NeuroQuantology| December 2022 | Volume 20 | Issue 20 |Page 3291-3300|doi: 10.48047/NQ.2022.20.20.NQ109326
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TOPOLOGICAL INSULATORS: FROM THEORY TO APPLICATIONS

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Abstract:

Topological insulators (TIs) are a novel class of materials characterized by insulating bulk properties and conductive surface states protected by time-reversal symmetry. This paper provides a comprehensive review of TIs, covering their theoretical foundations, material realizations, unique physical properties, and potential applications in spintronics, electronics, and optoelectronics. Despite the challenges in material synthesis, stability, and integration with existing technologies, TIs hold significant promise for advancing modern technology. The paper also discusses current challenges and future research directions, emphasizing the need for scalable synthesis methods and robust surface state protection.

Keywords: Topological insulators, Dirac cones, spin-momentum locking, molecular beam epitaxy, quantum computing, spintronics, optoelectronics, material synthesis, time-reversal symmetry, Berry phase, topological phases.

DOI Number: 10.48047/NQ.2022.20.20.NQ109326 NeuroQuantology2022;20(20):3291-3300

I. Introduction

A. Background

1. Definition of Topological Insulators

Topological insulators (TIs) are a class of materials that possess insulating bulk states but conductive surface states, which are protected by time-reversal symmetry. These surface states are characterized by their robustness against perturbations and impurities, making TIs a unique state of quantum matter (Hasan& Kane, 2010). The theoretical prediction and subsequent experimental confirmation of TIs have opened up new avenues in condensed matter physics and materials science (Qi & Zhang, 2011).

2. Historical Context and Discovery

The concept of TIs emerged from studies on the quantum Hall effect and the realization that topological invariants can characterize electronic states in condensed matter systems. The discovery of the quantum spin Hall effect in HgTe/CdTe quantum wells provided the first experimental evidence of a 2D TI (König et al., 2007). This was followed by the identification of 3D TIs such as Bi_2Se_3, Bi_2Te_3, and Sb_2Te_3, which exhibit strong spin-orbit coupling and topologically protected surface states (Xia et al., 2009; Chen et al., 2009).

B. Importance of Study

1. Unique Properties

TIs exhibit several unique properties that distinguish them from conventional insulators and metals. One of the most remarkable features is the presence of Dirac fermions on their surfaces, which lead to linear dispersion relations akin to those observed in graphene (Zhang et al., 2010). The spin-momentum locking in these surface states results in a helical spin texture, where the spin of an electron is perpendicular to its momentum. This property makes TIs highly resistant to backscattering from non-magnetic impurities, thereby preserving the coherence of spin states over long distances (Hsieh et al., 2009).

2. Potential Applications

The unique electronic and spintronic properties of TIs hold great promise for various technological applications. In spintronics, the efficient manipulation of spin without magnetic fields can lead to the development of low-power spin-based devices (Pesin& MacDonald, 2012). Moreover, the robust surface states of TIs can be harnessed in quantum computing to create fault-tolerant qubits based on Majorana fermions (Alicea, 2012). TIs are also being explored for applications in optoelectronics, such as in the development of novel photodetectors and solar cells (Ghaemi et al., 2013).

C. Objectives of the Paper

This paper aims to provide a comprehensive review of the theoretical foundations, material realizations, physical properties, and potential applications of topological insulators. The objectives are to:

- Summarize the fundamental theoretical concepts underlying the topological nature of these materials.
- Discuss the various experimental methods used to synthesize and characterize TIs.
- Highlight the unique electronic, optical, and magnetic properties that arise from their topological nature.
- Explore the current and potential future applications of TIs in various technological domains.
- Identify the challenges and future directions in the research and development of TIs.

II. Theoretical Foundation

A. Basic Concepts

1. Topology in Condensed Matter Physics

Topology, a branch of mathematics dealing with the properties of space that are preserved under continuous transformations, plays a crucial role in condensed matter physics. Topological phases of matter are characterized by global properties that remain invariant under local perturbations, making them robust against disorder and defects. This concept was revolutionary in understanding new states of matter beyond the traditional classification based on symmetry-breaking (Nakahara, 2003).

