Generalized − Closure and Interior

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ABSTRACT.

In this paper we defined and characterized the concept of generalized fuzzy closure($\mathrm{generalized}_{\mathcal{F}} - \mathrm{closure}$) andgeneralized fuzzy interior ($\mathrm{generalized}_{\mathcal{F}} - \mathrm{interior}$) and obtained some significant results in this context with help of various supporting examples.

Keywords: Fuzzy open set, fuzzy topological space, $\mathrm{generalized}_{\mathcal{F}} - \mathrm{topological\ space}$, $\mathrm{generalized}_{\mathcal{F}} - \mathrm{closure}$, $\mathrm{generalized}_{\mathcal{F}} - \mathrm{interior}$

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1. INTRODUCTION

Azad [1] has introduced the concept of fuzzy semi-open sets in fuzzy topological spaces. Bin Shahana [3-4] has introduced the concept of fuzzy pre-open sets and fuzzy α-open sets in fuzzy topological space. Thakur [9] has introduced the concept of fuzzy semi pre-open sets in fuzzy topological spaces. Csaszar [6] introduced the notions of generalized topological spaces.He also introduced the notions of continuous functions and associated interior and closure operators on generalized neighborhood systems and generalized topological spaces. PalaniCheety [7] introduced the concept of generalized fuzzy topology and investigates various properties.

In this paper, we have introduced the concept of $\mathrm{generalized}_{\mathcal{F}} - \mathrm{closure}$ and $\mathrm{generalized}_{\mathcal{F}} -$ interiorand verify the results with the help of some counter examples.Some require basic definitions, concepts of $\mathrm{generalized}_{\mathcal{F}} - \mathrm{topological\ space}$ and notations are discussed in Section 2. In section 3, we study the concept of $\mathrm{generalized}_{\mathcal{F}} - \mathcal{C}\mathrm{losure}$ and $\mathrm{generalized}_{\mathcal{F}} - \mathrm{Interior}$ in $\mathrm{generalized}_{\mathcal{F}}$ −topological space. Finally, Section 4 concludes the paper.

2. Preliminaries

Definition 2.1: Let X be a crisp set and let μbe a collection of fuzzy sets on X. Then μis called $\mathrm{generalized}_{\mathcal{F}} - \mathrm{topology}$on X if it satisfies following conditions

i) The fuzzy sets 0 and 1 are in μ where $0,1: X \to I$ are defined as $0(x) = 0$ and $1(x) = 1$ for all $x \in X$

ii) If $\{\lambda_j\}, j \in J$ is any family of fuzzy sets on X where $\lambda_j \in \mu$ then$\cup_{j \in J} \lambda_j \in \mu$

The pair (X, μ) is called $\mathrm{generalized}_{\mathcal{F}} - \mathrm{topological}\ \mathcal{S}\mathrm{pace}$

Definition 2.2: Let (X, μ)be $generalized_{\mathcal{F}} - topological\ \mathcal{S}pace$. The members of the collection μ are called $generalized_{\mathcal{F}} - \mathcal{O}pen\ \mathcal{S}et$ in $generalized_{\mathcal{F}} - topological\ \mathcal{S}pace$. The complement of $generalized_{\mathcal{F}} - \mathcal{O}pen\ \mathcal{S}et$ in X is called $generalized_{\mathcal{F}} - \mathcal{C}lose\ \mathcal{S}et$

Definition 2.3: Let (X, μ)be $generalized_{\mathcal{F}} - topological\ \mathcal{S}pace$. For a fuzzy setA in X the $\mathcal{C}losure$ of A is defined as $Cl_{\mu}(A) = \inf\{K : A \subseteq K, K^C \in \mu\}$. Thus$Cl_{\mu}(A)$ is the smallest $\mathcal{C}losed\ \mathcal{S}et$ in X containing the fuzzy $generalized_{\mathcal{F}} - \mathcal{O}pen\ \mathcal{S}et$ A. From the definition, if follows that $Cl_{\mu}(A)$is the intersection of all $generalized_{\mathcal{F}} - \mathcal{C}losed\ \mathcal{S}ets$ in X containing A.

