ξ-β-Continuousand ξ-β-generalized Continuous Maps

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Abstract.

In this paper we introduce the concept of ξ-β-continuous maps in ξ-topological spaces and investigate its various relationships with some other maps like totally ξ-β-continuous maps, perfectly ξ-β-continuous maps and stronglyξ-β-continuous maps. Further we introduce and study some ξ-β-generalized closed sets and ξ-β-generalized continuous maps inξ-topological spaces and investigate various relationship.

Keywords: *ξ-continuous maps, ξ-β-continuous maps, totally ξ-β-continuous maps, perfectly ξ-β-continuous maps, stronglyξ-β-continuous map, ξ-β-generalized closed, ξ-β-generalized continuous maps, ξ-β-generalized irresolute*

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1 Introduction

In both the pure and applied domains, the importance of general topology is quickly increasing. Information systems are fundamental instruments for generating information understanding in any real-life sector, and topological information collection structures are appropriate mathematical models for both quantitative and qualitative information mathematics.

Levine N. [11-13] studied the concept of semi-open sets, generalized closed sets and several types of continuities in topological spaces. Nour T.M [18], introduced the concept of totally semi-continuous functions and discussed several relationships by making the use of some counter examples. Singh D. [19] introduced and studied the concepts of almost perfectly continuous functions. Recently Pious Missier S. Anbarasi Rodrigo P. [24] introduced the concept of totally α^*-continuous functions in topological spaces. El-MaghrabiA.I and Nasef A.A. [25] introduced and studied semi-closed and GS-closed sets in 2009. In 2013 Narmadha A. [26], introduced and studied regular b-Open sets in topological spaces.Nithyanantha and Thangavelu[17] introduced the concept of binary topology between two sets and investigate some of the basic properties, where a binary topology from X to Y is a binary structure satisfying certain axioms that are analogous to the axioms of topology. Jamal M. Mustafa [9] studied binary generalized topological spaces and investigated the various relationships of the maps so discussed with some other maps.

As an outline, the concept of *ξ-β-continuous maps, totally ξ-β-continuous maps* and *strongly ξ-β-continuous maps*are introduced in ξ-topological spaces and investigate various relationships of these maps. Further we introduced *ξ-β-generalized closed sets, ξ-generalized β-closed sets, ξ-β-generalized continuous mapsand ξ-generalized β-ccontinuous maps* and establish the relationships of these sets and maps with some other sets and maps by making the use of some counter examples.

In this research paper, someof the required basic definitions, concepts of ξ-topological and notations are discussed in Section 2. In section 3, namely ***ξ-β-Continuous Maps***we have introduced several *ξ*-Continuousmaps and have discussed their relationships also. In section 4, headed by the concept of ***ξ-β-Generalized Closed Sets***we introduced several sets and studied their relationships. In section 5, headed by the concept of ***ξ-β-Generalized Continuous Maps***we introduced several ξ-continuous map and verify their relationships. Finally, Section 6 concludes the paper with possible scope of the concept. Throughout the paper ℘(Y) denotes the power set of Y.

2. Preliminaries

Some require and important definitions and concepts of *ξ-topological space* and notations have been given in this portion

Definition 2.1: Let Y_1 and Y_2 be any two non-void sets. Then ξ-topology (ξ_T) from Y_1 to Y_2 is a binary structure $\xi \subseteq \wp(Y_1) \times \wp(Y_2)$ satisfying the conditions i.e. $(\emptyset,\emptyset)$, $(Y_1, Y_2) \in \xi$ and If $\{(L_\alpha,\ M_\alpha)\ ;\ \alpha \in \Gamma\}$ is a family of elements of ξ, then $(\bigcup_{\alpha\in\Gamma} L_\alpha, \bigcup_{\alpha\in\Gamma} M_\alpha) \in \xi$. If ξ is ξ_T from $Y_1$to Y_2, then (Y_1, Y_2, ξ) is called a ξ-topological space ($\xi_T S$) and the elements of ξ are called the ξ-open subsets of (Y_1, Y_2, ξ). The elements of $Y_1 \times Y_2$ are called simply ξ-points.

Definition 2.2: Let Y_1 and Y_2 be any two non-void set and (L_1, M_1), (L_2, M_2) are the elements of $\wp(Y_1) \times \wp(Y_2)$. Then $(L_1, M_1) \subseteq (L_2, M_2)$ only if $L_1 \subseteq L_2$and $M_1 \subseteq M_2$.

