



AN INDUSTRIAL PRODUCTION INVENTORY MODEL WITH DETERIORATION UNDER NEUTROSOPHIC FUZZY OPTIMIZATION

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ABSTRACT

In this modern era, most of the industries are gradually shifting towards industry 4.0. This paper mainly gives a production inventory model with deterioration focusing on the paradigm shift towards smart production process that involves many new types of cost parameters on top of conventional inventory cost. This Industry 4.0 production inventory Model using Neutrosophic fuzzy system is discussed here and has been compared with deterministic model. The trapezoidal neutrosophic number representation of the parameters enhances the efficiency of the model in determining the optimal order time which minimizes the total costs. The model is highly comprehensive in nature and it is validated with a numerical example

KEYWORDS: - Neutrosophic sets, Industry 4.0, production inventory model, optimization.

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INTRODUCTION

Currently various sectors in the industries are acclimating the new approach of digitalization to meet the daily necessities of the demands that the customer's wants to account with. The production sectors practices new methods of production to ease out the process of production that amounts to several subsequent steps and new parameter costs. The optimizing principle of production companies is to minimize the costs of production and maximising the profit and the inventory models are promoted to take optimal decisions on order quantity and order time. The Economic Production Quantity (EPQ) model proposed by Taft [1], a basic production inventory model to manage the levels of inventory by the production sectors. This model is an accentuate model and it was advanced and extended based upon decision-making situations. The fundamental EPQ

model was further modified with the integration of the cost parameters of shortages, trade discount, imperfect items, supply chain, deteriorating items, remanufacturing, waste disposal and so on. The production inventory models are extended to cater the requirements of the production sectors. Presently, the fourth industrial revolution is gaining significance amidst the developed and developing nations. Industry 4.0 will certainly bring a paradigm transition at all the levels of organization and control over the different stages of the product's life. The entire process of product production beginning from product conception, product design, product development, initialization of product production, manufacturing of the product, product delivery and ending with product rework, recycle and disposal will get into the digitalized mode based on customer-centric approach.



The elements of Industry 4.0 are stepping into the production sectors of large, medium and small- sized and at all phases of production processes. Christian Decker et al [2] introduced a cost-benefit model for smart items in the supply chain which is an initial initiative in calculating the advantages of introducing smart items into the network of the supply chain. Andrew Kusiak [3] presented the benefits of smart manufacturing; its core components and the production pattern in future. Fei et al

[4] developed IT -driven assistance arranged shrewd assembling with its structure and attributes. Sameer et al [5] introduced a basic audit on keen assembling and Industry 4.0 development models and the suggestions for the entrance of medium and little enterprises. Xiulong et al [6] developed CPS-based smart production system for Industry 4.0 based on the review of the existing literature on smart production systems. Pietro et al [7] built up a digital flexibly chain through the powerful stock and smart agreements. Marc Wins[8] introduced a wide depiction of the highlights of a smart stock administration framework. Souvik et al [9] investigated the savvy stock administration framework dependent on the web of things (lot). Poti et al [10] introduced the prerequisite examination for shrewd flexibly chain the board for SMEs. Ghadge et al [11] tended to the effect of Industry 4.0 execution on flexibly chains; introduced the benefits and confinements of industry 4.0 in supply chain arrange alongside its cutting-edge headings; clarified the core Industry 4.0 innovations and their business applications and investigated the ramifications of Industry 4.0 with regards to operational and gainful proficiency. Iqra Asghar et al [12] presented a digitalized smart EPQ-based stock model for innovation subordinate items under stochastic failure and fix rate. The above examined stock models are deterministic in nature and the costs boundaries are traditional in nature and they don't mirror the real costs boundaries identified with industry 4.0 components.

