Using Potential Dependent Special Relativity, Gauss's Law and Schrodinger Equation to find Vacuum Quantized Energy and Elementary Particles Mass

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Abstract: Potential dependent special relativity and Gauss's law for electric field has been used for gravity by assuming vacuum energy to be generated when the energy is a minimum useful expression for vacuum energy has been found. This expression shows that elementary particles are generated by gravity vacuum filling their hollow balls; using Schrödinger equation for spherically symmetric particles vacuum energy is quantized. Treating mass as vibrating spheres thus solving Schrödinger, coordinate harmonic oscillator energy relation has been found.

Keywords: potential dependent special relativity, Gauss Law, gravity, vacuum energy, Schrödinger equation, spherical symmetry.

1. Introduction

Recent developments in cosmology and particle physics indicate that vacuum is not just an empty space devoiced from any physical meaning [1]. The Casimir effect shows clearly that vacuum has a definite energy.

According to the laws of quantum mechanics, vacuum energy results from the quantum fluctuations of fields [2]. Gauss's law describes the concept of the field using the flux notion. This notion has also been successfully used to describe the scattering process in quantum mechanics [3,4] ; Vacuum energy has been shown by many models to be responsible for solving many problems in physics. Some models proposed the so-called inflation by suggesting the existence of vacuum energy to solve many long-standing cosmological problems. These include singularity, horizon, flatness, and entropy problems [4], vacuum energy has also been suggested by many authors to explain the origin of elementary particles.

For instance, the work done by Mohammed Saeed [5], he uses generalized special relativity (GSR) to show that gravity potential field energy is transformed to elementary particle masses at the plank era [5].

Another work was done by Abeer Mohammed Khairy Ahmed et al utilized Maxwell's equations and GSR to find the photon and elementary particles, They showed that elementary rest mass has been found using Maxwell's equations, plank and De Broglie relations beside special relativity mass energy relationships [6]. The Saeed approach used the radial dependent gravity expressions, besides using very complex minimization relations. The Abeer work is based on classical GSR beside Maxwell's equation using also large number of complex relations.

The paper of S.A.orlov utilized the theory of vertex gravitation to prove that the photon must have a mass [8] he found the r gravity force on either vertex beside the centrifugal force to obtain useful expression for the photon in a dielectric medium has been takes by Nikolai B.chichkov, et al [16] in their work they utilized special relativity energy conservation principle, treating the photon as a quasi-particle, to obtain the photon mass.

Antonio Accioly and others [17] found the upper bound of the photon mass using general relativity theory; they utilized the dispersive deflection of the quantized massive electromagnetic radiation by the gravity field of the sun to perform this task [17]. All these work show the possibility of finding useful expression to obtain the photon mass.

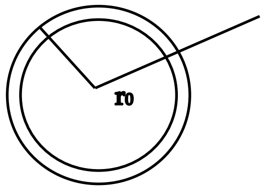
This motivates trying to use Gauss's law besides GSR and quantum laws to show that vacuum energy can generate the photon mass. International J. of Research Granth, Orlov, Vol.6 (Iss.3), March (2018).

2. Materials and Methods

Vacuum quantization and elementary particles mass generation, when we take the hollow particle model [6]. In this case, the surface density σ is related to the mass as follows:

$$M = 2\pi\, r^2 \sigma \qquad (1)$$

where : σ is mass per unit area



Thus the total potential per unit mass outside the mass is:

$$\Phi = \Phi_0 + \Phi_g$$

$$\Phi = \Phi_0 - \frac{Ma}{r} \tag{2}$$

With Φ_0 representing vacuum energy, relation can be deduced using Gausses law, According to Gauss's law outside the mass, the gravity flux is:

$$\Phi_g = \int D_g \, dA = \varepsilon_g \int E_g \, dA = M \tag{3}$$

However, inside the sphere, by consider the vacuum mass to be located at the center, Gauss law reads:

