The Monophonic Metric Dimension of The Total and The Middle Graphs of a Graph

[1]J. SUJI PRIYA AND [2]T. MUTHU NESA BEULA
[1] Research Scholar,(Reg. No:20113162092014),
Department of Mathematics,
Women's Christian College, Nagercoil.
email: sujimrthy@gmail.com
[2]Assistant Professor, Department of Mathematics,
Women's Christian College, Nagercoil.
email: tmnbeula@gmail.com
Affiliated to ManonmaniamSundaranar University, Abishekapatti,
Tirunelveli- 627 012

Abstract

Let $G = (V, E)$ be a simple graph and $M = \{v_1, v_2, \dots, v_k\} \subset V(G)$ be an ordered set and $v \in V(G)$. The representation $mr(v/M)$ of v with respect to $M$is the k-tuple $\left(d_m(v, v_1), d_m(v, v_2), \dots, d_m(v, v_k)\right)$. Then M is called a monophonic resolving set if different vertices of G have different representations with respect to M. A monophonic resolving set of minimum number of elements is called a minimum monophonic set for G and its cardinality is known as the monophonic *metric dimension*of G, represented by $mdim(G)$. In this article, we determined the monophonic metric dimension of the total and the middle graphs of a graphs.

Keywords, total graph,middle graph,chord, monophonic path, monophonic distance, metric dimension, monophonic metric dimension.
DOINumber:10.48047/nq.2022.20.22.NQ10335 NeuroQuantology2022;20(22):3373-3377

AMS Subject Classification: 05C12, 05C38.

1.Introduction

Let $G = (V, E)$ be a simple undirected connected graph. The *order* and *sizeof* G are denoted by n and m respectively.For basic graph theoretic terminology, we refer [1]. A path $P's$ chord is an edge that connects two of its non-adjacent vertices. If a path between two vertices u and v in a connected graph G lacks chords, it is referred to as *monophonic path*. The length of the longest $u - v$ monophonic path in G is the monophonic distance $d_m(u, v)$ between $u$and v. These concepts were studied in [4,5].

Let $G = (V, E)$ be a simple graph and $M = \{v_1, v_2, \dots, v_k\} \subset V(G)$ be an ordered set and $v \in V(G)$. The representation $mr(v/M)$ of v with respect to W is the k-tuple $\left(d_m(v, v_1), d_m(v, v_2), \dots, d_m(v, v_k)\right)$.Then M is called a monophonic resolving set if different vertices of G have different representations with respect to M. A monophonic resolving set of minimum number of elements is called a minimum monophonic set for G and its cardinality is known as the monophonic *metric dimension* of G, represented by $mdim(G)$.Any monophonic resolving set of cardinality $mdim(G)$ is called $mdim$ -set of G.These concepts were studied in [2,3,6,7]. In this article, we study a new metric dimension of the total and middle graphs of a graph.

2. The monophonic metric dimension of the total and the middle graphs of a graph

Total graph

The total graph $T(G)$ of the graph G is a graph such that the vertex set of T corresponds to the vertices and edges of G and two vertices are adjacent in T if and only if their corresponding elements are either adjacent or incident in G.

Theorem.2.1. Let $G = T(P_n)$be the total graph of $P_n, n \geq 3$.Then $mdim(G) = 2$.

Proof. Given path P_n is $v_1, v_2, \dots . v_n$and$u_i = v_{i,}v_{i+1}$, $1 \leq i \leq n-1$.Then $V\big(T(P_n)\big) = \{v_i, u_j \,/\, 1 \leq i \leq n, 1 \leq j \leq n-1\}$. In$T(P_n)$, v_1 is adjacent to v_2 and u_1; $v_n$is adjacent to v_{n-1}andu_{n-1}, v_i is adjacent to $v_{i-1,}v_{i+1,}u_{i-1}$andu_i, $2 \leq i \leq n-1$; $u_1$is adjacent to $v_1, v_2, and\ u_2$,u_i is adjacent to $u_{i-1,}u_{i+1,}v_i and\ v_{i+1}$ for $2 \leq i \leq n-2$ and $u_{n-1}$is adjacent to $v_{n-1,}v_n and\ u_{n-2}$. Clearly $\big|V\big(T(P_n)\big)\big| = 2n-1\ and$