In this paper, manufacturing inventory model incorporating a new range of smart costs is formulated, also in this industry 4.0 model, the cost parameters are characterized as

neutrosophic sets. This is the novelty of this research work and as for as the literature is concerned, industry 4.0 neutrosophic production inventory models have not been discussed so far and related literature does not exist. Smarandache [13] introduced neutrosophic sets that deal with truth, indeterminacy and falsity membership functions. Neutrosophic sets are widely applied to handle the situations of indeterminacy and it has extensive applications in diverse fields. Sahidul et al [14] developed neutrosophic goal programming for choosing the optimal green supplier, Abdel Nasser [15] used an integrated neutrosophic approach for supplier selection, Lyzbeth [16] constructed neutrosophic decision- making model to determine the operational risks in financial management, Ranjan Kumar et al [17,18] developed neutrosophic multi-objective programming for finding the solution to shortest path problem, Vakkas et al [19] proposed MADM method with bipolar neutrosophic sets.

Abdel-Basst, Mohamed et al [20] has developed neutrosophic decision-making models for effective identification of COVID-19; constructed bipolar neutrosophic MCDM for professional selection [21]; formulated a model to solve supply chain problem using best-worst method [22] and to measure the financial performance of the manufacturing industries [23]. Also, Abdel-Basset proposed presented a new framework for evaluating the innovativeness of the smart product – service systems [24]. As neutrosophic sets are highly viable, neutrosophic inventory models are formulated by many researchers. Chaitali Kar et al [25] developed inventory model with neutrosophic geometric programming approach. Mullai and Broumi[26] discussed neutrosophic inventory model without shortages, Mullai [27] developed neutrosophic model with price breaks. Mullai et al [28] constructed neutrosophic inventory model dealing with single-valued neutrosophic representation.

In all these neutrosophic inventory models, the cost parameters of the conventional inventory models are represented as neutrosophic sets or numbers, but these models did not discuss any



new kind of cost parameters reflecting the transitions in the production processes. But the proposed model reflects the paradigm shift towards smart production process and incorporates new kinds of costs to cater the requirements of smart production inventory model. The industry 4.0 neutrosophic production inventory model with the inclusion of the respective costs to the core elements of smart production systems is highly essential as the existing production sectors are adapting to the environment of smart production set up, but to the best of our knowledge such models are still uncovered. This model primarily focuses on increase productivity and high quality of the product within low investment of finance. The composition of several components of industry 4.0 production inventory model result in diverse costs parameters such as smart ordering cost , internet connectivity initialization cost, holding costs ,smart product design cost, data management cost, customer data analysis cost, supplier data analysis cost, smart technology cost, production monitoring cost, reworking cost, smart training work personnel cost, smart tools purchase cost , smart disposal costs , smart environmental costs, holding cost. The term smart refers to the costs incurred with the integration of digital gadgets to the respective production departments.

The article is structured into the following sections: section 2 consists of the preliminary definitions of neutrosophic sets and its arithmetic operation; section 3 presents the industry 4.0 production inventory model; section 4 validates the proposed model with neutrosophic parameters; section 5 discusses the results and the last section concludes the paper.

Basics of Neutrosophic sets and operations

This section presents the fundamentals of neutrosophic sets, arithmetic operations and defuzzification

Neutrosophic set [13]

A neutrosophic set is characterized independently by a truth-membership function $\alpha(x)$, an indeterminacy-membership

function $\beta(x)$, and a falsity-membership function $\gamma(x)$ and each of the function is defined from $X \rightarrow [0,1]$

Single valued Trapezoidal Neutrosophic Number

A single valued trapezoidal neutrosophic number $\tilde{A} = \langle (a, b, c, d): \rho_A, \sigma_A, \tau_A \rangle$ is a special neutrosophic set on the real number set R , whose truth –membership, indeterminacy-membership, and a falsity –membership is given as follows

$$\mu_{\tilde{A}}(x) = \begin{cases} (x-a)\rho_{\tilde{A}}/(b-a) & (a \leq x < b) \\ \rho_{\tilde{A}} & (b \leq x < c) \\ (d-x)\rho_{\tilde{A}}/(d-c) & (c < x \leq d) \\ 0 & \text{otherwise} \end{cases}$$

