alpha^{2n}} e^{-y},$$

$$\varphi_2(y) = \sum_{n=0}^{\infty} \frac{d_n(y)}{\alpha^{2n}} e^{y},$$

$$C_1 = \sum_{n=0}^{\infty} \frac{c_n}{\alpha^{2n}} \tag{19}$$

We substitute from (19) into (13)-(16) and (17), to obtain the zeroth order solutions of the problem

$$a_0(y) = 0, b_0(y) = 0, g_0(y) = K_0,$$

$$d_0(y) = K_0, \tag{20}$$

where K_0 is a non-zero constant. We find that

$$C_0 = 0 \tag{21}$$

The first order perturbation solutions are

$$a_1(y) = c_1 + c_2 y - \frac{i m^2}{P_2} (\alpha M + \chi y) K_0 \tag{22}$$

$$b_1(y) = c_3 + c_4 y - \frac{i m^2}{P_2} (\alpha M + y) K_0, \tag{23}$$

$$g_1(y) = c_5 + c_6 y - i \chi \left(C_1 - N_1 Q y\right) K_0, \tag{24}$$

$$d_1(y) = c_7 + c_8 y + i \left(C_1 - N_1 Q y\right) K_0. \tag{25}$$

Using the boundary conditions, we determine the eigenvalue C_1, given by

$$C_1 = \frac{i}{4P_2} \left(\frac{m}{1+m} \right) \left(2 \alpha M (\chi - 1) + (\chi - 1)^2 - 2 S \alpha^3 + 2 P_2 \left(\frac{1-m}{1+m} \right) \left(1 - \frac{m^2}{r} \right) - 2 P_2 S \alpha^3 \right) \tag{26}$$

It can be seen that at $O\left(\alpha^{-2}\right)$, the configuration will be stable or unstable depending on whether

$$\left(2 \alpha M (\chi - 1) + (\chi - 1)^2 - 2 S \alpha^3 - 2 P_2 S \alpha^3 + 2 P_2 \left(\frac{1-m}{1+m} \right) \left(1 - \frac{m^2}{r} \right) \right) < \text{or} > 0 \tag{27}$$

This shows that if S = 0, M = 0, $\nu = 0$ the configuration is always unstable provided $\chi \neq 1$, when the magnetic field does not vanish. If $M \neq 0$ it may have a stabilizing or destabilizing effect depending on the sign of M. The dimensional growth rate, correct to first order in $1/\alpha^2$ is given by

$$\alpha C = \left(\frac{s_2^2}{\mu_0 \rho_2}\right)^{1/2} \frac{1}{\alpha^2} \frac{i}{4P_2} \left(\frac{m}{1+m}\right) \Big(2\alpha\, M(\chi-1) + (\chi-1)^2 - 2S\alpha^3$$
$$+ 2P_2 \left(\frac{1-m}{1+m}\right)\left(1-\frac{m^2}{r}\right) - 2P_2 S \alpha^3 \Big) \qquad (28)$$

From equation (28) it can be seen that the growth rate vanishes when $s_2 = 0$. The condition for stability or instability given by (27) holds, provided the magnetic shears s_1 and s_2 are positive.

4. Results and Discussion

It is shown that an unbounded configuration of viscous electrically conducting parallel flows permeated by a sheared magnetic field is always unstable for short-wavelength disturbances (in the absence of surface tension and viscosity) if the magnetic field vanishes at the interface and the magnetic diffusivities of the two fluids are different.

The graphical representations of growth rate $Im(C_1)$ against the values of α for various values of the other parameters are shown in figures 2 , 3 and 4. Figure 2 shows that the maximum growth rate occurs for shorter wavelengths as χ increases. Figure 3 shows the growth rates for different magnetic Prandtl numbers P_2 with r = 1, m = 2, χ = 2, S = 0.001 and M = 0 . Figure 4 shows the growth rates for differen tviscosity ratios m for m = 0.1 and m = 10 with r = 1, χ = 2, S = 0.001, M = 0 and P_2 = 0.025. We find maximum growth rate for $\alpha = O(1)$ and is in agreement with equation (25) for short wave lengths.Figure 3 and 4 shows that the largest growth rate shifts to longer wavelengths with increase in P_2 or decrease in m.

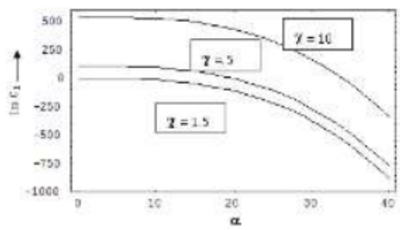

Fig 2. Growth rate Im (C_1) vs. wavenumber α for different values of magnetic diffusivity ratios χ with r = 1, m = 2, S = 0 , M = 0 and P_2 = 0.025.

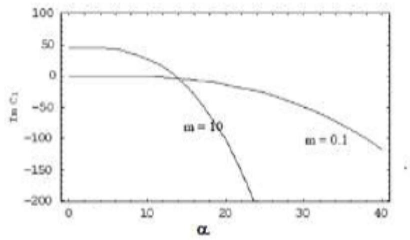

Fig 3. Growth rate Im (C_1) vs wavenumber α for different magnetic Prandtl numbers P_2 with r = 1, m = 2, χ = 2, S = 0.001 and M = 0

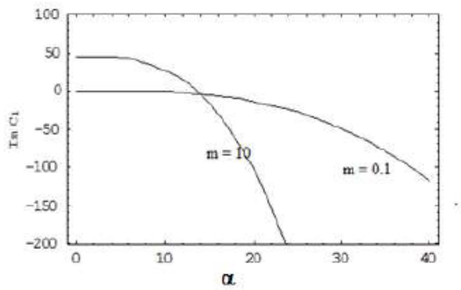

Fig 4.Growth rate Im (C_1) vs. wavenumber α for different viscosity ratios m with r = 1, χ = 2, S = 0.001 and M = 0 and P_2 = 0.025.

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