Remark 2.1: Let $\{T_\alpha\ ;\ \alpha \in \Lambda\}$ be the family of ξ_T from $Y_1$to Y_2. Then, $\bigcap_{\alpha\in\Lambda} T_\alpha$ is also ξ_T from $Y_1$to Y_2. Further $\cup_{\alpha\in\Lambda} T_\alpha$ need not be ξ_T.

Definition 2.3: Let (Y_1, Y_2, ξ) be a ξ_TSand $L \subseteq Y_1, M \subseteq Y_2$. Then (L, M) is called ξ-closed in (Y_1, Y_2, ξ)if $(Y_1 \backslash L, Y_2 \backslash M) \in \xi$.

Proposition 2.1: Let$(Y_1, Y_2, \xi)$is ξ_TS. Then (Y_1, Y_2) and $(\emptyset,\emptyset)$ are ξ-closed sets. Similarly if $\{(L_\alpha, M_\alpha): \alpha \in \Gamma\}$ is a family of ξ-closed sets, then $(\bigcap_{\alpha\in\Gamma} L_\alpha, \bigcap_{\alpha\in\Gamma} M_\alpha)$ is ξ-closed.

Definition 2.4: Let(Y_1, Y_2, ξ) is ξ_TSand $(L, M) \subseteq (Y_1, Y_2)$. Let $(L, M)^{1^*}{}_{\xi} = \bigcap\{L_\alpha: (L_\alpha\ ,\ M_\alpha)$is ξ-closed set and $(L, M) \subseteq (L_\alpha\ ,\ M_\alpha)\}$ and $(L, M)^{2^*}{}_{\xi} = \bigcap\{M_\alpha: (L_\alpha\ ,\ M_\alpha)$is ξ-closed set and $(L, M) \subseteq (L_\alpha\ ,\ M_\alpha)\}$.Then $(L, M)^{1^*}{}_{\xi}, (L, M)^{2^*}{}_{\xi})$ is ξ -closed set and $(L, M) \subseteq (L, M)^{1^*}{}_{\xi}, (L, M)^{2^*}{}_{\xi})$. The ordered pair

$\left((L,M)^{1^*}{}_{\xi}, (L,M)^{2^*}{}_{\xi})\right)$ is called *ξ-closure* of (L,M) and is denoted $Cl_\xi(L,M)$ in $\xi_T S(X,Y,\mu)$ where $(L,M) \subseteq (Y_1,Y_2)$.

Proposition 2.2: Let $(L,M) \subseteq (Y_1,Y_2)$. Then (L,M) is ξ-open in (Y_1,Y_2,ξ) iff $(L,M) = I_\xi(L,M)$ and (L,M) is ξ-closed in (Y_1,Y_2,ξ) iff $(L,M) = Cl_\xi(L,M)$.

Proposition 2.3: Let $(L,M) \subseteq (N,P) \subseteq (Y_1,Y_2)$ and (Y_1,Y_2,ξ) is $\xi_T S$. Then $Cl_\xi(\emptyset,\emptyset) = (\emptyset,\emptyset)$, $Cl_\xi(X,Y) = (X,Y)$, $(L,M) \subseteq Cl_\xi(L,M)$, $(L,M)^{1^*}{}_{\xi} \subseteq (N,P)^{1^*}{}_{\xi}$, $(L,M)^{2^*}{}_{\xi}) \subseteq (N,P)^{2^*}{}_{\xi}$, $Cl_\xi(L,M) \subseteq Cl_\xi(N,P)$ and $Cl_\xi(Cl_\xi(L,M)) = Cl_\xi(L,M)$

Definition 2.5: Let (Y_1,Y_2,ξ) is $\xi_T S$ and $(L,M) \subseteq (Y_1,Y_2)$. Let $(L,M)^{1^0}{}_{\xi} = \cup\{L_\alpha : (L_\alpha, M_\alpha)$ is ξ-open set and $(L,M) \subseteq (L_\alpha, M_\alpha)\}$ and $(L,M)^{2^0}{}_{\xi} = \cup\{M_\alpha : (L_\alpha, M_\alpha)$ is ξ-open set and $(L,M) \subseteq (L_\alpha, M_\alpha)\}$.Then $(L,M)^{1^0}{}_{\xi}, (L,M)^{2^0}{}_{\xi})$ is ξ -open set and $(L,M)^{1^0}{}_{\xi}, (L,M)^{2^0}{}_{\xi}) \subseteq (L,M)$. The ordered pair $\left((L,M)^{1^0}{}_{\xi}, (L,M)^{2^0}{}_{\xi})\right)$ is called *ξ-interior* of (L,M) and is denoted $I_\xi(L,M)$ in $\xi_T S(X,Y,\mu)$ where $(L,M) \subseteq (Y_1,Y_2)$.