$$\pi_{\tilde{A}}(x) = \begin{cases} (b-x+\sigma_{\tilde{A}}(x-a))/(b-a) & (a \leq x < b) \\ \sigma_{\tilde{A}} & (b \leq x < c) \\ (x-c+\sigma_{\tilde{A}}(d-x))/(d-c) & (c < x \leq d) \\ 1 & \text{otherwise} \end{cases}$$

$$\varphi_{\tilde{A}}(x) = \begin{cases} (b-x+\tau_{\tilde{A}}(x-a))/(b-a) & (a \leq x < b) \\ \tau_{\tilde{A}} & (b \leq x < c) \\ (x-c+\tau_{\tilde{A}}(d-x))/(d-c) & (c < x \leq d) \\ 1 & \text{otherwise} \end{cases}$$

Operations on Single valued Trapezoidal Neutrosophic Number

Let $\tilde{A} = \langle (a_1, b_1, c_1, d_1): \rho_{\tilde{A}}, \sigma_{\tilde{A}}, \tau_{\tilde{A}} \rangle$ and $\tilde{B} = \langle (a_2, b_2, c_2, d_2): \rho_{\tilde{B}}, \sigma_{\tilde{B}}, \tau_{\tilde{B}} \rangle$ be two single valued trapezoidal neutrosophic numbers and $\mu \neq 0$, then

1. $\tilde{A} + \tilde{B} = \langle (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2): \rho_{\tilde{A}} \wedge \rho_{\tilde{B}}, \sigma_{\tilde{A}} \vee \sigma_{\tilde{B}}, \tau_{\tilde{A}} \vee \tau_{\tilde{B}} \rangle$
2. $\tilde{A} - \tilde{B} = \langle (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2): \rho_{\tilde{A}} \wedge \rho_{\tilde{B}}, \sigma_{\tilde{A}} \vee \sigma_{\tilde{B}}, \tau_{\tilde{A}} \vee \tau_{\tilde{B}} \rangle$
3. $\tilde{A} \tilde{B} = \begin{cases} \langle (a_1 a_2, b_1 b_2, c_1 c_2, d_1 d_2): \rho_{\tilde{A}} \wedge \rho_{\tilde{B}}, \sigma_{\tilde{A}} \vee \sigma_{\tilde{B}}, \tau_{\tilde{A}} \vee \tau_{\tilde{B}} \rangle & (d_1 > 0, d_2 > 0) \\ \langle (d_1 d_2, b_1 c_2, c_1 b_2, d_1 a_2): \rho_{\tilde{A}} \wedge \rho_{\tilde{B}}, \sigma_{\tilde{A}} \vee \sigma_{\tilde{B}}, \tau_{\tilde{A}} \vee \tau_{\tilde{B}} \rangle & (d_1 < 0, d_2 > 0) \\ \langle (d_1 d_2, c_1 c_2, b_1 b_2, a_1 a_2): \rho_{\tilde{A}} \wedge \rho_{\tilde{B}}, \sigma_{\tilde{A}} \vee \sigma_{\tilde{B}}, \tau_{\tilde{A}} \vee \tau_{\tilde{B}} \rangle & (d_1 < 0, d_2 < 0) \end{cases}$
4. $\tilde{A}/\tilde{B} = \begin{cases} \langle (a_1/d_2, b_1/c_2, c_1/b_2, d_1/a_2): \rho_{\tilde{A}} \wedge \rho_{\tilde{B}}, \sigma_{\tilde{A}} \vee \sigma_{\tilde{B}}, \tau_{\tilde{A}} \vee \tau_{\tilde{B}} \rangle & (d_1 > 0, d_2 > 0) \\ \langle (d_1/d_2, c_1/c_2, b_1/b_2, a_1/a_2): \rho_{\tilde{A}} \wedge \rho_{\tilde{B}}, \sigma_{\tilde{A}} \vee \sigma_{\tilde{B}}, \tau_{\tilde{A}} \vee \tau_{\tilde{B}} \rangle & (d_1 < 0, d_2 > 0) \\ \langle (d_1/a_2, c_1/b_2, b_1/c_2, a_1/d_2): \rho_{\tilde{A}} \wedge \rho_{\tilde{B}}, \sigma_{\tilde{A}} \vee \sigma_{\tilde{B}}, \tau_{\tilde{A}} \vee \tau_{\tilde{B}} \rangle & (d_1 < 0, d_2 < 0) \end{cases}$
5. $\mu \tilde{A} = \begin{cases} \langle (\mu a_1, \mu b_1, \mu c_1, \mu d_1): \rho_{\tilde{A}}, \sigma_{\tilde{A}}, \tau_{\tilde{A}} \rangle & (\mu > 0) \\ \langle (\mu d_1, \mu c_1, \mu b_1, \mu a_1): \rho_{\tilde{A}}, \sigma_{\tilde{A}}, \tau_{\tilde{A}} \rangle & (\mu < 0) \end{cases}$
6. $\tilde{A}^{-1} = \langle (1/d_1, 1/c_1, 1/b_1, 1/a_1): \rho_{\tilde{A}}, \sigma_{\tilde{A}}, \tau_{\tilde{A}} \rangle \quad (\tilde{A} \neq 0)$



