

AN INVENTORY MODEL WITH PARTIAL BACKORDERING, WEIBULL DISTRIBUTION DETERIORATION UNDER TWO LEVEL OF STORAGE

Kamlesh Patil¹, Dr. Arihant Jain² and Dr. Ritesh Yadav³

^{1,2}Department of Mathematics, Dr. A. P. J. Abdul Kalam University, Indore ³Department of Physis, Dr. A. P. J. Abdul Kalam University, Indore Corresponding Author: -kamleshpatil405@gmail.com

ABSTRACT

Inventory management is a recurring issue that managers in a variety of organizations, such as manufacturing companies, distribution networks, or retail businesses, must deal with on a daily basis in this paper, it is presumed that both warehouses' rates of degradation vary. This notion is supported by the fact that each warehouse has a varied set of preservation facilities. Thus, the holding costs are seen as distinct and linearly time-dependent. It is assumed that the demand rate is constant. Shortages are permitted in the OW and partly backlogged during the next replenishment cycle, and salvage value is connected to the deteriorating units of inventory. This model's objectives are to determine the ideal order quantity and to reduce the overall cost of inventory. Using a numerical example, the model's applicability is examined. The shape parameter of the degradation rate in RW, which affects PCC value, is not sensitive to value.

Keywords: Inventory Model, Partial, Weibull Distribution Deterioration and Warehouse

DOI Number: 10.48047/NQ.2022.20.13.NQ88501 Neuroquantology 2022; 20(13): 4125-4133

INTRODUCTION

elSSN1303-5150

are several alternatives with more characteristics.

An inventory of raw materials aids in the planning of urthermore, it is impossible to precisely predict efficient production runs in a manufacturing systemdemand using a tried-and-true technique. By maintaining an appropriate inventory of items, Although there is a significant amount of shopkeeper may boost his sales and earnings. The dministration required, regional specialization, fact that it is physically impossible or economically periodic volatility, and the mismatch between unwise to have things on hand exactly when demand upply and demand need businesses to maintain arises is one of the primary justifications for keeping inventories of a variety of items in order to inventories of commodities. Customers often geoperate properly.

antsy when a stockout of a product happens until the inventory has to be kept in a warehouse in a next shipment of stock comes. Because of expectednanner that both protects and preserves its variations in the pricing of goods, inventories are hysical characteristics. The corporation may own sometimes kept on hand. Thus, stocks are kept to warehouse or it may be leased. The rental prevent frequent replenishments, to maintainwarehouse (RW) is a short distance from or close systematic production runs, to earn money, to the OW. The holding cost in RW is often higher arrange for flexibility in scheduling sequentiathan the OW. Furthermore, until the stock level of facilities, etc.

RW is depleted, the products from RW are sent in

No business can survive a stock-out scenario in abulk to OW to satisfy consumer demand. When an market when options are accessible since therebject is stored, its life is believed to be limitless in