Proposition 2.4: Let $(L,M) \subseteq (Y_1,Y_2)$. Then (L,M) is ξ-open set in (Y_1,Y_2,ξ) iff $(L,M) = I_\xi(L,M)$.

Proposition 2.5: Let $(L,M) \subseteq (N,P) \subseteq (Y_1,Y_2)$ and (Y_1,Y_2,ξ) is $\xi_T S$. Then $I_\xi(\emptyset,\emptyset) = (\emptyset,\emptyset)$, $I_\xi(X,Y) = (X,Y)$, $(L,M)^{1^0}{}_{\xi} \subseteq (N,P)^{1^0}{}_{\xi}$, $(L,M)^{2^0}{}_{\xi} \subseteq (N,P)^{2^0}{}_{\xi}$, $I_\xi(L,M) \subseteq I_\xi(N,P)$ and $I_\xi(I_\xi(L,M)) = I_\xi(L,M)$

Proposition 2.6: Let (Y_1,Y_2,ξ) is $\xi_T S$ and $(Z,\mathcal{T})$ be G_T. Then the map $\mathcal{F}:(Z,\mathcal{T}) \to Y_1 \times Y_2$ is called ξ-continuous if $\mathcal{F}^{-1}(L,M)$ is $\mathcal{T}$-open in $(Z,\mathcal{T})$ for every ξ-open set (L,M) in (Y_1,Y_2,ξ).

3.*ξ*-β-Continuous Maps (ξβCM)

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In this section, the concept of *ξ-β-continuous maps,totally ξ-β-continuous maps* and *strongly ξ-β-continuous maps* in $\xi_T S$ have been introduced and established the relationships between these maps and some other maps by making the use of some counter examples.

Definition 3.1: Let (Y_1,Y_2,ξ) is $\xi_T S$. Then $(L,M) \subseteq (Y_1,Y_2,\xi)$ is said to be *ξ-β-open set* ($\xi\beta OS$) if $(L,M) \subseteq Cl_\xi(I_\xi\left(Cl_\xi(L,M)\right))$.

Definition 3.2: Let (Y_1,Y_2,ξ) is $\xi_T S$ and $(Z,\mathcal{T})$ be G_T. Then the map $\mathcal{F}:(Z,\mathcal{T}) \to Y_1 \times Y_2$ is said to be *ξ-β-continuous map* ($\xi\beta CM$) $\mathcal{F}^{-1}(L,M)$ is $\mathcal{T}$-β-open set in $(Z,\mathcal{T})$ for every ξ-open set (L,M) in (Y_1,Y_2,ξ).

Example 3.1: Let $Z = \{1,2,3\}$, $Y_1 = \{m_1,m_2\}$ and $Y_2 = \{l_1,l_2\}$. Then $\mathcal{T} = \{\emptyset,\{1\},\{1,2\},\{1,3\},\{2,3\},Z\}$ and $\xi = \{(\emptyset,\emptyset),(\{m_1\},\{l_1\}),(\{\emptyset\},\{l_2\}),(\{m_1\},\{Y_2\}),(\{m_2\},\{l_1\}),(\{Y_1\},\{l_1\}),(\{m_2\},\{Y_2\}),(Y_1,Y_2)\}$. Clearly $\mathcal{T}$ is G_T on Z and ξ is ξ_T from Y_1 to Y_2. Now define $\mathcal{F}:(Z,\mathcal{T}) \to Y_1 \times Y_2$ by $\mathcal{F}(1) = (m_2,l_1)$ and $\mathcal{F}(2) = (m_1,l_1) = \mathcal{F}(3)$. Therefore $\mathcal{F}^{-1}(\emptyset,\emptyset) = \emptyset$, $\mathcal{F}^{-1}(\{m_1\},\{l_1\}) = \{2,3\}$, $\mathcal{F}^{-1}(\{\emptyset\},\{l_2\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{m_1\},\{Y_2\}) = \{2,3\}$, $\mathcal{F}^{-1}(\{m_2\},\{l_1\}) = \{1\}$, $\mathcal{F}^{-1}(\{Y_1\},\{l_1\}) = \{2,3\}$, $\mathcal{F}^{-1}(\{m_2\},\{Y_2\}) = \{1\}$ and $\mathcal{F}^{-1}(Y_1,Y_2) = Z$. This shows that the inverse image of every ξ-open set in (Y_1,Y_2,ξ) is $\mathcal{T}$-β-open set in $(Z,\mathcal{T})$. Hence $\mathcal{F}:(Z,\mathcal{T}) \to Y_1 \times Y_2$ is ξ-β-continuous map.

