"The Dirac Equation and its consequence"

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ABSTRACT

A scenic view is given of the main sequence of developments related to the Dirac Equation since it was proposed as a single particleequation in an external field. Attention is drawn to the fact that analternative form of the Dirac equation in terms of antisymmetrictensor fields (ATF) had first been proposed by Ivanenko and Landau. and much later by Kählar. a formulation which has found considerableuse in lattice QCD theory. Next, the sequenceof developments in respect of two-particle relativistic equations are formed—from the Bethe—Salpeter Equation on the one hand,tothe two-particle Dirac Equations with constrained Hamiltonians onthe other. together with their points of similarity and contrast.Finally.A brief sketch is given of the Dirac Equation Via the ATFformulation (using the language of external differential forms)whose main features include among other things a doubling ofthe spin content and a unifying role in the formulation of SUSYtheories.

Key words: Gauge field, Chiral invariance,Dirac equation, Goldstone pion field,Gluon field

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1. Introduction

The completion of sixty years of the Dirac equation is a good occasion forstocktaking of the vissicitudes in physical theory since the birth of quantummechanics. It has been a long Odessy, successively through Field Theory. QED.Covariant Formalism, and Yang-Mills gauge fields, followed by a temporary(decade old) eclipse due to the onslaught of the Bootstrap Philosophy characterizedby analyticity, UnitarityRegge trajectories and Nuclear Democracy. The subsequentrevival of Field theory was heralded by Spontaneous Symmetry breaking.Higgs mechanism and the Glashow-Weinberg-Salam Theory. This phase alsocoincided with the discovery of quark substructures, gluon fields, asymptoticfreedom and confinement, leading to QCD as the candidate theory of stronginteractions. On the other hand, the lack of an analytic solution of the QCDLagrangian theory for a formal understanding of Confinement led to the developmentof Lattice Gauge Theory with its ever-widening sophistications. A paralleland theoretically more provocative development has occurred in the direction ofSUSY. SUGRA. Strings and Super Strings together with their huge fallout andever expanding demand for pure intellectual exercises.In this emerging scenario.it is pertinent to ask what underlying role (if any)the Dirac equation (DE) and its subsequent incarnations (ramifications) have playedtowards the shaping of modem particle theory. For it isnot enough to pay a one-timelip-service to the DE for laying the foundations of field theory through the synthesisof Relativity with Quantum Theory. True. the profound concept of Anti-matterwhich was the direct result of this synthesis has been one of the biggest discoveries of this Century (followed only by Parity Violation and the Quark Hypothesis) andwould alone entitle this equation to a unique place under the Sun, even if it weretohave no further role in the subsequent developments. Nevertheless the subsequent incamations of the DE have proved no less relevant to the active developments in particle theory. In this report. I propose to give a panoramic view of the main sequence of developments related to the DE since it was first proposed.[1]

2. Singe Particle Dirac Equation

The DE for a single particle was conceived at an exact equation in a free field as well as in an extermale.m. field, viz.

$$-[-i\hbar\vec{\alpha}.\,\partial+mc^2\beta]\Psi=E\Psi \qquad (2.1)$$

$$H_A\Psi\equiv\left[-i\hbar\vec{\alpha}.\vec{\partial}-\left(\frac{ie}{c}\vec{A}\right)+mc^2\beta+e\emptyset\Psi=E\Psi\right] \qquad (2.2)$$

respectively. This equation had all the nice properties of Lorentz and gauge invariance but a closer examination soon revealed its fuller ramifications, viz., the impossibility of a consistent single particle equation which obeys both quantum theory andrelativity. This was manifested through the device of a negative energy sea (and holes in it) to overcome the problem of negative energy transitions.The probability density had to be reinterpreted as a "average" charge density which could locally have either (+) or (-) concentrations. And this knowledge was the forerunner of quantum Field Theory characterized by an infinite number of d.o.f.'s and non-conservation of particle number.