2. Quantum Hall Effect and Topological Invariants

The quantum Hall effect (QHE), observed in two-dimensional electron systems under strong magnetic fields, demonstrated the quantization of Hall conductance in integer multiples of e^2/h, where e is the elementary charge and h is Planck's constant (Klitzing et al., 1980). This quantization is understood through the Chern number, a topological invariant that characterizes the global properties of the electron wavefunctions over the Brillouin zone (Thouless et al., 1982). This discovery laid the foundation for understanding topological insulators, where similar invariants describe the robust edge states in the absence of magnetic fields.

B. Classification of Topological Insulators

1. 2D Topological Insulators

Two-dimensional topological insulators, also known as quantum spin Hall insulators, exhibit edge states that are protected by time-reversal symmetry. In these systems, electrons with opposite spins travel in opposite directions along the edges, forming helical edge states. The first experimental realization of a 2D TI was in HgTe/CdTe quantum wells, where the spin

Hall conductance was quantized due to the presence of these edge states (König et al., 2007).

2. 3D Topological Insulators

Three-dimensional topological insulators extend the concept of edge states to surface states. These materials have insulating bulk properties but conductive surfaces where the electrons are described by Dirac-like equations. The first 3D TIs discovered were Bi_2Se_3, Bi_2Te_3, and Sb_2Te_3, which have a single Dirac cone at the Γ point of their surface Brillouin zone (Xia et al., 2009). These surface states are protected by time-reversal symmetry and exhibit spin-momentum locking, where the electron's spin is perpendicular to its momentum (Chen et al., 2009).

C. Mathematical Framework

1. Berry Phase and Berry Curvature

The Berry phase is a geometric phase acquired over the course of a cycle, when the system is subjected to adiabatic processes. In the context of topological insulators, the Berry phase is integral to understanding the topological properties of the band structure. The Berry curvature, which is the field strength of the Berry connection, acts like a magnetic field in momentum space and plays a crucial role in defining the topological invariants of the system (Xiao et al., 2010).

2. Topological Band Theory

Topological band theory extends conventional band theory by incorporating topological aspects of electronic states. The key idea is that the electronic bands can be characterized by topological invariants, such as the Chern number in the quantum Hall effect or the Z_2 invariant in time-reversal symmetric systems (Kane &Mele, 2005). These invariants determine the presence of topologically protected edge or surface states, which are robust against perturbations that do not close the bulk band gap.

III. Material Realizations

A. Experimental Discovery

1. Bi_2Se_3, Bi_2Te_3, Sb_2Te_3 Compounds

Bi_2Se_3, Bi_2Te_3, and Sb_2Te_3 are among the first experimentally realized 3D topological insulators. These materials were chosen due to their strong spin-orbit coupling and simple band structure, which makes them ideal for studying topological properties. Bi_2Se_3, in particular, has a large bulk band gap (~0.3 eV) and a single Dirac cone on its surface, making it a model system for topological insulator studies (Xia et al., 2009; Zhang et al., 2009).

2. Transition Metal Dichalcogenides (TMDs)

Transition metal dichalcogenides (TMDs) are another class of materials that have shown promise as topological insulators. TMDs such as MoS_2, WS_2, and WTe_2 have layered structures and exhibit strong spin-orbit coupling. Recent studies have identified WTe_2 as a type-II Weyl semimetal, exhibiting unique topological properties (Soluyanov et al., 2015). These materials are also interesting due to their potential applications in valleytronics, where the valley degree of freedom in addition to spin can be manipulated.