the traditional inventory models. But when ithe last significant evaluation of this issue, was comes to the preservation of certain often usequiblished in 1913. The basic structure, modeling physical items like fruits, vegetables, and so forthapproach, capacity restrictions, trade credit, fuzzy the impact of degradation is quite essential. parameters, with shortages and diverse demand are In these situations, a portion of these commoditieshighlighted in the articles. We examine several have either been harmed or have degraded to the ublications from numerous journals that pertain to point that they can no longer be used to meetdeteriorating things with certain characteristics. We customer demand in the future. These unitsprovide an overview of the EGQ model of inventory. deteriorate over time continuously, and the Malik, A. et.al. (2017) To create a mathematical amount of inventory on hand often reflects this. Immodel for two warehouses is the goal of this paper. general, two aspects of the problem—itemDue to seasonal product for keeping the raw degradation and variations in demand rate overmaterial/products, we will assume two warehouse researchers' increasingystems, one of which is Own Warehouse (OW) and time—have drawn attention as they developed inventory models. the other is Rent Warehouse (RW). The suggested LITERATURE REVIEW research is designed for an environment with Srinivasa Reddy, Maram et.al. (2020). The two-quadratic demand, fluctuating holding costs, and no warehouse inventory model for deteriorating goodsroom for shortages. The result of the suggested is presented in this study where the rate ofmethod serves as an illustration of the ideal total deterioration can be controlled using preservationinventory cost and ideal amount of inventory. technology and the demand rate exhibits a non-Agarwal, Ritu et.al. (2021). In a real-world setting linear trend (i.e., quadratic form) with multiplewhere the lead time fluctuates over time, this deterioration rates under an acceptable paymentresearch proposes an inventory model for things delay. The system's ideal cost is estimated whilethat degrade with time. For calculating the overall accounting for shortages. The holding cost is alsocost and order quantity in a limited planning horizon assumed to be a linear function of time. A numerical with m number of cycles, a mathematical model has example is provided, and sensitivity analysis is usedbeen devised. Information technology and the implications of currency inflation, shortages, and to evaluate the model's robustness. (Dr.) Chandra (2017).lead times have all been taken into account. The Jaggi, Prof. et.al. Manufacturing processes can result in both ideal andtotal backlog and immediate degradation have their subpar products. The moment they are added toown special instances. An arbitrary log-concave inventory, the perfect objects begin to degrade. Thefunction of time is the demand rate. The study has a suppliers, on the other hand, put off payment in andemand function that uses the error function as an effort to encourage their customers to buy moreillustration. On the order amount and the overall goods. This study creates a two-warehousesupply chain cost for this function, the impact of inventory model that takes degradation, defectiveseveral factors including degradation rate and quality products, and one level of trade credit intobacklogging parameter is investigated. Utilizing account. The suggested inventory model maximizesMATLAB software, data analysis and sensitivity the overall profit per unit of time by optimizing theanalysis are performed. order quantity. Finally, a sensitivity analysis is Assumptions and Notations performed to demonstrate how the proposedThe following presumptions form the foundation inventory model responds to changes in parameter of the mathematical model designed to solve the values and numerical examples are used to test thetwo-warehouse inventory problem: proposed inventory model and its solution Assumptions technique. The demand rate is predictable and steady. Kumar, Kamal et.al, (2019) A key component of the. There is no starting inventory level and a research effort that links the earlier work to thevery short lead time.

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commodities' permitted.

There is no end to the replenishing pace.

The pace of deterioration differs between

Backorders and shortages are

present study is the examination of the literature.

This essay discusses current research on the.

perishable inventories. The Ford Harris EOQ Model, which was. the two warehouses. $$Q_{\!M}$$ Maximum ordered quantity T after the

Deterioration rate follows a two parametespecified amount of time

Weibull distribution and is time dependent. $\alpha \mathcal{K}_{\mathcal{C}}^{I}$ The following table gives the present value of 0denotescaleparameter and $\beta > 1$ the shapethe entire relevant inventory cost divided by the parameter should be indicated.

Notations t Deterioration o ver time, t>0

This PAPER makes use of the notations below: Rate of OW degradation on-the-spot

Units demanded per unit of time (constant $\Sigma(t) = \alpha \beta t^{\beta-1}$ where $0 < \alpha < 1$

W CapacityofOW Rate of immediate worsening in

 α Scale parameters of the pace of OWRWR(t)= $\mu 5t^{\gamma-1}$ where0< μ <1 degradation and $0<\alpha<1$ Development of Mathematical Model

the degradation rate in OW and its shape woware house system

parameter $\beta > 1$ The inventory system of OW is shown in Figure-1. μ The RW degradation rate's scalet may be broken down into three sections, as

 η Shape parameter of the deterioration rates utilized to address backlog shortages, with the in RW and η >1 remaining amount being added to the system.