At this stage.to keep the historical perspective right, it is necessary to draw attention to a lesser known form of the Dirac equation which was proposed by Ivanenko and Landau'. almost simultaneously with Dirac's seminal paper but had somehow been forgotten until it was rediscovered by Kähler 3 decades later. The usefulness of this alternative formulation in terms of antisymmetric tensor fields (ATF) was not immediately clear but has become particularly convincing in the more recent context of the lattice formulation of the DE in QCD theory. Indeed, anaive transcription of the usual DE on the lattice shows a higher degeneracy of the energy spectrum than in the Continuum, a disease which gets naturally cured in

the alternative ATF formulation. A brief description of the ATF formulation is given later in this report to bring out its main features, especially the closely parallel formulation it affords for both boson and fermion fields as well as several otherdesirable mathematical properties.

Before ending this summary of the single-particle DE it is worthwhile mentioningan important application of this equation in the more recent context of chiral potential models[4]. on the assumption that constituent quarks satisfy the Diracequation with "Scalar" (S) and for "vector" (V) potentials. Breaking of Chiralinvariance due to the S-term is sought to be balanced by a Goldstone pion fieldcoupled to the surface of the hadron through a suitable boundarycondition. Thusin the equation

$$[\gamma_\mu\delta_\mu+m(r)]\Psi(\vec{r})=0 \qquad (2.3)$$

the mass tem *m(r)* in general violates chiral invariance, since

$$\left[\delta^{\mu}\overline{\Psi}\gamma_\mu\gamma_5\frac{1}{2}\tau^{\lambda}\Psi=-im(r)\overline{\Psi}i\gamma_5\tau^{\lambda}\Psi: \qquad (2.4)\right]$$

To restore Chiral invariance. it is necessary to add a pseudoscalar field Ø (r) to give a odified axial current

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$$A_{\mu}^{\lambda} = \overline{\Psi} i \gamma_{\mu} \gamma_{5} \frac{1}{2} \tau^{\lambda} \Psi - i C_{\pi} \partial_{\mu} \emptyset^{\lambda} \qquad (2.5)$$

The latter satisfies $\partial_{\mu} A_{\mu}$ = 0 provided the source j^{2} of this field is identified as

$$\partial_{\mu}\partial^{\mu}\emptyset^{\lambda} = j^{\lambda} = \frac{m(r)}{C_{\pi}} \overline{\Psi} \gamma_{5} \tau^{\lambda} \Psi \qquad (2.6)$$

This formalism has had some limited success in the construction of certain low energy pionic couplings for hadrons.[4]

We now tum to the Dirac equation for two particles.

3. Two-Particle Dirac Equation: Bethe-Salpeter Equation

A two-particle DE was first attempted by Breif[5] through a generalization ofthe single particle DE as

$$[H_0^{(1)} + H_0^{(2)} + V_{12}]\, \Psi = E\, \Psi \qquad (3.1)$$

where H_0 is given by (1) and V_{12} is the MØller interaction energy, including retardation effects. However the attempt proved premature since it satisfied neitherLorentz nor gauge invariance. The next landmark in this direction was the Bethe-Salpeter Equation (BSE)[6]**|**as the first serious attempt to construct a consistentrelativistic two-particle equation satisfying both Lorentz and gauge invariance:

$$S_F^{-1}(P) S_F^{-1}(P_2)\, \Psi(P, q) = \int K_{12}(q,q')\Psi(q')d^4 q' \qquad (3.2)$$

where the interaction Kemel K_{12} plays the role of a generalized two-body potentialand can be viewed as a sum (finite or infinite) over a sequence of irreducible Feynman diagrams, while its successive iterations are represented by the integralequation (3.2) itself. The simplest form| of K_{12} is a one - boson exchange (MØller) interaction as in Eq. (3.1) but note the vast difference in its structure from the BSE. Eq. (3.2). In a sense the BSE may be regarded as a two-particle DE. were it not forthe non-uniqueness of the kernel born out of its lack of a closed form structure.