B. Methods of Synthesis

Table 1: Summary of Experimental Techniques for TI Synthesis

Technique	Materials	Advantages	Limitations
Molecular Beam Epitaxy (MBE)	Bi_2Se_3, Bi_2Te_3, Sb_2Te_3	High precision and control over layer thickness, excellent crystal quality	Expensive equipment, slow growth rate, limited scalability
Chemical Vapor Deposition (CVD)	MoS_2, WS_2, WTe_2	Large-area film growth, good crystallinity, scalable for industrial production	Requires high temperatures, potential for non-uniform films

Physical Vapor Deposition (PVD)	Various TIs (e.g., Bi_2Se_3)	Simplicity and versatility, good control over film composition	May produce films with defects, less control over layer thickness
Pulsed Laser Deposition (PLD)	Various TIs	High-quality film growth, ability to grow complex oxides	Limited to small-area substrates, high cost of equipment
Sol-Gel Processing	Oxide-based TIs	Cost-effective, scalable, ability to produce various morphologies	Potential for impurities, requires post-deposition annealing
Electrochemical Deposition	Bi_2Te_3, Sb_2Te_3	Low-cost, scalable, room temperature processing	Limited to specific materials, less control over film uniformity
Hydrothermal Synthesis	TMDs (e.g., MoS_2)	Simple, low-temperature process, ability to produce nanostructures	Limited to specific materials, potential for impurities

1. Molecular Beam Epitaxy (MBE)

Molecular beam epitaxy (MBE) is a highly controlled technique for growing high-quality thin films of topological insulators. MBE allows for the precise control of layer thickness and composition, which is crucial for studying the quantum mechanical properties of TIs. For example, MBE has been used to grow thin films of Bi_2Se_3 and Sb_2Te_3 with atomic precision, enabling the observation of quantum confinement effects and the tuning of topological surface states (Zhang et al., 2011).

2. Chemical Vapor Deposition (CVD)

Chemical vapor deposition (CVD) is another widely used method for synthesizing TIs, particularly for growing large-area films and nanostructures. CVD involves the chemical reaction of vapor-phase precursors to form solid thin films on a substrate. This technique has been successfully employed to synthesize high-quality films of TMDs and other topological materials. CVD-grown films often exhibit excellent crystalline quality and large domain sizes, which are beneficial for device applications (Xiao et al., 2015).

C. Characterization Techniques

1. Angle-Resolved Photoemission Spectroscopy (ARPES)

Angle-resolved photoemission spectroscopy (ARPES) is a powerful technique for probing the electronic structure of materials. ARPES can directly image the band structure and Fermi surface, making it an essential tool for studying topological insulators. Using ARPES, researchers have mapped the surface states of Bi_2Se_3 and observed the characteristic Dirac cone, providing direct evidence of the material's topological nature (Hsieh et al., 2009).

2. Scanning Tunneling Microscopy (STM)

Scanning tunneling microscopy (STM) provides atomic-scale resolution imaging and spectroscopy of surface states in TIs. STM studies have revealed the spin texture of topological surface states and their robustness against non-magnetic impurities. Moreover, STM can be used to manipulate and visualize individual atoms and molecules on the surface of TIs, offering insights into their topological protection and electronic properties (Roushan et al., 2009).

IV. Physical Properties

A. Electronic Properties

1. Surface States and Dirac Cones

Topological insulators (TIs) are characterized by their unique electronic properties, most notably the presence of surface states that are protected by time-reversal symmetry. These surface states form Dirac cones, where the energy of the electrons varies linearly with their momentum. The Dirac points, where the conduction and valence bands meet, are located at the surface Brillouin zone center. This linear dispersion is similar to that found in graphene, but with significant spin-orbit coupling (Zhang et al., 2009). The robustness of these states against non-magnetic impurities and defects arises from their topological nature (Xia et al., 2009).

2. Spin-Momentum Locking

One of the hallmark features of TIs is spin-momentum locking, where the spin of an electron is locked perpendicular to its momentum. This results in helical spin textures, meaning that electrons moving in opposite directions have opposite spins. This property leads to the suppression of backscattering, which is typically caused by impurities or defects, as it would require a spin flip, which is not energetically favorable in these systems (Hsieh et al., 2009). This unique spin structure has significant implications for spintronic applications, where the control of electron spin is essential (Pesin& MacDonald, 2012).