Definition 3.3: Let (Y_1,Y_2,ξ) is $\xi_T S$ and $(Z,\mathcal{T})$ be G_T. Then the map $\mathcal{F}:(Z,\mathcal{T}) \to Y_1 \times Y_2$ is said to be

i) Totally ξ-β-continuous map($T\xi\beta CM$) if $\mathcal{F}^{-1}$(L, M) is$\mathcal{T}$-β-clopen in (Z, $\mathcal{T}$)for every ξ-open set (L, M)in (Y_1, Y_2, ξ).
ii) Perfectly ξ-β-continuous map($T\xi\beta CM$) if $\mathcal{F}^{-1}$(L, M) is$\mathcal{T}$-clopen in (Z, $\mathcal{T}$)for every ⍰-⍰-open set (L, M)in (Y_1, Y_2, ξ).
iii) Strongly ξ-β-continuous map(⍰⍰⍰⍰⍰) if $\mathcal{F}^{-1}$(L, M) is⍰-⍰-clopen in (Z, ⍰)for every ⍰-set (L, M)in (Y_1, Y_2, ξ).

Proposition 3.1:
i) Every *totally ⍰-⍰-continuous map* in ξ_TS is *⍰-⍰-continuous*.
ii) Every *totally ⍰-continuous map* in ξ_TS is *totally ⍰-⍰-continuous*.
iii) Every *perfectly ⍰-⍰-continuous map* in ξ_TS is *totally ⍰-⍰-continuous*.
iv) Every *strongly ⍰-⍰-continuous map* in ξ_TS is *totally ⍰-⍰-continuous*.

Proof: Let (Y_1, Y_2, ξ) is ξ_TS and (Z, ⍰) be G_T and the map $\mathcal{F}$: (Z, ⍰) → $Y_1 \times Y_2$ is totally ξ-β-continuous map. Therefore $\mathcal{F}^{-1}$(L, M) is ⍰-β-clopen in (Z, ⍰). Since ⍰-β-clopen is ⍰-β-open in (Z, ⍰). Thus $\mathcal{F}^{-1}$(L, M) is ⍰-β-open in (Z, ⍰). Hence $\mathcal{F}$: (Z, ⍰) → $Y_1 \times Y_2$ is ⍰-β-*continuous map*. The proof of (ii), (iii) and (iv) are quite analogous.

Remark 3.1: Converse of Proposition 3.1 need not true in general shown in Example 3.2, Example 3.3, Example 3.4 and Example 3.5.

Example 3.2: In Example 3.1, define $\mathcal{F}$: (Z, ⍰) → $Y_1 \times Y_2$ by $\mathcal{F}(1) = (m_1, l_1) = \mathcal{F}(2)$and $\mathcal{F}(2) = (m_2, \emptyset)$. Therefore $\mathcal{F}^{-1}(\emptyset, \emptyset) = \emptyset$, $\mathcal{F}^{-1}(\{m_1\}, \{l_1\}) = \{1,2\}$, $\mathcal{F}^{-1}(\{\emptyset\}, \{l_2\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{m_1\}, \{Y_2\}) = \{1,2\}$, $\mathcal{F}^{-1}(\{m_2\}, \{l_1\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{Y_1\}, \{l_1\}) = \{1,2\}$, $\mathcal{F}^{-1}(\{m_2\}, \{Y_2\}) = \{\emptyset\}$ and $\mathcal{F}^{-1}(Y_1, Y_2) = Z$. This shows that the inverse image of every⍰-open set in (Y_1, Y_2, ξ) is⍰-β-open set in(Z, ⍰). Hence $\mathcal{F}$: (Z, ⍰) → $Y_1 \times Y_2$ is ξ-β-continuous map but not totally ξ-β-continuous map, because because {1,2} is ⍰ -β-open but not ⍰-β-clopen in (Z, ⍰)