The BSE has had a long history. While its non-relativistic limit correspondsto the ordinary Schrödinger equation.[6] its 4-D structure arising from the relativetime (X_0) dependence of the BS wave function has all along been an obstacle to itsnatural probability interpretation compatible with Lorentz invariance.Theinstantaneous approximation which amounts to putting X_0 = 0 was invoked[7] for a3-D reduction (without resort to an N-R. approximation), but the non-covariantnature of such an approximation was not conducive to applications beyond mildlyrelativistic situations. This circumstance led to alternative covariant 3-D formulationsunder the generic name of quasi-potential equations and variants thereof[9] all of which have acquired considerable significance in the quark-level context ofcompositeness. A particularly convenient form of the BSE is provided by the so-called null-plane or light-front coordinates[10] in which the role of time is played by the coordinate X = X_0-X_3, while that of the usual Z-component is taken over byX_+. = X_0 +X_3. A formal covariance may be shown to be preserved by interpreting the 3[rd]component of any 3-Vector $\vec{A}$ as A_3 = $A_+M|P_+$ where M is the mass of the composite and P_{μ} its total 4-momentum[11]. This last has the effect of constraining the internal 4-momentum q_{μ} by the condition qP = 0. A comparative discussion of various 3-D forms of the BSE, together with a fuller account of the null-plane language is given in a recent review.[12]In particular it is possible to argue[12] in favour of a formulation of the BSE on a two-tier basis[11] wherein the first stage consists in a 3-D reduction which suppresses the contribution of higher Fock states while the second stage consists in a restoration of the effect of such states through the full 4-D formulation. The first stage is appropriate for the prediction of hadron mass spectra which are hardly affected by the higher Fock states, while the second stage is suitable for the evaluation of various transition amplitudes for which the effect of the higher Fock states becomes progressively more important as the energy increases.

The nature of the inter connection between the 3-D and 4-D forms of the BSE may be illustrated" through the example of identical scalar particles interacting through an effective Kemel K:

$$i(2\pi)^4\Delta_1\Delta_2\emptyset(P, q) = \int d^4q'K(q-q')\emptyset(q') \quad (3.3)$$

$$\Delta_{1.2} = P_{1.2}^2 + m_q^2 \quad (3.4)$$

$$P_{1.2} = \frac{1}{2}P \pm q,\ P_{1.2'} = \frac{1}{2}P \pm q' \quad (3.5)$$

In the null-plane approximation (NPA), the 3-D wave function $\emptyset_-(\vec{q}_1, q_+)$ is obtained form the 4-D wave function $\emptyset$ as

$$\emptyset_-(\vec{q}_-, q_+) = \int \frac{1}{2}dq_-\emptyset(p,q) \quad (3.6)$$

Substitute of (3.6) gives for (3.3)

$$i(2\pi)^4\emptyset_- = \int\frac{1}{2}dq_-\frac{1}{\Delta_1\Delta_2}\int\frac{1}{2}dq'_-dq_+'d^2q_1'K(q-q')\emptyset(q') \quad (3.7)$$

Now the invariant argument $(q_\mu - q'_\mu)^2$ of K is effectively 3-D in content since[10] the on-shcll condition 0 = P. (q–q´) gives

$$q_- - q'_- = (q_+ - q_+')M^2 / p_+^2, \text{ and}$$

hence

$$(q_\mu - q'_\mu) = (\overrightarrow{q_1} - \overrightarrow{q_1}')^2 + (q_+ - q_+')^2M^2/P_+^2 \equiv (\overrightarrow{q} - \overrightarrow{q^1})^2 \quad (3.8)$$

Inserting (3.6) and (3.8) on the RHS of (3.7) gives the explicit 3-D form

$$D_+(\vec{q})\emptyset_-(\vec{q}) = \int d^2q_1'dq_+'K(|\vec{q} - \overrightarrow{q'}|)\emptyset_-(\overrightarrow{q'}) \quad (3.9)$$

Where $$2\pi i\, D_+^{-1} = \int\frac{1}{2}dq_-\Delta_1^{-1}\Delta_2^{-1} \quad (3.10)$$

defines the NPA denominator function D_+ which simplifies to

$$D_+(\vec{q}) = 2P_+(m_q^2 + q_1^2) - 2P_-P_{1+}P_{2+} \quad (3.11)$$

and is seen to be explicitly proportional to P_+ (as a signature of Lorentz covariance).
The *inverse* relation to (3.6) may be obtained by substituting (3.9) in (3.3) to yield

$$\emptyset(P, q) = \Delta_1^{-1}\Delta_2^{-1}\frac{1}{2m}D_+(\vec{q})\,\emptyset_-(\vec{q}) \quad (3.12)$$