B. Optical Properties

1. Optical Conductivity

The optical properties of TIs are governed by their electronic structure. Optical conductivity, which measures the response of a material to an external electromagnetic field, can provide insights into the nature of the surface states. TIs exhibit distinct features in their optical conductivity spectra, including interband transitions that are indicative of the Dirac-like dispersion of the surface states (Li et al., 2014). These transitions can be tuned by external parameters such as electric fields or magnetic doping, making TIs versatile materials for optoelectronic applications.

2. Faraday and Kerr Effects

The Faraday and Kerr effects, which involve the rotation of the polarization plane of light when it interacts with a material, are significantly enhanced in TIs due to their strong spin-orbit coupling. In TIs, these effects can be used to probe the surface states and their response to external magnetic fields. For instance, the Faraday effect in TIs can be used to measure the topological magnetoelectric effect, a phenomenon where an electric field induces a magnetic response and vice versa (Qi et al., 2009). These optical phenomena are promising for applications in non-reciprocal optics and optical isolation.

C. Magnetic Properties

1. Time-Reversal Symmetry Breaking

While TIs are protected by time-reversal symmetry, introducing magnetic impurities or applying an external magnetic field can break this symmetry, leading to new and exotic phases of matter. For example, breaking time-reversal symmetry can open a gap at the Dirac point, resulting in a quantum anomalous Hall effect where edge states carry a quantized Hall conductance without an external magnetic field (Chang et al., 2013). This effect has been observed in magnetically doped TIs, such as Cr-doped $(Bi,Sb)_2Te_3$, and offers new possibilities for topological phases and devices.

2. Magnetic Doping

Magnetic doping involves introducing magnetic atoms, such as Cr or Fe, into the TI lattice. This doping can induce ferromagnetism in the TI, leading to the opening of a gap at the Dirac point and enabling the observation of magnetic phenomena that are otherwise absent in pristine TIs (Checkelsky et al., 2012). The interplay between the topological surface states and magnetic order can result in novel magnetic and electronic properties, including the potential realization of magnetic monopoles and axion insulators.

V. Applications

A. Spintronics

1. Spintronic Devices

The spin-momentum locking in TIs makes them highly attractive for spintronic devices, which utilize the electron's spin rather than its charge for information processing. Spintronic devices based on TIs can achieve efficient spin injection and detection due to the robust and coherent nature of the topological surface states (Mellnik et al., 2014). This can lead to the development of spin-based transistors, memory devices, and other components that offer lower power consumption and higher speeds compared to conventional electronic devices.

2. Quantum Computing Implications

TIs are also promising candidates for quantum computing, particularly in the realization of fault-tolerant qubits. The surface states of TIs can host Majorana fermions when coupled with superconductors, which are particles that are their own antiparticles. These Majorana modes can be used to create topological qubits that are robust against local perturbations, thus offering a path toward scalable and stable quantum computers (Alicea, 2012). Research is ongoing to experimentally realize and manipulate these Majorana modes in TI-superconductor hybrid systems.

B. Electronics

1. Low-Power Electronic Devices

The unique electronic properties of TIs can be exploited to develop low-power electronic devices. The high mobility of surface states, combined with the suppression of backscattering, can lead to devices with reduced power consumption and improved performance. For example, TI-based field-effect transistors (FETs) have shown potential for high-speed and low-power operation, making them suitable for next-generation electronic circuits (Xu et al., 2014).

2. Topological Transistors

Topological transistors leverage the topological protection of surface states to achieve robust and efficient electronic switching. These devices can operate at lower voltages and with higher reliability compared to traditional transistors. The development of topological transistors could revolutionize the semiconductor industry by providing a new class of devices that combine the advantages of TIs with the functionality of conventional transistors (Saha et al., 2016).

C. Optoelectronics

1. Photodetectors

The strong spin-orbit coupling and unique surface states of TIs make them ideal materials for optoelectronic devices such as photodetectors. TI-based photodetectors can exhibit high sensitivity and fast response times due to the efficient interaction between light and the topological surface states. These devices can be tuned to operate over a wide range of wavelengths, offering versatile applications in communications, imaging, and sensing (Chen et al., 2013).