$$\big|E\big(T(P_n)\big)\big| = 4n-5.$$

Let $M = \{v_1, u_1\}$.Then

$$mr(v_1/M) = (0,1), mr(v_2/M) = (1,1), mr(v_3/M) = (3,3),$$

$$mr(v_4/M) = (4,4), \dots, mr(v_n/M) = (n,n), mr(u_1/M) = (1,0), mr(u_2/M) = (2,1),$$

$$mr(u_3/M) = (3,2), \dots, mr(u_{n-1}/M) = (n-1, n-2).$$

Since each representations are distinct,M is a monophonic resolving set of G. So that $mdim(G) \leq 2$. It is verified that no singleton subset of V is a monophonic resolving set of G, so that $mdim(G) = 2$. 3374

Theorem.2.2. Let $G = T\big(K_{1,n-1}\big)$, be the total graph of star, $K_{1,n-1}(n \geq 3)$.Then $mdim(G) = n-2$.

Proof. Let $V(K_1) = x$ and $\{v_1, v_2, v_3 \dots v_{n-1}\}$are the end vertices of$K_{1,n-1} and\ u_j = x, v_i$, $1 \leq i \leq n-1$.Then $V\big(T(G)\big) = \{x, v_i, u_j, 1 \leq i \leq n-1, 1 \leq j \leq n-1\}$. In G each$u_i$is adjacent tox, v_i and $u_j$for $1 \leq i \leq n-1, 1 \leq j \leq n-1$,and $i \neq j. x$ is adjacent to eachu_i and$v_i\ for\ 1 \leq i \leq n-1$. $v_i$is adjacent tou_j, $(1 \leq j \leq n-1)$,$i \neq j$.Hence it follows that $G(u_1, u_2 \dots u_{n-1})$is acomplete graph in G.Clearly $|V(G)| = 2n-1$ and $|E(G)| = \frac{(n-1)(n-4)}{2}$.

Let $M = \{v_1, v_2, \dots . v_{n-2}\}$.Then

$$mr(x/M) = (1,1 \dots ,1), mr(v_1/M) = (0,3,3 \dots 3), mr(v_2/M) = (3,0,3 \dots ,3),$$

$$mr(v_3/M) = (3,3,0, \dots 3), \dots, mr(v_{n-2}/M) = (\ 3,3, \dots ,0,3),$$

$$mr(v_{n-1}/M) = (\ 3,3, \dots ,3), mr(u_1/M) = (1,2,2 \dots 2),$$

$$mr(u_2/M) = (2,1,2, \dots 2), mr(u_3/M) = (2,2,1 \dots ,2), \dots,$$

$$mr(u_{n-2}/M) = (2,2, \dots 1,2), mr(u_{n-1}/M) = (2,2,2 \dots 2).$$

Since each representations are distinct M is a monophonic resolving set of G, so that $mdim(G) \leq n-2$. We prove that $mdim(G) = n-2$, On the contrary suppose that $mdim(G) \leq n-2$. Then there exist a $mdim$- set M' of G such that $|M'| \leq n-3$. Hence there exist $x, y \in V$ such that $x, y \notin M'$. Without loss of generality,let $x = v_{n-1}$ and $y = v_{n-2}$.Then $mr\big(x/M'\big) = mr\big(y/M'\big) = (3,3,3 \dots ,3)$, which is a contradiction.Therefore $mdim(G) = n-2$.