Definition 2.4: Let (X, μ)be $generalized_{\mathcal{F}} - topological\ \mathcal{S}pace$. For a fuzzy $\mathcal{S}et$ A in X, the $\mathcal{I}nterior$of A, is defined as $I_{\mu}(A) = Sup\{Q : Q \subseteq A, Q \in \mu\}$. Thus $I_{\mu}(A)$ is the largest $generalized_{\mathcal{F}} - \mathcal{O}pen\ \mathcal{S}et$ in X contained in the fuzzy $\mathcal{S}et$A. From the definition, if follows that $I_{\mu}(A)$is the union of all $generalized_{\mathcal{F}} - \mathcal{O}pen\ \mathcal{S}et$ in X contained in A.

Proposition 2.1: Let (X, μ) be $generalized_{\mathcal{F}} - topological\ \mathcal{S}pace$.Then:

i) 0 and 1 are fuzzy $generalized_{\mathcal{F}} - \mathcal{C}losed\ \mathcal{S}ets$ in X.
ii) Arbitrary intersection of $generalized_{\mathcal{F}} - \mathcal{C}losed\ \mathcal{S}ets$ in X is $generalized_{\mathcal{F}} - \mathcal{C}losed\ \mathcal{S}et$ in X.

3 $\mathbf{Generalized_{\mathcal{F}} - \mathcal{C}losure}$ and $\mathbf{Generalized_{\mathcal{F}} - Interior}$

Proposition 3.1: Let (X, μ) be$generalized_{\mathcal{F}} - topological\ \mathcal{S}pace$ and let A be a fuzzy set in X. Then A is $generalized_{\mathcal{F}} - \mathcal{C}losed\ \mathcal{S}et$ if and only if$Cl_{\mu}(A) = A$.

Proof: Suppose thatλ is $generalized_{\mathcal{F}} - \mathcal{C}losed\ \mathcal{S}et$ in X. then clearly the smallest $generalized_{\mathcal{F}} - \mathcal{C}losed\ \mathcal{S}et$ containing A is itself A. Hence $Cl_{\mu}(A) = A$. Conversely suppose $Cl_{\mu}(A) = A$ then by definition of $generalized_{\mathcal{F}} - \mathcal{C}losure$ it follow that$Cl_{\mu}(A)$is $generalized_{\mathcal{F}} - \mathcal{C}losed\ \mathcal{S}et$.

Proposition 3.2: Let (X, μ) be $generalized_{\mathcal{F}} - topological\ \mathcal{S}pace$ and let A and B are two $generalized_{\mathcal{F}} - \mathcal{O}pen\ \mathcal{S}et$ on X. Then following properties holds.

i) $Cl_{\mu}(0) = 0$.
ii) $Cl_{\mu}(1) = 1$.
iii) If $A \subseteq B$ then $Cl_{\mu}(A) \subseteq Cl_{\mu}(B)$.
iv) $Cl_{\mu}(A) \cup Cl_{\mu}(B) \subseteq Cl_{\mu}(A \cup B)$.
v) $Cl_{\mu}\left(Cl_{\mu}(A)\right) = Cl_{\mu}(A)$