DEFUZZIFICATION OF NEUTROSOPHIC SET

A single valued trapezoidal neutrosophic numbers of the form $\tilde{A} = \langle (a, b, c, d); \rho, \sigma, \tau \rangle$ can be defuzzified by finding its respective score value $K(\tilde{A})$

$$\frac{1}{K(\tilde{A})} = \frac{1}{16} [a + b + c + d][2 + \mu_{\tilde{A}} - \pi_{\tilde{A}} - \phi_{\tilde{A}}]$$

MODEL DEVELOPMENT:

Assumptions

Shortages are not allowed.
 Demand is not deterministic in nature.
 The products are of deteriorating type deteriorating at a rate θ
 Planning horizon is infinite

Notation:

P – Smart production rate per cycle
 D - Uniform demand rate per cycle

General Costs

O_s – Smart Ordering cost
 I_c – Initial Cost for Internet Connectivity

Costs for time period $0 \leq t \leq t_1$

PD_s – Smart Product design cost
 DM - Data management Cost
 CD – Customer Data Analysis cost
 SD – Supplier Data Analysis cost
 T_s - Smart Technology Cost
 M – Production Monitoring cost

r -defective rate

R – Reworking Cost

TR_s – Smart training work personnel cost

TO_s – Smart tools purchase cost

Costs for time period $t_1 \leq t \leq T$

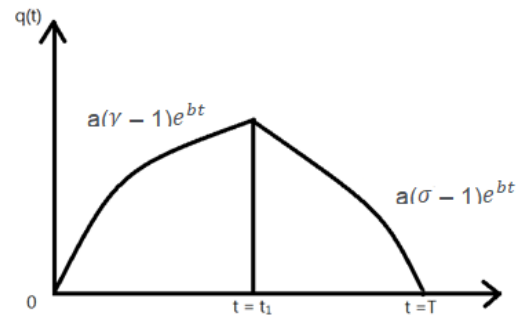
s – disposal rate

D_s – Smart disposal costs

E_s - Smart Environmental costs

Costs common for both the time periods

H - Holding costs



If $q(t)$ represents the inventory level at time $t \in [0, T]$, so the differential equation for the instantaneous inventory at any time over $[0, T]$ is

$$\frac{dq(t)}{dt} + \theta q = a(\gamma - 1)e^{bt} \quad 0 \leq t \leq t_1$$

$$= a(\sigma - 1)e^{bt} \quad t_1 \leq t \leq T$$

With the initial condition $q(0) = 0$, we get

$$q(t) = \frac{a(\gamma - 1)}{b + \theta} (e^{bt} - e^{-\theta t}) \quad (3)$$

and with the condition $q(T) = 0$, we get

$$q(t) = \frac{a(\sigma - 1)}{b + \theta} (e^{bt} - e^{(b + \theta)T - \theta t}) \quad (4)$$

from (3) and (4)