F Backordered portion of demand during the goods are held in OW in units of W, while the stock out period remaining goods are retained in RW. In Figure-2,

 C_{\circ} Cost per order for ordering the inventory level in the RW inventory system has

 C_d Cost of deterioration per item in each of been visually shown. Due to higher holding costs the two warehouses than OW, the inventory in RW is delivered first in $H_0 = bt_1$: maintaining cost per unit per unit time order to save inventory costs. Stock during the

in OW throughout T_1 time period; b > 0 time period in the RW T_1 until it approaches zero, $H_0 = bt_2$: maintaining cost per unit per unit time the resource depletes as a result of demand and

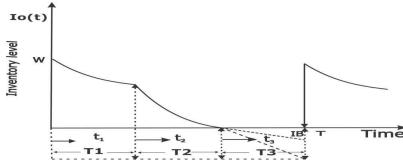
in OW throughout T_2 time period; b>0 wear. The sole cause of the inventory in OW's $H_R=at_1$: Cost per unit every unit of time held in decline over the timeframe is degradation. Due to

RW throughout T_1 timeperiodwere the combined effects of demand and degradation over time, the stock of OW decreases.

 C_s Cost of the shortage for each unit of time T_2 . Throughout the period T_3 , A portion of the L_c Cost of a shortage in terms of units, timeshortfall is on backorder for the next and lost sales replenishment, and both warehouses are bare.

Q_o The number of orders in OW

 Q_R The amount of the order in RW



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Figure-1Graphical representation for OWInventory System(InventoryLevelVs.CycleLength)

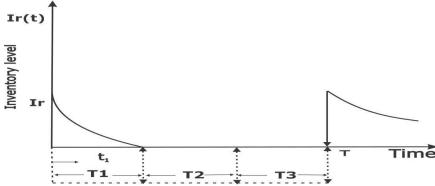


Figure-2 Graphical representation for RWInventory System (Inventory Level Vs. Cycle Length)

The products placed in RW are used to satisfy customer demand during the initial stage of the inventory cycle, which reduces the rate of change of inventory during periods of positive RW stock. T_1 the following differential equation provides

$$\frac{dI^{R}(t_{1})}{dt_{1}} = -\mu \eta \ t_{1}^{\eta-1} I^{R}(t_{1}) - d; \qquad 0 \le t \le T$$

the above equation's solution includes B.C. $I^R(0) = I^R$ is

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Where

$$I^{r} = d \int_{0}^{T_{1}} e^{\mu \mu^{\eta}} du = d \sum_{m=0}^{\infty} \frac{\mu^{m} T_{1}^{m\eta+1}}{m! (mn+1)}$$
$$\approx d(T_{1}) + \frac{\mu T_{1}^{n+1}}{n+1}$$

The rate of inventory change during the course of a positive stock in OW and the time period $T_1+T_2+T_3$ may be modeled by the differential equation below.

$$\frac{dI_1^0(t_1)}{dt_1} = -\alpha\beta t_1^{\beta-1}I_1^0(t_1); 0 \le t_1 \le T_1$$

$$\frac{dI_2^0(t_2)}{dt_2} = -\alpha\beta t_1^{\beta-1}I_2^0(t_2) - d; 0 \le t_2 \le T_2$$

Beginning at the time of stock out are shortages. T3 the differential equation may be used to describe in OW.

$$\frac{dI_3^0(t_3)}{dt_3} = -Fd; 0 \le t_3 \le T_3$$

Using boundary conditions, the aforementioned differential equation is solved. +* G (0) =W, +* G (=*) =W\$ %M." N =+I G (0) and +K G (0) =0 may be provided as

$$\begin{split} I_2^0(t_1) &= w e^{-at_1^\beta} 0 \le t_1 \le T_1 \\ I_2^0(t_1) &= w e^{-at_1^\beta} - d \int_0^{t_2} e^{au\beta} du) e^{-at_2^\beta} \ 0 \le t_2 \le T_2 \\ I_3^0(t_3) &= -F dt_3 0 \le t_3 \le T_3 \end{split}$$

DR indicates and reports the quantity of inventory that depreciated in RW during time period T_1 as follows:

$$\begin{split} D^R &= \int_0^{T_1} R(t) I^R(t) dt_1 \\ &\int_0^{T_1} \mu \eta \ t^{\eta-1} (I^r - d \int_0^{t_1} e^{\mu \mu^\eta} du) e^{-\mu t_1^\eta} \, dt_1 \\ &= \mu d \, \left(T_1 + \frac{\mu T_1^{\eta+1}}{n+1} \right) T_1^\eta \approx \mu dT_1^{\eta+1} \end{split}$$