Example 3.3: Let $Z = \{1, 2, 3\}$, $Y_1 = \{m_1, m_2\}$ and $Y_2 = \{l_1, l_2\}$. Then ⍰ = {∅, {1,3}, {2,3}, Z} and $\xi = \{(\emptyset, \emptyset), (\{m_1\}, \{l_1\}), (\{m_2\}, \{Y_2\}), (Y_1, Y_2)\}$. Clearly ⍰ is G_T on Z and ⍰ is ξ_T from $Y_1$to Y_2. Now define $\mathcal{F}$: (Z, ⍰) → $Y_1 \times Y_2$ by $\mathcal{F}(3) = (m_2, l_2)$ and $\mathcal{F}(1) = (m_1, l_1) = \mathcal{F}(2)$. Therefore $\mathcal{F}^{-1}(\emptyset, \emptyset) = \emptyset$, $\mathcal{F}^{-1}(\{m_1\}, \{l_1\}) = \{1,2\}$, $\mathcal{F}^{-1}(\{m_2\}, \{Y_2\}) = \{3\}$ and $\mathcal{F}^{-1}(Y_1, Y_2) = Z$. This shows that the inverse image of every⍰-open set in (Y_1, Y_2, ξ) is⍰-β-clopen set in(Z, ⍰). Hence $\mathcal{F}$: (Z, ⍰) → $Y_1 \times Y_2$ is totally ξ-β-continuous map but not totally ξ-continuous, because {3} and {1,2} are ⍰ -β-clopen but not ⍰-clopen sets in (Z, ⍰)

Example 3.4: In Example 3.3, $\mathcal{F}$: (Z, ⍰) → $Y_1 \times Y_2$ is totally ξ-β-continuous map but not perfectly ξ-continuous, because inverse image of every ξ-β-open set in (Y_1, Y_2, ξ) is ⍰-β-clopen but not ⍰-clopen sets in (Z, ⍰).

Example 3.5: In Example 3.3, define $\mathcal{F}$: (Z, ⍰) → $Y_1 \times Y_2$by $\mathcal{F}(1) = (\emptyset, l_2)$, $\mathcal{F}(2) = (m_2, \emptyset)$ and $\mathcal{F}(3) =$ (⍰$_2$, ⍰$_2$). Therefore $\mathcal{F}^{-1}(\emptyset, \emptyset) = \emptyset$, $\mathcal{F}^{-1}(\{m_1\}, \{l_1\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{m_2\}, \{Y_2\}) = \{3\}$ and $\mathcal{F}^{-1}(Y_1, Y_2) = Z$. This shows that the inverse image of every⍰-open set in (Y_1, Y_2, ξ) is⍰-β-clopen set in(Z, ⍰). Hence $\mathcal{F}$: (Z, ⍰) → $Y_1 \times Y_2$ is totally ξ-β-continuous map but not strongly ξ-β-continuous, because inverse image of every ξ-set in (Y_1, Y_2, ξ) is not ⍰-clopen sets in

Relationships of Various □ -continuous mapsthat we discussed in this section:

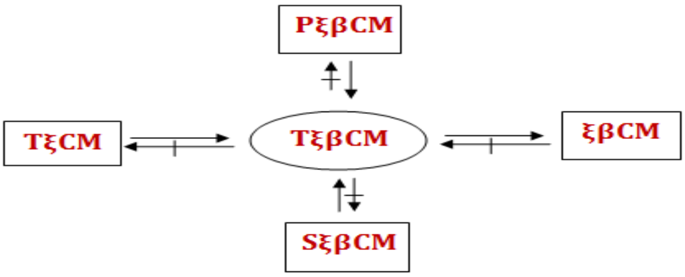

Figure-1

4.□-□-Generalized Closed Sets(□□□□□)

In this section, we have introduced and studied the concepts of*, □-□-generalized closed set*s *and □ -generalized □-closed sets.* Further,the relationships ofthese sets with some other sets have been established by making the use of some counter examples.