Eq. (3.12) brings out the Structure of the BS vertex function $\Gamma(\vec{q})$ ~ $D_+\emptyset_-$flankedby two Scalar quark propagators $\Delta_1^{-1}\Delta_2^{-1}$ (whichincorporate the higher Fock state effects through their 4-D structure). For further details, as well as applications of this twin BS formalism, see ref. (12).

4. Two-Particle Dirac Eq. Via Constrained Hamiltonian

An alternative formulation of the two-particle DE, which was pioneered by Todorov[9] and others[13-15], consists in the use of Dirac's constrained Hamiltonian formalism which helps in controlling the relative time *(t)* in a fully Covariant manner. *Constraints* (H_1) have a twin role of not only "constraining the motion" in phase space but also of "generating" it in their Hamiltonian capacity. [15] These are mutually compatible in the sense $[H_i, H_j] \approx 0$. Such compatibility restricts thedependence of the interaction on *(t)*, and requires a "reciprocity relation" betweenthe constitutent potentials something akin to Newton's III Law. Such descriptions are valid for both two-boson (S=0) and two-fermion (S $=\frac{1}{2}$) systems, in the sense of

Coupled Klein-Gordon and Coupled Dirac Equations respectively.

Further guiding principles are needed to construct mutually compatible constraints. E.g. for S=$\frac{1}{2}$ Dirac matrices must he regarded as elements of Grassmann algebra. In this way quasi-classical constraints can be converted into quantized versions by direct operator substitution e.g., ($\square^2 + m^2$) $\Psi = 0$ comes from $p^2 + m^2 = 0$.In this context a lesser known property of the Dirac Equation comes into play. viz.. it contains a "hidden symmetry" under a transformation that mixes γ_μ' S with p_μ' S. This essentially corresponds to a SUSY transformation mixing (x_μ , p_μ) with the Grassmann variables s_μ.

To illustrate the working of this method.consider first a free spinless (S = 0) particle with action

$$A = \int L d\tau = \int -m\sqrt{-\dot{x}^2}d\tau \qquad (4.1)$$

which gives

$$\boldsymbol{H = p^2 + m^2 \approx 0 \text{ (mass shell condition)}} \qquad \textbf{(4.2)}$$

Interaction with an external Scalar field S (x) is introduced through

$$\boldsymbol{m \to M = m + S(x)} \qquad \textbf{(4.3)}$$

imilarly for two S = 0 particles, the corresponding constrained Hamiltonians are

$$\boldsymbol{H_1 \equiv P_1^2 + M_1^2 \approx 0,\ H_2^2 \equiv P_2^2 + M_2^2 \approx 0}$$

$$\boldsymbol{M_i = m_i + S_i(X).}$$

More general external fields are introduced Via:

$$m \to \boldsymbol{M = m + S(x);\ P_\mu \to H_\mu = P_\mu - A_\mu(X)} \qquad \textbf{(4.5)}$$

If $H_{1.2}$are first class constraints they satisfy the weak equality condition

$$\{\boldsymbol{H_1}, \boldsymbol{H_2}\} \approx 0 \qquad \textbf{(4.6)}$$

These relations are made more transparent through the introduction of canonical relative coordinates and momenta as[9]

$$\boldsymbol{x = x_1 - x_2, q = \frac{1}{W}(\varepsilon_2 P_1 - \varepsilon_1 P_2} \qquad \textbf{(4.7)}$$

$$\boldsymbol{P = p_1 + p_2, \qquad P^2 = -W^2} \qquad \textbf{(4.8)}$$

$$\boldsymbol{\varepsilon_i = \varepsilon_i(P^2),\ \varepsilon_1 + \varepsilon_2 = W} \qquad \textbf{(4.9)}$$