2. Solar Cells

TIs also show promise in the field of photovoltaics. The surface states can enhance light absorption and carrier separation, improving the efficiency of solar cells. Additionally, the topological protection of the surface states can lead to more stable and durable solar cell materials. Research is exploring the integration of TIs with traditional photovoltaic materials to create hybrid systems that combine the benefits of both (Zhang et al., 2015).

D. Other Emerging Applications

1. Topological Quantum Computation

Beyond the use of Majorana fermions for qubits, TIs are being investigated for their broader applications in topological quantum computation. The inherent robustness of topological phases against local perturbations can be utilized to create quantum gates and circuits that are less prone to errors, thereby enhancing the feasibility of practical quantum computers (Nayak et al., 2008).

2. Metamaterials

TIs can also be used to design novel metamaterials with unique electromagnetic properties that are not found in nature. These metamaterials can exhibit negative refractive indices, cloaking, and other exotic phenomena due to the interplay between their topological surface states and the engineered structure of the material (Lu et al., 2014). The ability to manipulate electromagnetic waves in unprecedented ways opens up new possibilities for advanced optics and communication technologies.

VI. Current Challenges and Future Directions

A. Challenges in Material Synthesis and Scalability

One of the primary challenges in the field of topological insulators (TIs) is the synthesis of high-quality materials on a large scale. Techniques like molecular beam epitaxy (MBE) and chemical vapor deposition (CVD) can produce high-quality thin films, but scaling these methods for industrial applications remains a significant hurdle (Zhang et al., 2011; Xiao et al., 2015). Additionally, controlling the stoichiometry and doping levels precisely is critical for maintaining the topological properties of these materials, which adds complexity to the synthesis process (Taskin et al., 2012).

B. Stability and Robustness of Surface States

The stability and robustness of the surface states in TIs are crucial for their practical applications. While these states are theoretically protected by time-reversal symmetry, in practice, they can be affected by external perturbations such as magnetic fields, impurities, and structural defects (Liu et al., 2012). Understanding and mitigating these effects are essential to ensure the reliable performance of TI-based devices. Research is ongoing to develop methods to enhance the robustness of these surface states, including the use of protective capping layers and advanced fabrication techniques (Miao et al., 2013).