Definition 4.1: Let $(Y_1, Y_2, \xi)$is $\xi_T S$. Then $(L, M) \subseteq (Y_1, Y_2, \xi)$ is said to be

i) ξ-pre-open if $(L, M) \subseteq I_\xi(Cl_\xi(A, B))$.

ii) ξ-β-open if $(L, M) \subseteq Cl_\xi(I_\xi(Cl_\xi(A, B)))$

iii) ξ-b-closed if $\left(Cl_\xi(L, M)\right) \cap Cl_\xi\left(I_\xi(L, M)\right) \subseteq (L, M)$.

Definition 4.2: Let (Y_1, Y_2, ξ) is ξ_TSand $(L, M) \subseteq (Y_1, Y_2, \xi)$, then

i) $pCl_\xi(A, B) = (A, B) \cup Cl_\xi(I_\xi(A, B))$.

ii) $\beta Cl_\xi(A, B) = (A, B) \cup I_\xi(Cl_\xi\left(I_\xi(A, B)\right))$.

Definition 4.3: Let (Y_1, Y_2, ξ) is ξ_TSand $(L, M) \subseteq (Y_1, Y_2, \xi)$, then

i) $(L, M)$is *□-pre-generalized closed set* (□□□□□) if $pCl_\xi(L, M) \subseteq (U, V)$ whenver $(L, M) \subseteq (U, V)$ and (U, V) is ξ-pre-open setin (Y_1, Y_2, ξ)

ii) (L, M) is *□-generalized pre-closed set* (□□□□□) if $pCl_\xi(L, M) \subseteq (U, V)$ whenver $(L, M) \subseteq (U, V)$ and (U, V) is ξ-open setin (Y_1, Y_2, ξ)

iii) $(L, M)$is *□-□-generalized closed set*(□□□□□)if $\beta Cl_\xi(L, M) \subseteq (U, V)$ whenver $(L, M) \subseteq (U, V)$ and (U, V) is ξ-open setin (Y_1, Y_2, ξ)

iv) (L, M) is *□ -generalized □-closed set* (□□□□□) if $\beta Cl_\xi(L, M) \subseteq (U, V)$ whenver $(L, M) \subseteq (U, V)$ and (U, V) is ξ-β-open setin (Y_1, Y_2, ξ)

v) $(L, M)$is *□-b-generalized closed set*(□□□□□)if $b(L, M) \subseteq (U, V)$ whenver $(L, M) \subseteq (U, V)$ and (U, V) is ξ-open setin $(\Upsilon_1, \Upsilon_2, \xi)$

vi) (L, M) is *ξ -generalized b-closed set*($\xi GbCS$) if $bCl_\xi(L, M) \subseteq (U, V)$ whenver $(L, M) \subseteq (U, V)$ and (U, V) is ξ-b-open setin $(\Upsilon_1, \Upsilon_2, \xi)$

Proposition 4.1:

i) Every ξ-generalized pre-closed set in$\xi_T S$ is ξ-pre-generalized closed

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ii) Every ξ-generalized β-closed set in$\xi_T S$ is ξ-β-generalized closed
iii) Every ξ-generalized b-closed setin$\xi_T S$ is ξ-b-generalized closed

Proof: Obvious

Remark 4.1: The Converse of Proposition 4.1 is not true in general shown in Example 4.1, Example 4.2 and Example 4.3.

Example 4.1: Let $\Upsilon_1 = \{m_1, m_2, m_3\}$ and $\Upsilon_2 = \{l_1, l_2, l_3\}$. Then $\xi = \{(\emptyset, \emptyset), (\{m_1\}, \{l_1, l_2\}), (\{m_2, m_3\}, \{l_3\}), (\{l_1, l_3\}, \{\Upsilon_2\}), (\Upsilon_1, \Upsilon_2)\}$. Clearly ξ is ξ_T from Υ_1 to Υ_2. Now consider, $(\{m_1, m_2\}, \{l_2, l_3\}) \subseteq (\{m_1, m_2\}, \{\Upsilon_2\})$. Therefore $pCl_\xi(\{m_1, m_2\}, \{l_2, l_3\}) = (\{m_1, m_2\}, \{l_2, l_3\}) \subseteq (\{m_1, m_2\}, \{\Upsilon_2\})$, where $(\{m_1, m_2\}, \{\Upsilon_2\})$ is ξ -pre-open. Therefore $(\{m_1, m_2\}, \{l_2, l_3\})$ is ξ -pre-generalized closed but not ξ-generalized pre-closedbecause $(\{m_1, m_2\}, \{\Upsilon_2\})$ is ξ-pre-open but not ξ-open.