The value of depreciated objects in RW is documented and provided as $CD^R = C_d \mu dT_1^{\eta+1}$

Inventory degradation throughout the time period T₁ + T₂ in OW is indicated by DO and is provided as

$$D^{O} = \int_{0}^{T_{1}} Z(t)wdt_{1} + (we^{-aT_{1}^{\beta}} - \int_{0}^{T_{2}} d dt_{2})$$

$$= \int_{0}^{T_{1}} \alpha \beta t^{\beta - 1} Wdt_{1} + (we^{-aT_{1}^{\beta}} - \int_{0}^{T_{2}} d dt_{2})$$

$$= W(\alpha T_{1}^{\beta} + e^{-aT_{1}^{\beta}}) - dT_{2}$$

CDO indicates and provides the cost of damaged things in OW as

$$CD^{O} = C_{d} \{ W(\alpha T_{1}^{\beta} + e^{-aT_{1}^{\beta}}) - dT_{2} \}$$

 $CD^0=C_d\{W(\alpha T_1^\beta+e^{-aT_1^\beta})-dT_2\}$ The term "MQ" stands for "maximum ordered quantity" and is used to indicate

$$M_Q = d\left(T_1 + \frac{\mu T_1^{\eta + 1}}{\eta + 1}\right) + W + Fd\frac{T_3^2}{2}$$

IHO indicates and provides the following value for the inventory holding cost in OW for the time period T₁ + T₂:

$$\begin{split} IH^O &= \int_0^{T_1} H_O I_1^O(t_1) dt_1 + \int_o^{T_2} H_O I_2^O(t_2) dt_2 \\ &= \int_0^{T_1} bt_1 \, W e^{-aT_1^{\beta}}; dt_1 + \int_o^{T_2} bt_2 \, (W \, e^{-aT_1^{\beta}} - d \int_o^{t_2} e^{au^{\beta}} du) \, e^{-aT_2^{\beta}} dt_2 \\ &= \left[bW \left\{ \frac{T_1^2}{2} - \frac{\alpha T_1^{\beta+2}}{\beta+2} \right\} + bW \left\{ \frac{T_1^2}{2} \left(1 - \alpha T_1^{\beta} \right) - \frac{\alpha T_1^{\beta+2}}{\beta+2} \right\} - bd \left\{ \frac{T_2^3}{3} - \frac{\alpha \beta T_2^{\beta+3}}{(\beta+1)(\beta+3)} \right\} \right] \end{split}$$

Single Ware-house System

The graphical depiction of one warehouse inventory system is shown in Figure-3. We calculated the inventory level throughout time periods T_1 and T_2 for a single warehouse inventory system, which is represented by the differential equation below.

$$\frac{dI_1^0(t_1)}{dt_1} = -\alpha \beta_1^{\beta - 1} W - d; \qquad 0 \le t_1 \le T_1$$

$$I_1^0(0) = W$$

Using boundary conditions, the aforementioned differential equation is solved.

$$dI_{1}^{0}(t_{1}) = W\left(1 - \alpha t_{1}^{\beta}\right) - dt_{1}0 \le t_{1} \le T_{1}$$

During the time period, there are shortages $[0 T_1]$. The current cost of shortages is

$$S_c = C_S \left\{ \int_0^{T_2} (Fdt_2) dt_2 \right\}$$
$$= \frac{C_S Fd}{2} T_2^2$$