The conditions (4.4) lead to

$$\boldsymbol{H_1 - H_2 = 2P.q \approx 0, \varepsilon_1 - \varepsilon_2 = \frac{1}{W}(m_1^2 - m_2^2)} \qquad \textbf{(4.10)}$$

The other independent combination of the constraints is

$$\boldsymbol{H = \frac{\varepsilon_1}{W}H_1 + \frac{\varepsilon_2}{W}H_2 \approx H_1 \approx H_2 = q^2 - l^2 + \emptyset} \qquad \textbf{(4.11)}$$

Where $\boldsymbol{b^2 = \varepsilon_1^2 - m_1^2 = \varepsilon_2^2 - m_2^2 = \lambda(m_1^2, m_2^2, W^2)/4W^2}$ **(4.12)**

$$\boldsymbol{\emptyset = 2m_1 S_1 + S_1^2 = 2m_2, S_2 + S_2^2} \qquad \textbf{(4.13)}$$

A similar treatment is possible for two S =$\frac{1}{2}$particles. We note in passing thatthe 3-D form (3.9) of the BSE has a very similar. Structure to the Todorov equation(4.13) while its 4-D form (3.3) which incorporates higher Fock states. has no obvious counterpart in the latter.[12]

5. DE Via ATF Formulation

We end this review with a brief discussion of the alternative ATF formulation[2] of the DE which. as already noted in 2, was historically almost coincident with theoriginal Dirac formulation. This formalism has been developed quite rapidly in recent years and in the context of recent field theory models (QCD, dual strings.gauge theory of gravity, etc.). The natural language of ATF theories[16-18] is that ofgauge fields and differential forms, wherein the coordinate invariance is automatically ensured by the "exterior form" formalism onx^μ manifolds, thus facilitating a global definition of spin structure. Further, the SUSY generalization

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is also natural[18] since both Boson and Fermion fields are formulated with equal ease via ATF, and can be unified in a single geometrical object—differential form. Perhaps the most important advantage of ATF is that it can be unambiguously transferred on the lattice[19], without facing the problem of degeneracy which characterizes the ordinary Dirac field.

To illustrate the basic language (as well as the symbolism) of the ATF theory.an ATF field is best expressed as an external differential form[16]

$$\emptyset = \emptyset(x)+\emptyset_{\mu}dx^{\mu}+\frac{1}{2!}\emptyset_{\mu\nu}dx^{\mu}\wedge dx^{\nu}+ \ldots = \sum_{H}\emptyset(x,H)dx^{H} \quad (5.1)$$

(H= order set of indices).

An external differential d is expressed by

$$d\emptyset= dx^{\mu}\wedge \partial_{\mu}\emptyset \quad (5.2)$$

while the adjoint (Co) differential δ is defined by

Where the Hodge operator * has the following action $\delta = -*^{-1} d^{*}$ **(5.3)**

$$*1= dx^1\wedge d^2 \wedge dx^3 \wedge dx^4$$

$$*dx^{\mu}= (1/3!)\,\epsilon^{\mu}_{\nu p\sigma}dx^{\nu}\wedge dx^{p} \wedge dx^{\sigma}$$

$$*dx^{\mu} \wedge dx^{\nu}= -\frac{1}{2}!\varepsilon^{\mu\nu}_{p\sigma}dx^{p} \wedge dx^{\sigma}$$

Further $** = 1.d^2 = \delta^2=0$ **(5.4)**

The Ivancnko - - Landau - - Kähler Equation has the form

$$(d-\delta + m)\emptyset = 0 \quad (5.5)$$

To see the significance of the operator $(d-\delta + m)$ note that

$$(d-\delta)^2= -(d\delta + \delta d) =\square^2 = \partial_{\mu}\partial^{\mu} \quad (5.6)$$

byvirtuc of (5.4). In this sense, d-δ may be regarded as the "square root" of $\square^2$ a property which it shares with the more familiar Dirac operator $\partial=\partial_{\mu}\gamma_{\mu}$.