Example 4.2: In Example 4.1 the set $(\{m_1, m_2\}, \{l_2, l_3\})$ is ξ-β-generalized closed but not ξ-generalized β-closed because $(\{l_1, l_3\}, \{\Upsilon_2\})$ is ξ-β-open but not ξ-open set.

Example 4.3: In Example 4.1 the set $(\{m_1, m_2\}, \{l_2, l_3\})$is ξ-b-generalized closed but not ξ-generalized b-closed because $(\{l_1, l_3\}, \{\Upsilon_2\})$ is ξ-b-open but not ξ-open set.

Relationships of Various ξ -continuous maps that we discussed in this section

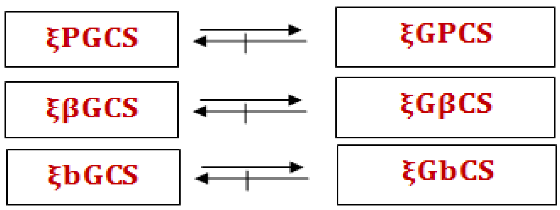

Figure-2

5. ξ-β-Generalized Continuous Maps (ξβGCM)

In this section, we have introduced and studied the concepts of *ξ-β-generalized continuous mapsand ξ -generalized β-ccontinuous maps.* Further, the relationships of these maps with some other maps have been established by making the use of some counter examples.

Definition 5.4: Let $(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$ and $(Z, \mathcal{T})$be G_T. Then the map $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is said to be

i) *ξ-β-Continuous Map(ξβCM)*$\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}$-β-closed in $(Z, \mathcal{T})$ for every ξ-closed set (L, M)in $(\Upsilon_1, \Upsilon_2, \xi)$.
ii) *ξ-β-Generalized Continuous Map(ξβGCM)*$\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}$-β-generalized closed in $(Z, \mathcal{T})$ for every ξ-closed set (L, M)in $(\Upsilon_1, \Upsilon_2, \xi)$.
iii) *ξ-β-Generalized Irresolute(ξβGI)*$\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}$-β-closed in $(Z, \mathcal{T})$ for every ξ-β-closed set (L, M)in $(\Upsilon_1, \Upsilon_2, \xi)$.
iv) *ξ-β-Irresolute(ξβI)*$\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}$-β-generalized closed in $(Z, \mathcal{T})$ for every ξ-β-generalized closed set (L, M)in $(\Upsilon_1, \Upsilon_2, \xi)$.
v) *ξ-b-Continuous Map(ξβCM)*$\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}$-b-closed in $(Z, \mathcal{T})$ for every ξ-closed set (L, M)in $(\Upsilon_1, \Upsilon_2, \xi)$.

Proposition 5.1:

i) Every ξ-β-Continuous Map in $\xi_T S$ is ξ-β-Continuous Map
ii) Every ξ-β-Continuous Map in $\xi_T S$ is ξ-β-Irresolute
iii) Every ξ-β-Continuous Map in $\xi_T S$ is ξ-β-Generalized Irresolute
iv) Every ξ-β-Irresolute in $\xi_T S$ is ξ-β-Generalized Continuous Map
v) Every ξ-β-Irresolute in $\xi_T S$ is ξ-β-Generalized Irresolute

Proof: Follows from definitions

Remark 5.1: The Converse of Proposition 5.1 is not true in general shown in Example 5.1, Example 5.2, Example 5.3, Example 5.4 and Example 5.5.

Example 5.1: Let $Z = \{1,2,3,4\}$, $\Upsilon_1 = \{m_1, m_2, m_3\}$ and $\Upsilon_2 = \{l_1, l_2, l_3\}$. Then $\mathcal{T} = +\{\emptyset, \{1\}, \{3,4\}, \{1,2,4\}, \{1,3,4\}\, Z\}$ and $\xi = \{(\emptyset, \emptyset), (\{m_1\}, \{l_1\}), (\{m_2\}, \{l_2\}), (\{m_2\}, \{\Upsilon_2\}), (\Upsilon_1, \Upsilon_2)\}$. Clearly $\mathcal{T}$ is G_T on Z and ξ is ξ_T from Υ_1 to Υ_2. Now define $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ by $\mathcal{F}(1) = (m_1, l_1) = \mathcal{F}(2)$ and $\mathcal{F}(3) = (m_2, \emptyset) = \mathcal{F}(4)$. Therefore $\mathcal{F}^{-1}(\emptyset,\ \emptyset) = \emptyset$, $\mathcal{F}^{-1}(\{m_1\},\ \{l_1\}) = \{1,2\}$, $\mathcal{F}^{-1}(\{m_2\}, \{l_2\}) = \{\emptyset\}$ and $\mathcal{F}^{-1}(\Upsilon_1,\ \Upsilon_2) = Z$.This shows that the inverse image of everyξ-closed set in $(\Upsilon_1, \Upsilon_2, \xi)$ is $\mathcal{T}$ - β -generalized closed setin $(Z, \mathcal{T})$. Hence $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is ξ - β -generalized continuous map but notξ-β-continuous, because the set$\{1,2\}$ are $\mathcal{T}$ -β-generalized closed but not $\mathcal{T}$-β-closed set in $(Z, \mathcal{T})$

