Radial Radio Geometric Mean Graceful Labeling Of Graphs

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Abstract

Let G be a connected graph. For any two distinct vertices x and y of G, define a 1 − 1 mapping φ : V (G) → N such that d(x, y) + $\lceil\sqrt{\varphi(x)\varphi(y)}\,\rceil \geq 1 + r(G)$, where r(G) is the radius of G. The maximum number assigned to any vertex of G is the radial radio geometric mean number of φ and it is denoted by $rr_{gmn}(\varphi)$. The minimum value of $rr_{gmn}(\varphi)$ taken overall radial radio geometric mean labelings of G is the radial radio geometric mean number of G and it is denoted by $rr_{gmn}(G)$. If $rr_{gmn}(G) = |V(G)|$, we call such graphs as radial radio geometric mean graceful graphs. In this paper, we investigate the radial radio geometric mean graceful labeling of Closed helm graph CH_n, Jelly fish graph $J(n,n)$, web graph Wb_n and Gear graph G_n.

Keywords: Graceful labeling, Distance, Eccentricity, Radius, Closed helm graph, Jelly fish graph, Web graph, Gear graph.
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1.Introduction Throughout this paper we consider the graphs are simple, finite, and connected. Let *V* (*G*) and *E*(*G*) respectively denote the vertex set and edge set of G. Ponrajet al.[3] introduced the notion of radio mean labeling of graphs. In [4], Hemalatha et al introduced the concept of radio geometric mean labeling of graphs. In this sequel,we introduce the radial radio geometric mean graceful labeling of graphs. For standard terminology and notations we follow Harary[1] and Gallian[2].

Definition 1.1 [5] A function f is called graceful labeling of a graph *G*(*V,E)* if $f : V \to \{0,1,...,q\}$ is injective and the induced function $f^*: E \to \{0,1,2,...,q\}$ defined as $f^*(e) = |f(x) - f(y)|$ is bijective. A graph which admits graceful labeling is called a graceful graph.

Definition 1.2 [6] The distance *d*(*u,v)* from a vertex u to a vertex v in a connected graph G is the minimum of the lengths of the *u* − *v* paths in G.

Definition 1.3 [6] The eccentricity *e*(*v*) of a vertex v in a connected graph G is the distance between v and a vertex farthest from v in G.

Definition 1.4[7] The Radius $r(G)$is the smallest eccentricity among the vertices of G. **Definition 1.5** [8] A Closed helm CH_n is the graph obtained from a helm

H_n and adding edges between the pendant vertices.

Definition 1.6 [9]The Jelly fish graphJ(n,n) is obtained by joining a 4-cycle whose vertices are x,y,z,w with vertices x and w defined by an edge and appending n pendant edges to y and z.

Definition 1.7 [8]The Web Wb_n is the graph obtained by joining the pendant vertices of helm H_n to form a cycle and then adding a pendant edge to each vertex of outer circle.

Definition 1.8 [10]The Gear graph $G_n$is obtained from the wheel by adding a vertex between every pair of adjacent vertices of the cycle.

2. Main Results

Theorem 2.1The closed helm graph$CH_n$is radial radio geometric mean graceful for $n \geq 3$.

Proof. Let $x_i, 1 \leq i \leq n$and *x* be the vertices of wheel in which *x* is the center vertex. Let $y_i, 1 \leq i \leq n$ *be the pendant vertices which are joined to* $x_i, 1 \leq i \leq n$,we obtain H_n. Join $y_i y_{i+1}, 1 \leq i \leq n-1$and $y_n y_1$. The resultant graph is CH_nwhose edge set is $E(CH_n) = \{xx_i, x_i y_i \leq i \leq n\} \cup \{x_i x_{i+1}, y_i y_{i+1} \leq i \leq n-1\} \cup \{x_n x_1, y_n y_1\}$ and radius of $CH_n$is *r* = 2. Define a function $\emptyset: V(CH_n) \rightarrow N$by $\emptyset(x) = 1$;

$\emptyset(x_i) = n + i + 1, 1 \leq i \leq n$;