Sales are lost during the T_2 timeframe. The cost of missing sales at OW present value is stated as

$$CL_S = L_C \{ \int_0^{T_2} (1 - F) dt_2$$

= $L_C (1 - F) dT_2$

 $=L_{\mathcal{C}}(1-F)dT_{2}$ Cost of depreciated units over time [0 T_{1}] is stated as

$$CD^R = C_d \alpha W d T_1^{\beta}$$

The most items that may be ordered per order is

$$M_Q = W + \frac{Fd}{2}T_2^2$$

Salvages value per unit time of degraded units is

$$SV = \gamma W \alpha T_1^{\beta}$$

Inventory holding expenses over time T_1 is



$$\begin{split} IH^{O} &= \int_{O}^{T_{1}} H_{O}I_{1}^{O}(t_{1})dt_{1} \\ &= \int_{O}^{T_{1}} bt_{1} \left(W \left(1 - aT_{1}^{\beta} \right) - dt_{1} \right) dt_{1} \\ &= bW \left\{ \frac{T_{1}^{2}}{2} - \frac{\alpha\beta T_{1}^{\beta+2}}{\beta+2} \right\} \end{split}$$

Noting that $T = T_1 + T_2$, The ordering cost, holding cost, shortages cost, lost sales cost, and salvage value of degraded units are added together to provide the total present value of the entire relevant cost per unit time within the cycle.

$$T_{C}^{1}(T_{1}T_{2}) = \frac{1}{T} \left[O_{C} + bW \left\{ \frac{T_{1}^{2}}{2} - \frac{\alpha T_{1}^{\beta+2}}{\beta+2} \right\} + C_{d}\alpha W dT_{1}^{\beta} + C_{S}F d\frac{T_{2}^{2}}{2} \right] + L_{C}(1-F)dT_{2} - \gamma W \alpha T_{1}^{\beta}$$
In the written as Minimize:

The ideal issue might be written as Minimize:

$$T_C^1(T_1T_2)$$

Subject to: $T_1 \ge 0$, $T_2 \ge 0$;

Getting rid of equation (3.33) T_1 , T_2 according to the values of T_1 , T_2 and T^* a Equation (3.32) is used to get the total minimum inventory cost using the values that were obtained.

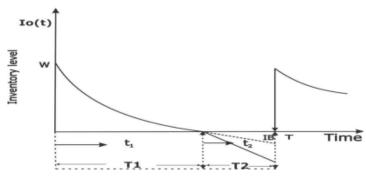


Figure-3 Graphical representation for a single warehouse Inventory System (Inventory Level Vs. Cycle Length)

Numerical results

Table-1 lists the decision variables that were derived for the models.

Table-1: Representing value of decision variables and inventory cost

Decision Variables	Value obtained for	Value obtained for One
	Two Ware-house	Warehouse system
	model	
$\check{\Tau}_1$	0.4003150	0.0426341
$\check{\Tau}_2$	3.5014900	0.0426341
$\check{\Tau}_3$	0.0859854	
T*	3.9877904	0.1668881
$T^{\mathrm{I}*}_{C}$	1487.8800	1794.0300

Table-1 shows that the best solution is found when when there is only one warehouse with a limited all the requirements and restrictions are met. The number of W units, the minimal present value of the choice variables' respective optimum values in this total relevant inventory cost per unit of time in an case are 1487.88 for the minimum present value of appropriate unit is 1794.03, and the optimal time total relevant inventory cost per unit time in apperiods for positive and negative inventory levels are applicable unit. T_1 , T_2 , T_3 and T^* are 0.40031500.0426341, 0.0426341, and 0.1668881 respectively. 3.5014900, 0.0859854 and 3.9877904 respectively. Comparing the cost incurred in two warehouse systems to the cost incurred in the corresponding

inventory, there has been an increase of \$306.15. The model under the two-warehouse system, there is an system lacks the capacity to keep more units, and ncrease of 634.78.

holding costs and shortage costs drive up the overalFigure-4 provides a visual illustration of the convexity relevant inventory cost. of inventory cost for a two-warehouse system. There

Ť1, Ť2, Ť3 and T* are, respectively, 6.65684, 1.78748js a point when the overall relevant inventory cost is 0.212266, and 8.656586. Comparing the totahegligible, as shown in Figs. 4 relevant inventory cost to the partial backlogging

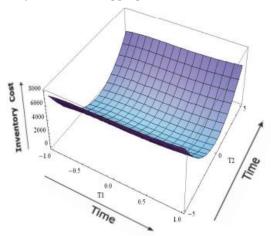


Figure-4: Graphical representation of cost function (When Ť3*= 0.0859854) depicting convexity w.r.t. T1 and T2 for Two Warehouse System