Cost for designing the smart product

$$= PD_s \int_0^{t_1} q(t) dt$$

$$= PD_s \left[\left(\frac{a(\gamma - 1)}{b + \theta} \right) \left(\frac{e^{bt_1} - 1}{b} + \frac{e^{-\theta t_1} - 1}{\theta} \right) \right]$$

Cost for Data Management

$$= DM \int_0^{t_1} q(t) dt$$

$$= DM \left[\left(\frac{a(\gamma - 1)}{b + \theta} \right) \left(\frac{e^{bt_1} - 1}{b} + \frac{e^{-\theta t_1} - 1}{\theta} \right) \right]$$

Cost for Customer's Data Analysis

$$= CD \int_0^{t_1} q(t) dt$$

$$= CD \left[\left(\frac{a(\gamma - 1)}{b + \theta} \right) \left(\frac{e^{bt_1} - 1}{b} + \frac{e^{-\theta t_1} - 1}{\theta} \right) \right]$$

Cost for Supplier's Data Analysis

$$= SD \int_0^{t_1} q(t) dt$$

$$= SD \left[\left(\frac{a(\gamma - 1)}{b + \theta} \right) \left(\frac{e^{bt_1} - 1}{b} + \frac{e^{-\theta t_1} - 1}{\theta} \right) \right]$$

Cost for Smart Technology

$$= T_s \int_0^{t_1} q(t) dt$$



$$= T_s \left[\left(\frac{a(\gamma-1)}{b+\theta} \right) \left(\left(\frac{e^{bt_1}-1}{b} \right) + \left(\frac{e^{-\theta t_1}-1}{\theta} \right) \right) \right]$$

Cost for Production Monitoring

$$= M \int_0^{t_1} q(t) dt$$

$$= M \left[\left(\frac{a(\gamma-1)}{b+\theta} \right) \left(\left(\frac{e^{bt_1}-1}{b} \right) + \left(\frac{e^{-\theta t_1}-1}{\theta} \right) \right) \right]$$

Cost for Reworking

$$= R \int_0^{t_1} q(t) dt$$

$$= R \left[\left(\frac{a(\gamma-1)}{b+\theta} \right) \left(\left(\frac{e^{bt_1}-1}{b} \right) + \left(\frac{e^{-\theta t_1}-1}{\theta} \right) \right) \right]$$

Cost for Smart Training Personal Work

$$= TR_s \int_0^{t_1} q(t) dt$$

$$= TR_s \left[\left(\frac{a(\gamma-1)}{b+\theta} \right) \left(\left(\frac{e^{bt_1}-1}{b} \right) + \left(\frac{e^{-\theta t_1}-1}{\theta} \right) \right) \right]$$

Cost for Smart tool Purchase

$$= TO_s \int_0^{t_1} q(t) dt$$

$$= TO_s \left[\left(\frac{a(\gamma-1)}{b+\theta} \right) \left(\left(\frac{e^{bt_1}-1}{b} \right) + \left(\frac{e^{-\theta t_1}-1}{\theta} \right) \right) \right]$$

Cost for Disposing smart items

$$= D_s \int_{t_1}^T q(t) dt$$

$$= D_s \left[\left(\frac{a(\sigma-1)}{b+\theta} \right) \left(\left(\frac{e^{bt_1}-1}{b} \right) + e^{(b+\theta)T} \left(\frac{e^{-\theta t_1}-1}{\theta} \right) \right) \right]$$

Cost for Smart Environment

$$= E_s \int_{t_1}^T q(t) dt$$

$$= E_s \left[\left(\frac{a(\sigma-1)}{b+\theta} \right) \left(\left(\frac{e^{bt_1}-1}{b} \right) + e^{(b+\theta)T} \left(\frac{e^{-\theta t_1}-1}{\theta} \right) \right) \right]$$