$\emptyset(y_i) = i + 1, 1 \leq i \leq n.$

Now we verify the radial radio geometric mean condition d(x, y) + $\lceil\sqrt{\varphi(x)\varphi(y)}\,\rceil \geq 1 + r(CH_n)$ for every pair of vertices of CH_n.

case(i): verify the pair $(x, x_i), 1 \leq i \leq n. d(x, x_i) + \left\lceil\sqrt{\emptyset(x)\emptyset(x_i)}\right\rceil \geq 1 + \left\lceil\sqrt{(1)(n+i+1)}\right\rceil \geq 3 = 1 + r(CH_n)$**case(ii)**: verify the pair $(x, y_i), 1 \leq i \leq n.$ $d(x, y_i) + \left\lceil\sqrt{\emptyset(x)\emptyset(y_i)}\right\rceil \geq 2 + \left\lceil\sqrt{(1)(i+1)}\right\rceil \geq 3$**case(iii)**: verify the pair $(x_i y_j), 1 \leq i, j \leq n.$ $d(x_i, y_j) + \left\lceil\sqrt{\emptyset(x_i)\emptyset(y_j)}\right\rceil \geq 1 + \left\lceil\sqrt{(n+i+1)(j+1)}\right\rceil \geq 5$**case(iv)**: verify the pair $(x_i x_j), i \neq j, 1 \leq i, j \leq n.$ $d(x_i, x_j) + \left\lceil\sqrt{\emptyset(x_i)\emptyset(x_j)}\right\rceil \geq 1 + \left\lceil\sqrt{(n+i+1)(n+j+1)}\right\rceil \geq 9$**case(v)**: verify the pair $(y_i y_j), i \neq j, 1 \leq i, j \leq n. d(y_i, y_j) + \left\lceil\sqrt{\emptyset(y_i)\emptyset(y_j)}\right\rceil \geq 1 + \left\lceil\sqrt{(i+1)(j+1)}\right\rceil \geq 3$

Thus the radial radio geometric mean condition is satisfied for all pairs of vertices.Hence, $\emptyset$is a valid radial radio geometric mean graceful labeling of CH_n.

Therefore, $rr_{gmn}(CH_n) = 2n + 1$ for $n \geq 3$.Clearly,$|V(CH_n)| = 2n + 1, n \geq 3$

Thus,$rr_{gmn}(CH_n) = |V(CH_n)|$

Hence,the closed helm graph $CH_n$is Radial Radio Geometric mean graceful.

Example 2.1 The Radial Radio Geometric mean graceful labeling of $CH_6$is in Figure 2.1

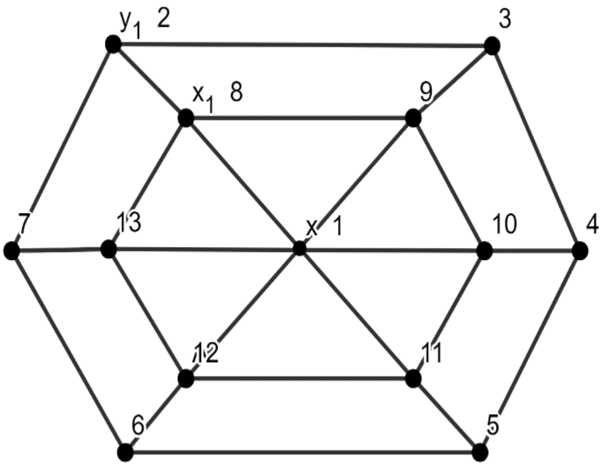

Figure 2.1

Theorem 2.2 J(n,n) is radial radio geometric mean graceful for $n \geq 1$.