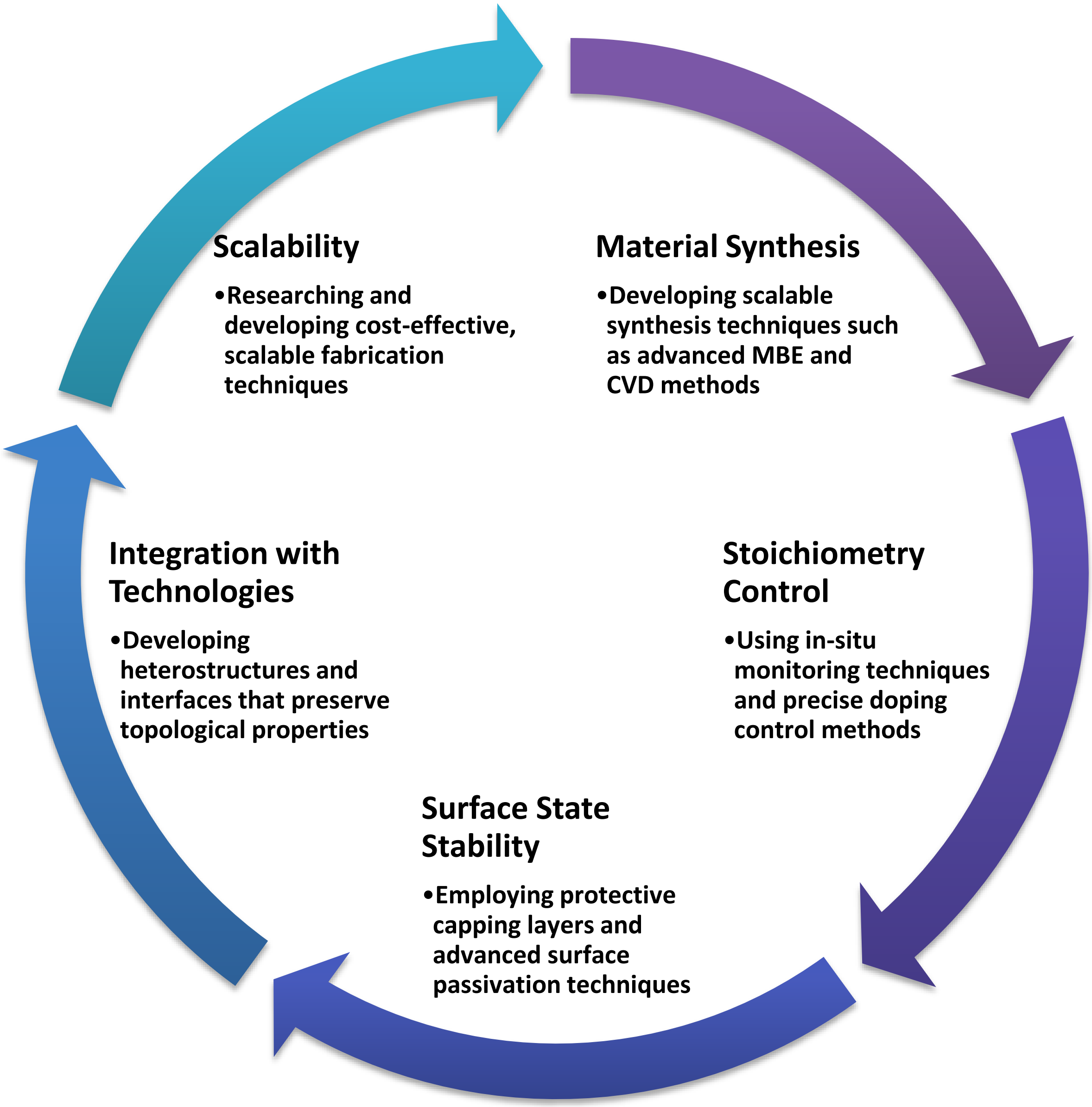

Figure 1: Challenges and Potential Solutions in TI Research

C. Integration with Existing Technologies

Integrating TIs with existing electronic and spintronic technologies presents another set of challenges. The unique properties of TIs, such as spin-momentum locking and topological protection, need to be harnessed in conjunction with conventional semiconductor materials and devices. This requires the development of heterostructures and interfaces that can preserve the topological properties while enabling efficient charge and spin transfer (Lee et al., 2014). Moreover, the compatibility of TIs with current fabrication processes must be addressed to facilitate their adoption in mainstream technology (Garate& Franz, 2010).

D. Potential Future Research Areas

The field of TIs is rapidly evolving, and several promising research areas are emerging. One area of interest is the exploration of new topological phases and materials, such as topological superconductors and Weyl semimetals, which can exhibit even more exotic properties (Wang et al., 2012). Another promising direction is the use of TIs in quantum information processing, where their robust surface states can be used to create qubits and quantum gates (Nayak et al., 2008). Additionally, the study of interaction effects in TIs, such as electron-electron and electron-phonon interactions, can provide deeper insights into their physical properties and potential applications (Hohenadler&Assaad, 2013).

VII. Conclusion

Topological insulators represent a fascinating and rapidly developing area of condensed matter physics. Their unique electronic, optical, and magnetic properties, arising from their topologically protected surface states, offer a wealth of opportunities for both fundamental research and technological innovation. Despite the challenges in material synthesis, stability, and integration, the potential applications of TIs in spintronics, electronics, optoelectronics, and quantum computing are highly promising. Continued research into new materials, topological phases, and interaction effects will further our understanding of these remarkable materials and pave the way for their practical applications in various fields. As the study of TIs progresses, it is expected to lead to significant advancements in both fundamental physics and applied technologies, ultimately contributing to the development of new and improved electronic, spintronic, and quantum devices.

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