$$\Phi_g = \int D_g \, dA = \varepsilon_g \int E_g \, dA = 0 \tag{4}$$

This means that,

$$E_g = 0$$

$$\Phi_g = \int E_g . dr = 0 + c_0 = c_0 = \Phi_0 \tag{5}$$

Thus, one can write,

$$\Phi = \Phi_g + \Phi_0 = -\frac{GM}{r} + \Phi_0 \tag{6}$$

But According to GSR for $v \ll c$

$$E = \left(1 + \frac{2\Phi}{c^2}\right)^{\frac{1}{2}} \tag{7}$$

However, the energy is not imaginary, hence

$$1 + \frac{2\Phi}{c^2} \geq 0$$

The minimum value for E is when,

$$1 + \frac{2\Phi}{c^2} \tag{8}$$

Is minimum this takes place when,

$$1 + \frac{2\Phi}{c^2} = 0 \tag{9}$$

This requires:

$$\frac{2\Phi}{c^2} = -1$$

$$\Phi = -\frac{c^2}{2} \tag{10}$$

Consider the potential inside the sphere. According to Gauss law the contribution of the mass M vanishes, thus according to equations (4) and (6),($\Phi_g = 0$).

$$\Phi = \Phi_0 \tag{11}$$

Thus equation (10) gives:

$$\Phi_0 = \frac{c^2}{2} \tag{12}$$

Assuming that vacuum energy is attractive (see eqn 6):

$$\Phi_0 = \Phi_v \tag{13}$$

Thus:

$$\Phi_v = \frac{c^2}{2} \tag{14}$$

However, according to quantum laws all particles have dual nature, thus the photon can behave as electromagnetic wave as observed in our day life. Sine c is the maximum speed ($c_m = c$) thus the average speed c_e is related to c according to the relation:

$$c_e = c = \sqrt{2}c_e$$

Hence,

$$\Phi_v = \frac{2}{2}c_e^2 = c_e^2 \quad (15)$$

Thus, the vacuum potential is given by:

$$V_v = m_0\Phi_v = m_0 c_e^2 \quad (16)$$

Which is the rest mass of the particle.

Thus, according to the hollow mass model, there is no singularity, but instead one have vacuum energy inside .fortunately this vacuum energy is itself the self-mass of the particle.

Since the self-energy of elementary particles are small, thus one can also use another approach by assuming Φ to be small in eqn (7) then use the identity:

$$(1+x)^n = 1 + nx \quad (17)$$

To get E see eqn (7),

$$E = (1 + \frac{2\Phi}{c^2})^2 = \left(1 + \frac{1}{2}\frac{2\Phi}{c^2}\right) = \left(1 + \frac{\Phi}{c^2}\right) \quad (18)$$

Sine inside the hollow sphere [see eqn (11) & (13):

$$\Phi = \Phi_0 = -\Phi_v \quad (19)$$

In addition, since the self-energy is positive thus the minimum value of E is zero again,

$$E = 1 + \frac{\Phi}{c^2} = 0 \qquad \Phi = -c^2 \quad (20)$$

There for:

$$\Phi_v = -\Phi_0 = -\Phi = c^2 \quad (21)$$

Thus the vacuum potential is:

$$V_v = m_0 c^2 \quad (22)$$

For vacuum state, one can use the ordinary Schrödinger equation to see how the wave function look like and how the vacuum energy appear.

The solution of the wave function can be done by bearing in mind that the spherical particle possess spherical symmetry. Thus, one can split the wave equation to radial part R (r) and angular part $y(\theta, \Phi)$, in this case the wave function become (13).

$$\psi = \psi(r,\theta,\Phi) = R(r)y(\theta,\Phi) = Ry \quad (23)$$

$$\frac{-\hbar^2}{2m}\nabla^2 + v\psi = E\psi \quad (24)$$

In spherical coordinate, Schrödinger equation takes the form:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial\theta}\left(sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2 sin^2\theta}\frac{\partial^2\psi}{\partial\Phi^2} + \frac{2m}{\hbar^2}(E-v)\psi = 0 \quad (25)$$

m ,v and E standing for the mass, potential and total energy. Separating variables the radial part becomes,