Proof. Let *x,y,z,w*be the vertices of the cycle $C_4.Join\, xw.\ Lety_i\ and\ z_i\ , 1 \leq i \leq n$ be the pendant vertices which are joined to y and z respectively. The resultant graph is J(n,n)whose edge set is $E(J(n,n)) = \{yy_i, zz_i, 1 \leq i \leq n\} \cup \{wx, wy, wz, xy, xz\}$ and radius of J(n,n)is r = 2. Define a function $\emptyset: V(J(n,n) \rightarrow N$by $\emptyset(w) = 2n + 1$;

$$\emptyset(x) = 2n + 2; \emptyset(y) = 2n + 3; \emptyset(z) = 2n + 4; \emptyset(y_i) = 2i - 1, 1 \leq i \leq n; \emptyset(z_i) = 2i, 1 \leq i \leq n.$$

Now we verify the radial radio geometric mean condition d(x, y) + $\lceil\sqrt{\varphi(x)\varphi(y)}\,\rceil \geq 1 + r(J(n,n))$ for every pair of vertices of J(n,n).

case(i): verify the pair $(w, x). d(w, x) + \left\lceil\sqrt{\emptyset(w)\emptyset(x)}\right\rceil \geq 1 + \left\lceil\sqrt{(2n+1)(2n+2)}\right\rceil \geq 3 = 1 + r(J(n,n))$**case(ii)**: verify the pair $(w, y). d(w, y) + \left\lceil\sqrt{\emptyset(w)\emptyset(y)}\right\rceil \geq 1 + \left\lceil\sqrt{(2n+1)(2n+3)}\right\rceil \geq 11$**case(iii)**: verify the pair $(w, z). d(w, z) + \left\lceil\sqrt{\emptyset(w)\emptyset(z)}\right\rceil \geq 1 + \left\lceil\sqrt{(2n+1)(2n+4)}\right\rceil \geq 11$**case(iv)**: verify the pair $(x, y). d(x, y) + \left\lceil\sqrt{\emptyset(x)\emptyset(y)}\right\rceil \geq 1 + \left\lceil\sqrt{(2n+2)(2n+3)}\right\rceil \geq 11$**case(v)**: verify the pair $(y, z). d(y, z) + \left\lceil\sqrt{\emptyset(y)\emptyset(z)}\right\rceil \geq 1 + \left\lceil\sqrt{(2n+3)(2n+4)}\right\rceil \geq 13$**case(vi)**: verify the pair $(x, z). d(x, z) + \left\lceil\sqrt{\emptyset(x)\emptyset(z)}\right\rceil \geq 1 + \left\lceil\sqrt{(2n+2)(2n+4)}\right\rceil \geq 11$**case(vii)**: verify the pair $(y, y_i), 1 \leq i \leq n.$ $d(y, y_i) + \left\lceil\sqrt{\emptyset(y)\emptyset(y_i)}\right\rceil \geq 1 + \left\lceil\sqrt{(2n+3)(2i-1)}\right\rceil \geq 4$**case(vii)**: verify the pair $(z, z_i), 1 \leq i \leq n.$ $d(z, z_i) + \left\lceil\sqrt{\emptyset(z)\emptyset(z_i)}\right\rceil \geq 1 + \left\lceil\sqrt{(2n+4)(2i)}\right\rceil \geq 6$**case(ix)**: verify the pair $(w, y_i), 1 \leq i \leq n.$ $d(w, y_i) + \left\lceil\sqrt{\emptyset(w)\emptyset(y_i)}\right\rceil \geq 2 + \left\lceil\sqrt{(2n+1)(2i-1)}\right\rceil \geq 5$**case(ix)**: verify the pair $(w, z_i), 1 \leq i \leq n.$ $d(w, z_i) + \left\lceil\sqrt{\emptyset(w)\emptyset(z_i)}\right\rceil \geq 2 + \left\lceil\sqrt{(2n+1)(2i)}\right\rceil \geq 6$**case(x)**: verify the pair $(y_i, z_j), 1 \leq i, j \leq n.$ $d(y_i, z_j) + \left\lceil\sqrt{\emptyset(y_i)\emptyset(z_j)}\right\rceil \geq 4 + \left\lceil\sqrt{(2i-1)(2j)}\right\rceil \geq 5$**case(xi)**: verify the pair $(y, z_i), 1 \leq i \leq n.$ $d(y, z_i) + \left\lceil\sqrt{\emptyset(y)\emptyset(z_i)}\right\rceil \geq 3 + \left\lceil\sqrt{(2n+3)(2i)}\right\rceil \geq 8$**case(xii)**: verify the pair $(z, y_i), 1 \leq i \leq n.$ $d(z, y_i) + \left\lceil\sqrt{\emptyset(z)\emptyset(y_i)}\right\rceil \geq 3 + \left\lceil\sqrt{(2n+4)(2i-1)}\right\rceil \geq 6$**case(xiii)**: verify the pair $(x, y_i), 1 \leq i \leq n.$