Proof: Since 0 and 1 are $generalized_{\mathcal{F}} - \mathcal{C}losed\ \mathcal{S}et$ from let (X, μ) be $generalized_{\mathcal{F}} - topological\ \mathcal{S}pace$ and let A be $generalized_{\mathcal{F}} - \mathcal{O}pen\ \mathcal{S}et$ in X. then A is $generalized_{\mathcal{F}} - \mathcal{C}losed\ \mathcal{S}et$ if and only if $Cl_{\mu}(A) = A$we have $Cl_{\mu}(0) = 0$and $Cl_{\mu}(1) = 1$. Suppose $A \subseteq B$ in X. Since $B \subseteq Cl_{\mu}(B)$ and $A \subseteq B$ we have $A \subseteq CI_{\mu}(B)$. Now $Cl_{\mu}(B)$ is a $generalized_{\mathcal{F}} - \mathcal{C}losed\ \mathcal{S}et$ we have, $Cl_{\mu}(A) \subseteq Cl_{\mu}(B)$ because $Cl_{\mu}(A)$ is the smallest $generalized_{\mathcal{F}} - \mathcal{C}loaed\ \mathcal{S}et$containing A. As $A \subseteq A \cup B$, $B \subseteq A \cup B$ we have $Cl_{\mu}(A) \subseteq Cl_{\mu}(A \cup B)$ and $Cl_{\mu}(B) \subseteq Cl_{\mu}(A \cup B)$ this implies $Cl_{\mu}(A) \cup Cl_{\mu}(B) \subseteq Cl_{\mu}(A \cup B)$. Since $Cl_{\mu}(A)$ is $generalized_{\mathcal{F}} - \mathcal{C}losed\ \mathcal{S}et$ in X. if follow that $Cl_{\mu}\left(Cl_{\mu}(A)\right) = Cl_{\mu}(A)$.

Proposition 3.3: Let X be $generalized_{\mathcal{F}} - topological\ \mathcal{S}pace$ and $\{A_j\}_{j \in J}$be a family of fuzzy subsets of X. Then

i) $\cup_{j \in J} Cl_{\mu}(A_j) \subseteq Cl_{\mu}(\cup_{j \in J} A_j)$.

ii) $Cl_\mu(\cap_{j\in J} A_j) \subseteq \cap_{j\in J} Cl_\mu(A_j)$.

Proposition 3.4: Let A be $\text{generalized}_{\mathcal{F}} - \mathcal{O}\text{pen}\ \mathcal{S}\text{et}$in $\text{generalized}_{\mathcal{F}} - \text{topological}\mathcal{S}\text{pace}$ (X, μ). Then A is $\text{generalized}_{\mathcal{F}} - \mathcal{O}\text{pen}\ \mathcal{S}\text{et}$ if and only if$I_\mu(A) = A$

Proof: Suppose that A is $\text{generalized}_{\mathcal{F}} - \mathcal{O}\text{pen}\ \mathcal{S}\text{et}$ in X. Since $I_\mu(A)$is the union of all $\text{generalized}_{\mathcal{F}} - \mathcal{O}\text{pen}\ \mathcal{S}\text{et}$ in X contained in A and $A \subseteq A$ follows that $A \subseteq I_\mu(A)$. As we know that $I_\mu(A) \subseteq A$, we find that $I_\mu(A) = A$.

Conversely, suppose that $I_\mu(A) = A$. Then by the definition of $\text{generalized}_{\mathcal{F}} - \mathcal{I}\text{nterior}$ of $\text{generalized}_{\mathcal{F}} - \mathcal{O}\text{pen}\ \mathcal{S}\text{et}$ it follows that $I_\mu(A)$ is $\text{generalized}_{\mathcal{F}} - \mathcal{O}\text{pen}\ \mathcal{S}\text{et}$. Thus A is $\text{generalized}_{\mathcal{F}} - \mathcal{O}\text{pen}\ \mathcal{S}\text{et}$ in X.

Proposition 3.5: Let (X, μ) be $\text{generalized}_{\mathcal{F}} - \text{topological}\ \mathcal{S}\text{pace}$ and A, B are two fuzzy $\mathcal{S}\text{ets}$ in X. Then

i) $I_\mu(0) = 0$.
ii) $I_\mu(1) = 1$.
iii) If $A \subseteq B$ then $I_\mu(A) \subseteq I_\mu(B)$.
iv) $I_\mu(A \cup B) = I_\mu(A) \cup I_\mu(B)$.
v) $I_\mu(A \cap B) \subseteq I_\mu(A) \cap I_\mu(B)$.
vi) $I_\mu\left(I_\mu(A)\right) = I_\mu(A)$.