Example 5.2: Let $Z = \{1,2,3,4\}$, $\Upsilon_1 = \{m_1, m_2, m_3\}$ and $\Upsilon_2 = \{l_1, l_2, l_3\}$. Then $\mathcal{T} = \{\emptyset, \{1\}, \{3,4\}, \{1,2,4\}, \{1,3,4\}\, Z\}$ and $\xi = \{(\emptyset, \emptyset), (\{m_1\}, \{l_1\}), (\{m_2\}, \{\Upsilon_2\}), (\Upsilon_1, \Upsilon_2)\}$. Clearly $\mathcal{T}$ is G_T on Z and ξ is ξ_T from Υ_1 to Υ_2. Now define $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ by $\mathcal{F}(1) = (m_2, l_2) = \mathcal{F}(3)$ and $\mathcal{F}(2) = (\emptyset, l_1) = \mathcal{F}(4)$. This shows that the inverse image of everyξ-β-closed set in $(\Upsilon_1, \Upsilon_2, \xi)$ is$\mathcal{T}$-β-closed set in $(Z, \mathcal{T})$. Hence $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is ξ-β-Irresolute but not ξ-β-continuous, because $\mathcal{F}^{-1}(\{m_2\}, \{l_2\}) = \{1,3\}$, where the set $(\{m_2\}, \{l_2\})$ is ξ-β-closed set but not ξ-closed in $(\Upsilon_1, \Upsilon_2, \xi)$

Example 5.3: In Example 5.2, $\mathcal{F}$ is ξ-β-generalized irresolute but not ξ-β-continuous because $\mathcal{F}^{-1}(\{a_2\}, \{b_2\}) = \{1,3\}$, where the set $(\{a_2\}, \{b_2\})$ is ξ-β-generalized closed but not ξ-closed.

Example 5.4: In Example 5.1, $\mathcal{F}$ is ξ-β-generalized continuous but not ξ-β-irresloute because the set $\{1,2\}$ is $\mathcal{T}$-β-generalized closed but not $\mathcal{T}$-β-closed.

Example 5.5: In Example 5.2, $\mathcal{F}$ is ξ-β-generalized irresolute but not ξ-β-irresolute because $\mathcal{F}^{-1}(\{a_2\}, \{b_2\}) = \{1,3\}$, where the set $(\{a_2\}, \{b_2\})$ is ξ-β-generalized closed but not ξ-β-closed.

Relationships of Various ξ -continuous maps that we discussed in this section:

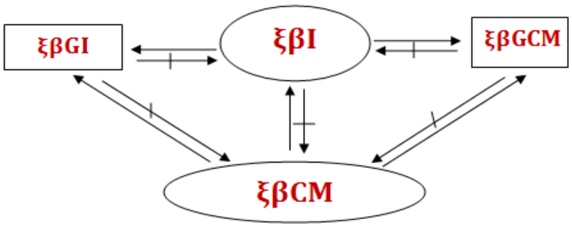

Figure-3

6. Conclusion

In this paper, a very useful concept of*ξ-β-continuous maps,totally ξ-β-continuous maps* and *strongly ξ- β-continuous maps* in ξ-topological spaces have been introduced and established the relationships between these maps and some other maps. Further we introduced the concepts of *ξ-β-generalized closed sets,ξ-generalized β-closed sets, ξ-β-generalized maps and ξ- β-irresolutes with* the relationships of these particular types of sets and maps in ξ-topological spaces. All the relationships have been verified by making the use of some examples.

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