$d(x, y_i) + \lceil\sqrt{\emptyset(x)\emptyset(y_i)}\rceil \geq 2 + \lceil\sqrt{(2n+4)(2i-1)}\rceil \geq 5$ **case(xiv)**: verify the pair $(x, z_i), 1 \leq i \leq n.$ $d(x, z) + \lceil\sqrt{\emptyset(x)\emptyset(z_i)}\rceil \geq 2 + \lceil\sqrt{(2n+4)(2i)}\rceil \geq 5$ **case(xv)**: verify the pair $(y_i, y_j), i \neq j, 1 \leq i, j \leq n. d(y_i, y_j) + \left\lceil\sqrt{\emptyset(y_i)\emptyset(y_j)}\right\rceil \geq 2 + \lceil\sqrt{(2i-1)(2j-1)}\rceil \geq 3$ **case(xvi)**: verify the pair $(z_i, z_j), i \neq j, 1 \leq i, j \leq n. d(z_i, z_j) + \left\lceil\sqrt{\emptyset(z_i)\emptyset(z_j)}\right\rceil \geq 2 + \lceil\sqrt{(2i)(2j)}\rceil \geq 4$

Thus the radial radio geometric mean condition is satisfied for all pairs of vertices.
Hence, $\emptyset$ is a valid radial radio geometric mean graceful labeling of $J(n, n)$.
Therefore, $rr_{gmn}(J(n, n)) = 2n + 4$ for $n \geq 1$. Clearly, $|V(J(n, n))| = 2n + 4, n \geq 1$
Thus, $rr_{gmn}(J(n, n)) = |V(J(n, n)|$
Hence, the Jelly fish graph J(n,n) is a Radial Radio Geometric mean graceful.

Example 2.2 The Radial Radio Geometric mean graceful labeling of $J(4,4)$ is in Figure 2.2

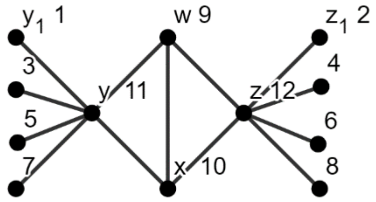

Figure 2.2

Theorem 2.3 The Web graph Wb_n is radial radio geometric mean graceful for *n* ≥ 3.

Proof. Let y_i *and* $z_i, 1 \leq i \leq n$ *be the vertices of a helm* H_n with center vertex x. Join $y_i z_i, 1 \leq i \leq n$. *Let* $x_i, 1 \leq i \leq n$ *be the pendant vertices which joined with* $y_i, 1 \leq i \leq n$ The resultant graph is Wb_n whose edge set is $E(Wb_n) = \{xz_i, z_iy_i, y_ix_i / 1 \leq i \leq n\} \cup \{z_iz_{i+1}, y_iy_{i+1} \leq i \leq n-1\} \cup \{z_nz_1, y_ny_1\}$ and radius of Wb_n is *r* = 3. Define a function $\emptyset: V(Wb_n) \rightarrow N$ by $\emptyset(x) = n + 1; \emptyset(x_i) = i, 1 \leq i \leq n; \emptyset(y_i) = n + i + 1, 1 \leq i \leq n; \emptyset(z_i) = 2n + i + 1, 1 \leq i \leq n;$

Now we verify the radial radio geometric mean condition d(x, y) + $\lceil\sqrt{\varphi(x)\varphi(y)}\rceil \geq 1 + r(Wb_n)$ for every pair of vertices of Wb_n.