Sensitivity Analysis

Sensitivity analysis is carried out for the numericalensitivity analysis's findings along with a related example in order to investigate the impact of change graphical illustration. The percentage Change in Cost in parameter values after the best solution. The bestPCC) representing the change in the entire relevant bargains of \check{T}_1 , \widecheck{T}_2 , \widecheck{T}_3 and T_C^1 are obtained when inventory cost is provided by $PCC = \frac{T_C^1 - T_C^1}{T_C^1} \times 100$ one parameter in subset S grows by 10% while all other values stay constant. Table-2 displays the

Table-2: Representing Sensitivity Analysis in relation to the parameters to investigate the percentage change in cycle duration and cost function

a	$\check{\mathrm{T}}_{1^*}$	$\check{\mathrm{T}}_{2^*}$	Ť ₃ *	T_{C}^{IC}	PCC (%)	PCC
30	0.3732	3.4991	0.0866	1492.72	0.32	02
27.5	0.3859	3.5002	0.0863	1490.43	0.17	0.2 22, 24 26 28 30 % changed
22.5	0.4169	3.5029	0.0856	1485.08	-0.19	0.4
20	0.4362	3.6668	0.0853	1475.85	-0.80	0.6
						0.0
b	Ť ₁ *	Ť _{2*}	Ť _{3*}	T_{C}^{IC}	PCC (%)	PCC
24	0.4515	3.5220	0.1370	1896.03	27.43	20
22	0.4268	3.5123	0.1116	1692.90	13.78	10 schanged
18	0.3714	3.4894	0.0607	1280.54	-13.94	10 18 20 22 24
16	0.3393	3.4756	0.0338	1070.75	-28.04	20

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C _o 120 110 90 80	Ť _{1*} 0.4008 0.4013 0.3998 0.3993	Ť _{2*} 3.5002 3.4989 3.5027 3.5040	Ť _{3*} 0.0863 0.0866 0.0857 0.0853	T _C ^{1C} 1492.90 1490.39 1485.38 1482.87	PCC (%) 0.34 0.17 -0.17 -0.34	PCC 0.3 0.2 0.1 0.1 90 100 110 120 1 changed
C _d	Ť _{1*}	Ť _{2*}	Ť _{3*}	T _C ^{IC} 818.951	PCC (%)	PCC
1	1		0.002	010.751	-44.90	20
11	0.3307	3.4653 3.5402	0.0448	1185.25	-20.34 21.61	20 9 10 11 12 changed

(1) The value of PCC is directly proportional to thenstantaneously makes use of this model since values of the parameters W (capacity of ownnventory costs are indirectly related to demand. warehouse), Y (Salvages value incurred or REFERENCES

deteriorating products), and b (Holding cost of. inventory in OW).

- (2) The Value of PCC is indirectly proportional to the values of Cd (Cost of Deterioration), d (Demand of Inventory), and α (Scale Parameter of the Deterioration Rate in OW). It is very sensitive to these values.
- (3) The value of PCC is directly proportional to the values of β Cs (Cost of Shortages), Lc (Cost of Lost Sale), Co (Ordering Cost), and a (Holding Cost in RW) and is somewhat sensitive to the values of μ (The shape parameter of deterioration rate in RW) and (scale parameter of deterioration rate in OW). 3. (4) The value of PCC is indirectly proportional to F (Number of shortages backlogged) and is only marginally sensitive to its values.
- (5) The shape parameter of the degradation rate in RW, which affects PCC value, is not sensitive to value.

CONCLUSION:

For two warehouse inventory problems with varied rates of degradation and partial backlogs, an inventory model is described in this Paper to find the best replacement cycle. The model assumes that the distributors' warehouse has a finite capacity. By5. minimizing the overall relevant cost of the inventory system, the optimal replenishment strategy is determined using the optimization approach. An example using numbers is provided to demonstrate the model's viability. The overall relevant cost per unit time of the inventory system is greater when just one warehouse is present than when there are two warehouses. Weibull distribution6. degradation rate for things degrade

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