For an alternative demonstration of the same idea, Kähler uses the Clifford product "V" with the following properties

$$1V\,1 = 1,\ 1\,Vd^{\mu}_{x} = dx^{\mu}\,V\,1 = dx^{\mu}$$

$$dx^{\mu}\,V\,dx^{\nu}= g^{\mu\nu}.1+dx^{\mu} \wedge dx^{\nu}$$

Then

$$(d-\delta)\,\emptyset = dx^{\mu}V\partial_{\mu}\emptyset \quad (5.7)$$

Thus the Clifford product "*V*" may be compared with the corresponding multiplication with γ-matrices, viz…,

$$\gamma^{\mu}\gamma^{\nu} = \delta_{\mu\nu} + i\sigma_{\mu\nu} \quad (5.8)$$

The precise mapping is represented by

$$\gamma^{\mu} \leftrightarrow dx^{\mu}V, \quad (5.9)$$

Which translates Eq. (5.5) as

$$(d - \delta + m)\,\emptyset = (dx^{\mu}V\partial_{\mu} + m)\,\emptyset \Rightarrow (\gamma^{\mu}\partial_{\mu} + m)\ \psi = 0 \quad (5.10)$$

Again the precise correspondence between an ATF ∅Eq. (5.1) and a Dirac second order spinor**ψ** (a 4 X 4 matrix) is[18]

$$\emptyset_{\mu 1\ldots\mu 2} = (-1)^{\frac{1}{2}}k(k-1)\quad Tr\,(\psi\Gamma_{\mu_1\ldots\mu_k}) \quad (5.11)$$

$$\Gamma_{\mu_1\ldots\mu_K} = \gamma_{[\mu_1}\gamma_{\mu_2}\ldots\gamma_{\mu_K]},\ k = 1, 2, 3, 4 \quad (5.12)$$

and similarly for ∅ versus $\overline{\psi} \equiv \gamma^{\circ}\psi^{+}\gamma^{\circ}$. Under a Lorentz transformation $\emptyset^{[\mu]}$ behaves like a rank –k tensor, while transforms as

$$\psi\rightarrow \psi^{\prime} = S\psi S^{-1},\ S^{-1}\gamma^{\mu}S = \wedge^{\mu}_{\nu}\gamma^{\nu} \quad (5.13)$$

Again, as usual, the invariance of the Lagrangian under the Lorentz group implies the conservation of spin. However the "spin" now includes not only the standardDirac spin ("left-spin") but also the "right" spin which would be defined via the transformation[18]

$$\Psi \rightarrow \psi' = \psi M, \; \bar{\psi} \rightarrow \frac{-1}{\psi} = M^{-1}\bar{\psi} \qquad (5.14)$$

where "*M*" realizes the representation of the conformal group SO (4.2). Note however that the bi-spinor (4x4) representation is *not*bosonic but *fermionic*, since only anticommutation rules are consistent in this picture. Thus in the ATF picturewe have *four* Dirac fields which arc decoupled in flat space-time but coupled incurved space-time thus bringing in the role of gravity in a natural manner. For fullerdetails see ref (16- 18).

6. Conclusion

In this short report it has been attempted to sketch some of the landmarkdevelopments following from, and related to, the Dirac equation since its birth. Inproviding a perspective we have refrained from the obvious generalities whichwould merely regard the DE as a language of quantum field theory, and insteaddwelt on the more specific aspects of its developments at the quantum mechanicallevel characterised by a*finite* number of d.o.f. In this respect particular emphasis has been sought to be given to the problem of relativistic formulation of *two* particles in mutual interaction in which the Dirac Equation has played a crucial role from two independent directions, viz, the Bethe-Salpeter Equation on the one hand and Dirac's Constrained hamiltonianformalism on the other. Another, less known, aspect of the Dirac equation concerns the ATF formalism which has not only played a major unifying role in the formulation of SUSY etc, theories, but more importantly in providing a stronger theoretical basis of the formulation of lattice QCD. The Dirac equation which is the cornerstone of relativistic quantum mechanics has
maintained a continued dynamical relevance in the changing scenario of theoreticalphysics ever since it was born.

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