Theorem.2.3. Let $G = T(C_n)$ be the totalgraph of cycle $C_n, n \geq 5$. Tthen $mdim(G) = 3$

Proof. Let cycle$C_n$isC_n: $v_1, v_2, \dots, v_n, v_1$, $u_i = v_i v_{i+1}, 1 \leq i \leq n$ and $u_n = v_n v_1$.Then

$$V\big(T(C_n)\big) = \{v_i, u_i / 1 \leq i \leq n\}. \text{Also} \big|V\big(T(C_n)\big)\big| = 2n \text{ and} \big|E\big(T(C_n)\big)\big| = 4n.$$

Case (i) Let n be even

Let $M = \{u_1, u_2, u_3\}$. Then

$$mr(v_1/M) = (1, n, n-1), mr(v_2/M) = (1,1,n), mr(v_3/M) = (n,1,1),$$

eISSN1303-5150 www.neuroquantology.com

$(v_4/M) = (n-1,n,1), \dots, mr\left(v_{\frac{n}{2}}/M\right) = (n,1,1), \dots mr\left(v_{\frac{n}{2}+1}/M\right) = (n-1,n,1),\ mr(v_{n-1}/M) = (n-1,n-1,n), mr(v_n/M) = (n,n-1,n-1), mr(u_1/M) = (0,1,n), mr(u_2/M) = (1,0,1), \dots, mr\left(u_{\frac{n}{2}}/M\right) = (n-1,1,0), mr\left(u_{\frac{n}{2}+1}/M\right) = (n-2,n-1,1), \dots, mr(u_{n-1}/M) = (n-1,n-2,n-1), mr(u_n/M) = (1,n-1,n-2)$.

Since each representations are distinct M is a monophonic resolving set of G,swo that $mdim(G) \leq 3$. We prove that $mdim(G) = 3$. On the contrary, suppose that $mdim(G) = 2$.Then there exist $mdim$ - set $M'$of G, such that $|M'| = 2$. Without loss of generality, let$M' = \{u_1, u_2\}$. Then $mr(u_5/M') = mr(v_5/M') = (5,5)$,which is a contradiction. Therefore $mdim(G) = 3$.

Case (ii) $Let\ n$be odd

Let $M = \{u_1, u_2, u_3\}$. Then

$mr(v_1/M) = (1,n,n-1)$,$mr(v_2/M) = (1,1,n)$,$mr(v_3/M) = (n,1,1),\dots,$

$$mr\left(v_{\frac{n+1}{2}}/M\right) = (n-1,n,1), mr\left(v_{\frac{n+2}{2}}/M\right) = (n-2,n-1,n), \dots,$$

$mr(v_{n-1}/M) = (n-1,n-2,n-1), mr(v_n/M) = (n,n-1,n-2),$

$$mr(u_1/M) = (0,1,n-1), mr(u_2/M) = (1,0,1), mr(u_3/M) = (n-1,1,0), \dots,$$

$mr\left(u_{\frac{n+1}{2}}/M\right) = (n-2,n-1,1), mr\left(u_{\frac{n+2}{2}}/M\right) = (n-2,n-2,n-1), \dots,$

$mr(u_{n-1}/M) = (n-1,n-2,n-2), , mr(u_n/M) = (1,n-1,n-2)$.

Since each representations are distinct M is a monophonic resolving set of G,so that $mdim(G) \leq 3$. We prove that $mdim(G) = 3$.On the contrary suppose that $mdim(G) = 2$.Then there exist $mdim\text{-}set\ M'$of G, such that $|M'| = 2$.Without loss of generality, let$M' = \{u_1, u_2\}$.Then$mr(u_5/M') = mr(v_5/M') = (5,5)$,which is a contradiction. Therefore $mdim(G) = 3$. 3375

Middle graph

The middle graph $M(G)$ of a graph G is the graph in which the vertex set is $V(G) \cup E(G)$, and two vertices are adjacent if and only if either they are adjacent edges of G or one is a vertex of G and the other is an edge incident with it.

Theorem.2.4. Let $G = M(P_n)$ be the middle graph of P_n, $(n \geq 5)$.Then $mdim\,(G) = 2$.