Holding Cost

$$= C_1 \left[\int_0^{t_1} q(t) dt + \int_{t_1}^T q(t) dt \right]$$

$$= C_1 \left[\left[\left(\frac{a(\gamma-1)}{b+\theta} \right) \left(\left(\frac{e^{bt_1}-1}{b} \right) + \left(\frac{e^{-\theta t_1}-1}{\theta} \right) \right) \right] + \left[\left(\frac{a(\sigma-1)}{b+\theta} \right) \left(\left(\frac{e^{bt_1}-1}{b} \right) + e^{(b+\theta)T} \left(\frac{e^{-\theta t_1}-1}{\theta} \right) \right) \right] \right]$$

Cost of Deterioration

$$= C_d \theta \left[\int_0^{t_1} q(t) dt + \int_{t_1}^T q(t) dt \right]$$

$$= \theta C_d \left[\left[\left(\frac{a(\gamma-1)}{b+\theta} \right) \left(\left(\frac{e^{bt_1}-1}{b} \right) + \left(\frac{e^{-\theta t_1}-1}{\theta} \right) \right) \right] + \left[\left(\frac{a(\sigma-1)}{b+\theta} \right) \left(\left(\frac{e^{bt_1}-1}{b} \right) + e^{(b+\theta)T} \left(\frac{e^{-\theta t_1}-1}{\theta} \right) \right) \right] \right]$$

Total Average Cost (TAC) = $\frac{1}{T}$ [Smart Ordering cost + Internet Connectivity Initialization Cost+ Holding Costs + Deterioration Cost + Smart Product design cost+ Data management Cost+ Customer Data Analysis cost+ Supplier Data Analysis cost+ Smart Technology Cost+ Production Monitoring cost+ Reworking Cost+ Smart training work personnel cost+ Smart tools purchase cost + Smart disposal costs + Smart Environmental costs]

$$= \frac{1}{T} \left[(O_s + I_c + PD_s + DM + SD + CD + T_s + M + R + TR_s + TO_s) \left[\left(\frac{a(\gamma-1)}{b+\theta} \right) \left(\left(\frac{e^{bt_1}-1}{b} \right) + \left(\frac{e^{-\theta t_1}-1}{\theta} \right) \right) \right] + (D_s + E_s) \left[\left(\frac{a(\sigma-1)}{b+\theta} \right) \left(\left(\frac{e^{bt_1}-1}{b} \right) + e^{(b+\theta)T} \left(\frac{e^{-\theta t_1}-1}{\theta} \right) \right) \right] + (C_1 + \theta C_d) \left[\left[\left(\frac{a(\gamma-1)}{b+\theta} \right) \left(\left(\frac{e^{bt_1}-1}{b} \right) + \left(\frac{e^{-\theta t_1}-1}{\theta} \right) \right) \right] + \left[\left(\frac{a(\sigma-1)}{b+\theta} \right) \left(\left(\frac{e^{bt_1}-1}{b} \right) + e^{(b+\theta)T} \left(\frac{e^{-\theta t_1}-1}{\theta} \right) \right) \right] \right] \right]$$

The problem is

Minimize TAC

Subject to I_{max}

By the help of Lingo Software, Minimisation is done and is shown in the following numerical

ILLUSTRATION

To validate the developed model, an inventory system with the below characteristics is taken into consideration

Rate of deterioration $\theta = 0.4$,

Smart production rate per cycle = Rs.500unit/per month ,

Uniform demand rate per cycle = Rs.250/month,



Smart Ordering cost =Rs.310/run,
 Internet Connectivity Initialization Cost =
 Rs.370/year,
 Smart Product design cost = Rs.25/unit,
 Data management Cost = Rs.50/unit,
 Customer Data Analysis cost = Rs.45/unit,
 Supplier Data Analysis cost = Rs.25/unit,
 Smart Technology Cost = Rs.15/unit,
 Production Monitoring cost = Rs.45/unit,
 defective rate = Rs.1, Reworking Cost =
 Rs.22/run,
 Smart training work personnel cost =
 Rs.30/unit,
 Smart tools purchase cost= Rs.10/unit,
 Disposal rate = Rs. 3/unit,
 Smart disposal costs = Rs. 5/unit,
 Smart Environmental costs = Rs.7/unit,
 Holding costs = Rs.1/unit/year.
 $I_{max} = 20$ units
 Find the time interval and find the total
 average cost.
 The value of T^* and $TAC(T^*)$ is 1.598558 and Rs.
 31466.29 respectively