Proof: Since 0 and 1 are $\text{generalized}_{\mathcal{F}} - \mathcal{O}\text{pen}\ \mathcal{S}\text{ets}$ in $\text{generalized}_{\mathcal{F}} - \text{topological}$ $\mathcal{S}\text{pace}(X, \mu)$and let A be $\text{generalized}_{\mathcal{F}} - \mathcal{O}\text{pen}\ \mathcal{S}\text{et}$ in X. Then A is $\text{generalized}_{\mathcal{F}} - \mathcal{O}\text{pen}\ \mathcal{S}\text{et}$ if and only if $I_\mu(A) = A$we have $I_\mu(0) = 0$and $I_\mu(1) = 1$. Suppose $A \subseteq B$ in X. Since $I_\mu(A) \subseteq A$ and $A \subseteq B$ we have $I_\mu(A) \subseteq B$. Now $I_\mu(B)$ is $\text{generalized}_{\mathcal{F}} - \mathcal{O}\text{pen}\ \mathcal{S}\text{et}$ we have $I_\mu(A) \subseteq I_\mu(B)$ because $I_\mu(B)$ is the largest $\text{generalized}_{\mathcal{F}} - \mathcal{O}\text{pen}\ \mathcal{S}\text{et}$ contained in B. As $A \subseteq A \cup B$, $B \subseteq A \cup B$ we have $I_\mu(A) \subseteq I_\mu A \cup B)$ and $I_\mu(B) \subseteq I_\mu(A \cup B)$. This implies $I_\mu(A) \cup I_\mu(B) \subseteq I_\mu(A \cup B)$. Since $I_\mu(A)$ is $\text{generalized}_{\mathcal{F}} - \mathcal{O}\text{pen}\ \mathcal{S}\text{et}$ in X, it follow that $I_\mu\left(I_\mu(A)\right) = I_\mu(A)$ 4669

Proposition 3.6: Let X be $\text{generalized}_{\mathcal{F}} - \text{topological}\ \mathcal{S}\text{pace}$ and $\{A_j\}_{j\in J}$be a family of subsets of X. Then

i) $\cup_{j\in J} I_\mu(A_j) \subseteq I_\mu(\cup_{j\in J} A_j)$.
ii) $I_\mu(\cap_{j\in J} A_j) \subseteq \cap_{j\in J} I_\mu(A_j)$.

Proposition 3.7: Let (X, μ) be $\text{generalized}_{\mathcal{F}} - \text{topological}\ \mathcal{S}\text{pace}$ and λ be a fuzzy set in X. Then

i) $Cl_\mu(1 - A) = 1 - I_\mu(A)$.
ii) $I_\mu(1 - A) = 1 - Cl_\mu(A)$.

Proof (i): We have $I_\mu(A) = \cup_j A_j$ where A_j are$\text{generalized}_{\mathcal{F}} - \mathcal{O}\text{pen}\ \mathcal{S}\text{ets}$ in X. and $A_j \subseteq A$ for all $j \in J$. This implies $1 - I_\mu(A) = 1 - \cup_j A_j = \cap_j A_j^c$, where $\{A_j^c\}$ is the family of $\text{generalized}_{\mathcal{F}} - \mathcal{C}\text{losed}\ \mathcal{S}\text{ets}$ containing $1 - A$. Hence, by definition of $\text{generalized}_{\mathcal{F}} - \mathcal{C}\text{losure}$ of fuzzy set we have $Cl_\mu(1 - A) = 1 - I_\mu(A)$.

(ii): Further, we have $\mathrm{Cl}_\mu(\mathrm{A}) = \mathrm{Cl}_\mu\big(1-(1-\mathrm{A})\big) = \mathrm{Cl}_\mu\big(1-\mathrm{A}_j^c\big) = 1-\mathrm{I}_\mu\big(\mathrm{A}_j^c\big)$. This implies $\mathrm{I}_\mu(1-\mathrm{A}) = 1-\mathrm{Cl}_\mu(\mathrm{A})$.

5. Conclusion

In this Paper we have studied the concept of $\mathrm{generalized}_{\mathcal{F}} - \mathrm{closure}$ and $\mathrm{generalized}_{\mathcal{F}} - \mathrm{interior}$ and verify the results with the help of some examples.

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