Proof. Let $v_1, v_2, \dots. v_n$ be the vertices of path P_n and let $u_1, u_2, \dots u_{n-1}$ be the added vertices corresponding to the edges $e_1, e_2, \dots e_{n-1}$ of$P_n$to obtain $M(P_n)$.Then $|V(M(P_n)| = 2n-1$,and $E(M(P_n)) = 3n-4$.Let $M = \{v_1, v_n\}$.Then

$mr(v_1/M) = (0,3), mr(v_2/M) = (1,2), mr(v_3/M) = (2,1), mr(v_4/M) = (3,0), \dots, mr(v_{n-1}/M) = (n-2,1), mr(v_n/M) = (n-1,0)$,$mr(u_1/M) = (1,4)$,

$$mr(u_2/M) = (1,3), mr(u_3/M) = (2,2), mr(u_4/M) = (3,1), \dots,$$

$$mr(u_{n-1}/M) = (n-2,1), mr(u_n/M) = (n-1,1).$$

Since each representation are distinct M is a monophonic resolving set of G, so that $mdim(G) \leq 2$. It is verified that no singleton subset of G is monophonic resolving set of G.Therefore $mdim(G) = 2$

Theorem.2.5. Let $G = M(K_{1,n-1})(n \geq 3)$be the middle graph of the star $K_{1,n-1}$,.Then $mdim(G) = n-2$

Proof. Let $x, v_1, v_2, \dots. v_{n-1}$ be the vertices of star $K_{1,n-1}$ and let $u_1, u_2, \dots u_{n-1}$ be the added vertices corresponding to the edges $e_1, e_2, \dots e_{n-1}$ of$K_{1,n-1}$to obtain $M(K_{1,n-1})$.Then $\left|V\left(M(K_{1,n-1})\right)\right| = 2n-1$,and $\left|E\left(M(K_{1,n-1})\right)\right| = \frac{(n+2)(n-1)}{2}$.

Let $M = \{v_1, v_2 \dots, v_{n-2}\}$. Then

$mr(x/M) = (2,2,\dots,2)$,$mr(v_1/M) = (0,3,3\dots 3)$,$mr(v_2/M) = (3,0,3\dots,3)$,

$$mr(v_3/M) = (3,3,0,\dots 3), \dots, mr(v_{n-2}/M) = (3,3,\dots 0,3),$$

$mr(v_{n-1}/M) = (3,3,\ldots.3), mr(u_1/M) = (1,2,2\ldots 2),$
$mr(u_2/M) = (2,1,2,\ldots 2), mr(u_3/M) = (2,2,1\ldots,2),\ldots,$
$mr(u_{n-2}/M) = (2,2,\ldots 1,2), mr(u_{n-1}/M) = (2,2,2\ldots 2)$

Since each representations are distinct M is a monophonic resolving set of G. So that $mdim(G) \le n-2$. We prove that $mdim(G) = n-2$. On the contrary suppose that $mdim(G) \le n-2$.Then there exist $mdim\ set\ M'$ of G.Such that $|M'| \le n-3$. Hence there exist $x, y \in V$ Such that $x, y \notin M'$, without loss of generality. Let $x = v_n$and $y = u_{n-1}$. Then $mr(x/M') = mr(y/M') = (3,3,\ldots 3)$,which is a contradiction. Therefore $mdim(G) = n-2$.

Theorem.2.6. Let the middle graph $G = M(G)$ of cycle $C_n, n \ge 5$. Then $mdim(G) = 2$

Proof. Let $v_1, v_2, \ldots\ldots v_n$ be the vector of cycle C_n and let $u_1, u_2, \ldots . u_n$ be the added vertices corresponding to the edges $e_1, e_1 \ldots\ldots e_n$ of C_n to obtain $M(G)$.Then $|V(M(G)| = 2n$ and $|E(M(G))| = 3n$.