This model can be validated with the single
 valued neutrosophic trapezoidal fuzzy value
 representations as follows,
 $D = \langle (200,300,400,500):0.6,0.25,0.15 \rangle$
 $O_s = \langle (300,325,350,400):0.9,0.4,0.2 \rangle$
 $I_c = \langle (500,650,800,950):0.8,0.3,0.4 \rangle$
 $PD_s = \langle (20,30,40,50):0.8,0.4,0.3 \rangle$
 $DM = \langle (65,75,85,95):0.85,0.15,0.2 \rangle$
 $CD = \langle (52,62,72,82):0.7,0.3,0.2 \rangle$
 $SD = \langle (25,35,45,55):0.9,0.2,0.1 \rangle$
 $T_s = \langle (15,18,22,24):0.7,0.1,0.2 \rangle$
 $M = \langle (50,70,90,110):0.9,0.3,0.5 \rangle$
 $r = \langle (1,1.5,2.5,3):0.9,0.1,0.2 \rangle$
 $R = \langle (20,27,34,41):0.9,0.3,0.2 \rangle$
 $TR_s = \langle (32,35,38,41):0.85,0.4,0.3 \rangle$
 $TO_s = \langle (5,6,7,8):0.6,0.5,0.2 \rangle$
 $S = \langle (4,6,8,10):0.8,0.4,0.3 \rangle$
 $D_s = \langle (5,7,9,11):0.7,0.2,0.3 \rangle$
 $E_s = \langle (6,9,12,15):0.8,0.2,0.3 \rangle$
 $C_1 = \langle (1,1.5,2.5,3):0.9,0.3,0.2 \rangle$
 And for this Model under Neutrosophic
 Environment,
 $T^* = 2.024798$ and $TAC^*(T^*) = 26860.00$

DISCUSSION

An inventory model with deterioration under
 neutrosophic environment incorporating the

costs parameters of industry 4.0 with
 deterioration is build-up together with the
 presentation of its conceptual framework.
 Numerous key advantages of this production
 inventory model under neutrosophic
 environment have been emphasized in this
 paper, together with the additional cost
 parameters. Also, the usage of this
 neutrosophic inventory model is to find the
 feasible time to place orders that reduces the
 total expenses. The illustration of these cost
 parameters in a single valued trapezoidal
 neutrosophic number tackles the situation of
 uncertainty. This inventory model is validated
 with deterministic and neutrosophic
 parameters. The optimal time that yields
 minimum costs is almost equal in both the
 deterministic and neutrosophic cases. The
 neutrosophic representation makes this
 inventory model with deterioration more
 comprehensive. Here we have considered
 shortages are not allowed, the products are of
 deteriorating type and planning horizon is
 infinite. The developed model can be extended
 to neutrosophic production inventory model
 with shortages. This model mainly focuses on
 increases productivity of high-quality smart
 products with deterioration within low
 investments. This proposed model will
 certainly assist the production sectors to
 incorporate or reduce new types of costs.

CONCLUSION

The proposed industry 4.0 inventory model
 with deterioration is a typical approach in
 uniting the concept of smart production
 principles, and neutrosophic illustration of cost
 parameters. This is an elementary model
 showing smart production with deterioration
 with neutrosophic development and this model
 can be further advanced according to the
 demand of the production sectors. The
 proposed model is realistic in nature and it can
 be extended by including the various concepts
 of further realistic environmental condition and
 customer acquisition. These models will
 certainly disclose the new exigencies of
 production scheme to meet the necessities of
 the customers in this information age. The
 model constructed in this paper represents the
 daily need of the production environment and



it will certainly collaborate the decision makers to optimize profit.

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