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial R}{\partial r}\right) + \frac{2m}{\hbar^2}(E-v)r^2 R = \frac{2m_e}{\hbar^2}R \quad (26)$$

Where,

$$c = \frac{\hbar\, l(l+1)}{2m}$$

Where l is the orbital quantum number. This eqn can be simplified by considering vanishing orbital angular momentum ($l = 0$) and defining anew function v to be,

$$U = rR \tag{27}$$

Thus, eqn (27) becomes,

$$\frac{d^2u}{dr^2} = \frac{2m}{\hbar^2}(v - E)u \tag{28}$$

Consider now constant uniform vacuum energy V_v , according to equations (19) & (20)

$$E = 0$$

$$V = m\Phi = -m\Phi_v = -V_v \tag{29}$$

Thus eqn (28) reads,

$$\frac{d^2u}{dr^2} = \frac{-2m_v}{\hbar^2}u \tag{30}$$

$$u = sin\alpha\, r \tag{31}$$

$$-\alpha^2 = \frac{-2m}{\hbar^2}v_v$$

$$V_v = \frac{\hbar^2\alpha^2}{2m} \tag{32}$$

For a single particle just outside the hollow vacuum filled body no matter exist.

$$u(r_0) = \sin\alpha\, r_0 = 0 \tag{33}$$

$$\alpha r_0 = 2n\pi$$

$$n = 0,1,2,3, \ldots\ldots\ldots\ldots\ldots$$

$$\alpha = \frac{2\pi n}{r_0} \tag{34}$$

$$V_v = \frac{4\pi^2\hbar^2n^2}{2mr_0} = \frac{\hbar^2n^2}{2mr_0} \tag{35}$$

Equation (25) can be rearrange to be the form:

$$-\frac{\hbar^2}{2m}\frac{d^2u}{dr^2} + vu = Eu \tag{36}$$

Treating the particle membrane as vibrating membrane, the potential takes the form:

$$v = \frac{1}{2}kr^2 \tag{37}$$

Where the vibration take place along the radius. Thus, eqn (36) takes the form:

$$-\frac{\hbar^2}{2m}\frac{d^2u}{dr^2} + \frac{1}{2}kr^2 = Eu \tag{38}$$

This equation is typical to that of linear harmonic oscillator along the x-axis, with x replace by r.

$$u = H\, e^{-\frac{k^2}{2}r^2}$$

$$H = \sum a_s y^s \tag{39}$$

$$y = \alpha\, r \tag{40}$$

The energy in this case becomes,

$$E = \left(n + \frac{1}{2}\right)\hbar\omega \tag{41}$$

3. Discussion:

According to eqn (6) the total flux is the sum of mass flux and vacuum flux, for particle at rest vacuum energy per unit mass is equal to the square of the average speed of light as shown by equation (15) & (16) indicates that vacuum potential is itself the rest mass self-energy. This means that gravity vacuum energy background is responsible for generating the elementary particles masses [see eqn (16)]. The value of vacuum energy in equation (16) is obtain for any potential, assuming that the energy cannot be an imagining quantity, then finding the minimum energy for positive $\boldsymbol{\Phi}$ .one can also obtain the same results for weak fields there ($\boldsymbol{\Phi} < c^2$) enables simplifying energy relation (18) to get the same results in equation (22) using Schrödinger equation in spherical coordinate, assuming the particles are in the form of spheres [see eqn (25)] vacuum energy is shown to be quantized as shown in eqn (35).

Considering the particles as hollow spherical vibrating membrance the string energy is found to be that of the harmonic oscillator as shown by equation (41).

4. Conclusion:

The masses of elementary particles are generated by gravitational vacuum at early stages and at black holes where quantum laws are dominant gravity, vacuum energy is quantized.

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