Case (i)n be odd, let $n = 2k+1, k \ge 2$.Let $M = \{u_1, v_1\}$.Then

$mr(u_1/M) = (0,1), mr(u_2/M) = (1,2), mr(u_3/M) = (2,3),$
$mr(u_4/M) = (3,4),\ldots, mr(u_k/M) = (k-1,k), mr(u_{k+1}/M) = (k,\mathrm{k}+1),\ldots,$
$mr(u_{2k}/M) = (2k-1,2\mathrm{k}), mr(u_{2k+1}/M) = (1,2\mathrm{k}+1), mr(v_1/M) = (1,0),$
$mr(v_2/M) = (1,2), mr(v_3/M) = (2,3), mr(v_4/M) = (3,4),\ldots,$
$mr(v_k/M) = (k-1,k), mr(v_{k+1}/M) = (k,k+1),$
$mr(v_{k+2}/M) = (k+1,k+2),\ldots, mr(v_{2k}/M) = (2k-1,2k),$
$mr(v_{2k+1}/M) = (1,2k+1).$

Since each representation are distinct,$M$is a monophonic resolving set of G. So that $mdim(G) \le 2$. It is verified that no singleton subset of G is monophonic resolving set of G.Therefore $mdim(G) = 2$.

Case (ii)n is even, Let $n = 2k, k \ge 2$.
Let $M = \{u_1, v_1\}$.Then

$mr(u_1/M) = (0,1), mr(u_2/M) = (1,2), mr(u_3/M) = (2,3),$
$mr(u_4/M) = (3,4),\ldots, mr(u_k/M) = (k-1,\mathrm{k}), mr(u_{k+1}/M) = (k,\mathrm{k}+1),$
$mr(u_{k+2}/M) = (k+1,\mathrm{k}+2),\ldots, mr(u_{2k-1}/M) = (k+3,k+4),$
$mr(u_{2k}/M) = (k+4,2\mathrm{k}),, mr(v_1/M) = (1,0), mr(v_2/M) = (1,2),$
$mr(v_3/M) = (2,3), mr(v_4/M) = (3,4),\ldots, mr(v_k/M) = (k-1,k),$
$mr(v_{k+1}/M) = (k,k+1), mr(v_{k+2}/M) = (k+1,k+2),\ldots,$
$mr(v_{2k-1}/M) = (k+3,k+4), mr(v_{2k}/M) = (k+4,2k).$

Since each representation are distinct M is a monophonic resolving set of G. So that $mdim(G) \le 2$. It is verified that no singleton subset of G is monophonic resolving set of G.Therefore $mdim(G) = 2$.

Conclusion: We investigated the idea of monophonic metric dimension of graphs in this article. We expand on this idea to create more special graphs.

References

[1] F.Buckley,andF.Harary, *Distance in graphs*, Addison-Wesley, Redwood City,CA,(1990).

[2] G.Chartrand, L. Eroh, M.A. Johnson and O.R.Oellermann,Resolvability in graphs and the metric dimension of a graph, Discrete Appl. Math. 105 (1-3), (2000),99-113.

[3] M.C.Dourado, F.Protti, and J.L.Szwarcfiter,Complexity results related to monophonicconvexity, *Discrete Applied Mathematics*,158,(2010),1268-1274.

[4] J.John and S.Panchali, The upper monophonic number of agraph, *International Journal of Mathematical Combinatorics*, 4,(2010),46-52

[5] J.John, The forcing monophonic and forcing geodetic numbers of a graph, *Indonesian Journal of Combinatorics,*4 (2) (2020), 114–125.

[6] E. M. Paluga, and S. R.

CanoyJr.Monophonic numbers of the join and Composition ofconnectedgraphs,*DiscreteMathematics*,307,(2007),1146–1154.

[7] B.Sooryanarayana,Shreedhar K. and Narahari N. On the metric dimension of the total graph of a graph,Notes on Number Theory and Discrete Mathematics,4 (22) (2016), 82-95.

[8] J.SujiPriya and.T. Muthu NesaBeula, The monophonic metric dimension of a graph, (communicated)

[9] J.SujiPriya and T.MuthuNesaBeula, The forcing monophonic metric dimension of a graph. (communicated)

[10] J.SujiPriya and.T. Muthu NesaBeula, The monophonic metric dimension of some graphs under join operation. (communicated)

.,