case(i): verify the pair $(x, x_i), 1 \leq i \leq n. d(x, x_i) + \lceil\sqrt{\emptyset(x)\emptyset(x_i)}\rceil \geq 3 + \lceil\sqrt{(n+1)(i)}\rceil \geq 4 = 1 + r(Wb_n)$ **case(ii)**: verify the pair (x, y_i), $1 \leq i \leq n.$ $d(x, y_i) + \lceil\sqrt{\emptyset(x)\emptyset(y_i)}\rceil \geq 2 + \lceil\sqrt{(n+1)(n+i+1)}\rceil \geq 8$ **case(iii)**: verify the pair (x, z_i), $1 \leq i \leq n.$ $d(x, z_i) + \lceil\sqrt{\emptyset(x)\emptyset(z_i)}\rceil \geq 1 + \lceil\sqrt{(n+1)(2n+i+1)}\rceil \geq 9$ **case(iv)**: verify the pair $(x_i, y_j), 1 \leq i, j \leq n.$ $d(x_i, y_j) + \left\lceil\sqrt{\emptyset(x_i)\emptyset(y_j)}\right\rceil \geq 1 + \lceil\sqrt{(i)(n+j+1)}\rceil \geq 4$ **case(v)**: verify the pair $(x_i, z_j), 1 \leq i, j \leq n.$ $d(x_i, z_j) + \left\lceil\sqrt{\emptyset(x_i)\emptyset(z_j)}\right\rceil \geq 2 + \lceil\sqrt{(i)(2n+j+1)}\rceil \geq 5$ **case(vi)**: verify the pair $(y_i, z_j), 1 \leq i, j \leq n.$ $d(y_i, z_j) + \left\lceil\sqrt{\emptyset(y_i)\emptyset(z_j)}\right\rceil \geq 1 + \lceil\sqrt{(n+i+1)(2n+j+1)}\rceil \geq 10$ **case(vii)**: verify the pair $(x_ix_j), i \neq j, 1 \leq i, j \leq n.$ $d(x_i, x_j) + \left\lceil\sqrt{\emptyset(x_i)\emptyset(x_j)}\right\rceil \geq 3 + \lceil\sqrt{ij}\rceil \geq 4$ **case(viii)**: verify the pair $(y_i, y_j), i \neq j, 1 \leq i, j \leq n. d(y_i, y_j) +$

$\left\lceil\sqrt{\emptyset(y_i)\emptyset(y_j)}\right\rceil \geq 1 + \lceil\sqrt{(n+i+1)(n+j+1)}\rceil \geq 8$**case(ix)**: verify the pair $(z_i, z_j), i \neq j, 1 \leq i, j \leq n. d(z_i, z_j) + \left\lceil\sqrt{\emptyset(z_i)\emptyset(z_j)}\right\rceil \geq 1 + \lceil\sqrt{(2n+i+1)(2n+j+1)}\rceil \geq 13$

Thus the radial radio geometric mean condition is satisfied for all pairs of vertices. Hence, ∅is a valid radial radio geometric mean graceful labeling of Wb_n.

Therefore, $rr_{gmn}(Wb_n) = 3n + 1$ for $n \geq 3$. Clearly,$|V(Wb_n)| = 3n + 1, n \geq 3$. Thus,$rr_{gmn}(Wb_n) = |V(Wb_n)|$

Hence, the Web graph Wb_n is Radial Radio Geometric mean graceful. **Example 2.3** The Radial Radio Geometric mean graceful labeling of $Wb_5$is in Figure 2.3

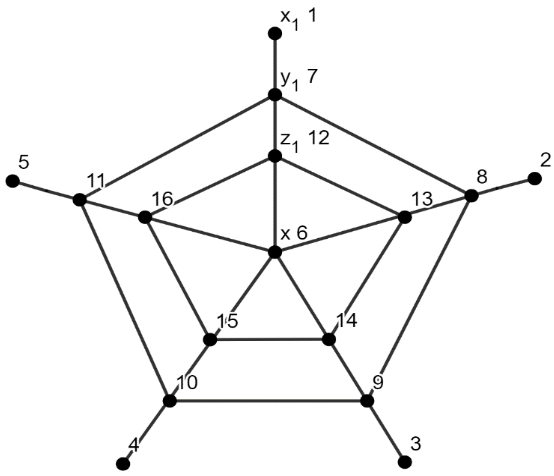

Figure 2.3

Theorem 2.4 The Gear graph$G_n$is radial radio geometric mean graceful for $n \geq 3$.**Proof.** Let $x_i, 1 \leq i \leq n$be the vertices joined to the center vertex z. Subdivide the edges $x_i x_{i+1}, 1 \leq i \leq n-1$ and $x_n x_1$ $with$ $y_i, 1 \leq i \leq n$. The resultant graph is $G_n$whose edge set is $E(G_n) = \{zx_i, x_i y_i / 1 \leq i \leq n\} \cup \{y_i x_{i+1}, 1 \leq i \leq n-1\} \cup \{y_n x_1\}$ and radius of G_n is $r(G_n) = 2$. Define a function $\emptyset: V(G_n) \to N$by $\emptyset(x_i) = i, 1 \leq i \leq n; \emptyset(y_i) = n + i, 1 \leq i \leq n; \emptyset(z) = 2n + 1$.

Now we verify the radial radio geometric mean condition d(x, y) + $\lceil\sqrt{\varphi(x)\varphi(y)}\rceil \geq 1 + r(G_n)$ for every pair of vertices of G_n.

case(i): verify the pair $(z, x_i), 1 \leq i \leq n. d(x, x_i) + \left\lceil\sqrt{\emptyset(x)\emptyset(x_i)}\right\rceil \geq 1 + \left\lceil\sqrt{(2n+1)(i)}\right\rceil \geq 3 = 1 + r(G_n)$**case(ii)**: verify the pair (z, y_i),$1 \leq i \leq n$. $d(x, y_i) + \left\lceil\sqrt{\emptyset(z)\emptyset(y_i)}\right\rceil \geq 1 + \left\lceil\sqrt{(2n+1)(n+i)}\right\rceil \geq 10$**case(iii)**: verify the pair $(x_i y_j), 1 \leq i, j \leq n$. $d(x_i, y_j) + \left\lceil\sqrt{\emptyset(x_i)\emptyset(y_j)}\right\rceil \geq 1 + \lceil\sqrt{(i)(n+j)}\rceil \geq 3$**case(iv)**: verify the pair $(x_i x_j), i \neq j, 1 \leq i, j \leq n$. $d(x_i, x_j) + \left\lceil\sqrt{\emptyset(x_i)\emptyset(x_j)}\right\rceil \geq 2 + \lceil\sqrt{(i)(j)}\rceil \geq 3$**case(v)**: verify the pair $(y_i, y_j), i \neq j, 1 \leq i, j \leq n. d(y_i, y_j) + \left\lceil\sqrt{\emptyset(y_i)\emptyset(y_j)}\right\rceil \geq 2 + \lceil\sqrt{(n+i)(n+j)}\rceil \geq 8$

Thus the radial radio geometric mean condition is satisfied for all pairs of vertices. Hence, ∅is a valid radial radio geometric mean graceful labeling of G_n. Therefore, $rr_{gmn}(G_n) =$

$2n+1$ for $n \geq 3$. Clearly,$|V(G_n)| = 2n+1, n \geq 3$Thus,$rr_{gmn}(G_n) = |V(G_n)|$ Hence, the Gear graph$G_n$is a Radial Radio Geometric mean graceful.

Example 2.4 The Radial Radio Geometric mean graceful labeling of $G_6$is in Figure 2.4

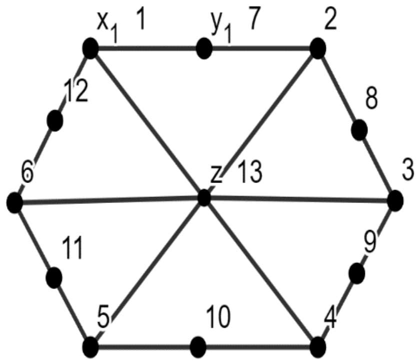

Figure 2.4

Conclusion

In this paper,we investigate the existence of radial radio geometric mean graceful labeling of Closed helm graph, Jelly fish graph, Web graph and Gear graph.

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