COMPLEMENTARY TRIPLE CONNECTED SUBSTANTIAL INDEPENDENCE NUMBER FOR STRONG PRODUCT OF PATHS AND CYCLES

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Abstract:

A Complementary triple connected substantial independent set is a non-empty subset $S \subset V$ of a graph G = (V, E) if S is a substantial independent set and the induced subgraph < V - S > is triple connected. The Complementary triple connected substantial independence number is the maximum cardinality among all Complementary triple connected substantial independent sets and is indicated by β_{ctcs}. In this paper, we determine the value of Complementary Triple connected substantial independence number for strong product of paths and cycles.

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1. Introduction and Preliminaries:

By a graph we mean a finite, undirected connected graph without loops or multiple edges. Terms not defined here are used in the sense of Haynes et. al. and Harary [1]. A substantial independent set is a non-empty subset $S \subset V$ in a connected graph G = (V, E) if S is an independent set of G and any vertex in V - S is join by an edge to atmost one vertex in S. The substantial independence number of G is the supremum cardinality among all substantial independent sets in G and is indicated by $\beta_s(G)$[3]. The Strong product of two graphs is a graph with vertex set $V(G \otimes H) = V(G) \times V(H)$ and $((u_1, v_1)(u_2, v_2)) \in E(G \otimes H)$ if one of the following holds: (i) $u_1u_2 \in E(G)$ and $v_1v_2 \in E(H)$, (ii) $u_1 = u_2$ and $v_1v_2 \in E(H)$ and (iii) $u_1u_2 \in E(G)$ and $v_1 = v_2$[4]. In this paper, we determine the value of Complementary Triple connected substantial independence number for strong product of paths and cycles.

2. Main Results

2.1 COMPLEMENTARY TRIPLE CONNECTED SUBSTANTIAL INDEPENDENCE NUMBER:

Definition : 2.1

A Complementary triple connected substantial independent set is a non-empty subset $S \subset V$ of a graph G = (V, E) if S is a substantial independent set and the induced subgraph < V - S > is triple connected. The Complementary triple connected substantial independence number is the maximum cardinality among all Complementary triple connected substantial independent sets and is indicated by β_{ctcs}.

Example : 2.2

The figure :1 reveals the complete graph K_5 and its β_{ctcs}.

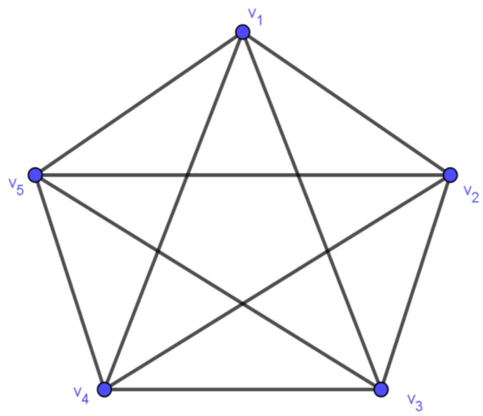

Figure.1: K_5

Here $\{v_1\}$, $\{v_2\}$, $\{v_3\}$, $\{v_4\}$ and $\{v_5\}$ are Complementary triple connected substantial independent sets with maximum cardinality.

Therefore, $\beta_{ctcs}(K_5) = 1$

Theorem : 2.3

For any two Paths P_m and P_n, $\beta_{ctcs}(P_m \otimes P_n) = \left\lceil \frac{m}{3} \right\rceil \left\lceil \frac{n}{3} \right\rceil$.

Proof:

The graph $P_m \otimes P_n$ includes m rows and n columns.

To form a maximal complementary triple connected substantial independent set, we can select vertices $v_{1,1}, v_{1,4}, v_{1,7}, \ldots$ from the first row of vertices.

Then we cannot select any vertices from the second row of vertices, because the first row of vertices is adjacent to the second row of vertices.

Also, we can select vertices $v_{4,1}, v_{4,4}, v_{4,7}, \ldots$ from the fourth row of vertices.

Then we cannot select any vertices from the third and the fifth row of vertices, because the fourth row of vertices is adjacent to the third and the fifth row of vertices.

Then we can select vertices from the seventh row and so on.

Hence the Substantial independent set S is $\{ v_{1,1}, v_{1,4}, v_{1,7}, \ldots, v_{4,1}, v_{4,4}, v_{4,7}, \ldots \}$

Also, < V - S > is triple connected.

Thus the maximal Complementary triple connected substantial independent set is of the form

$$S = \left\{ v_{1+3i,} v_{1+3j} : i = 0, 1, 2, \ldots, \left(\left\lceil \frac{m}{3} \right\rceil - 1 \right), j = 0, 1, 2, \ldots, \left(\left\lceil \frac{n}{3} \right\rceil - 1 \right) \right\}$$

Consider the set $S' = S \cup \{v_{i,j}\}$ where $v_{i,j} \in V - S$ is never a Complementary triple connected Substantial independent set, because the vertex $v_{i,j} \in V - S$ is either join by an edge to a vertex in S or join by an edge to a neighbor vertex of S or both.

But it is a contradiction to the definition of the Complementary triple connected substantial independent set, so it is never possible.

Hence S is 1-maximal, as Complementary triple connected substantial independent is hereditary.

Therefore, we may conclude that, S is a maximal Complementary triple connected substantial independent set.

Therefore, $\beta_{ctcs}(P_m \otimes P_n) = |S| = \left\lceil \frac{m}{3} \right\rceil \left\lceil \frac{n}{3} \right\rceil$.

Illustration: 2.4

Consider the graph $(P_4 \otimes P_5)$ in figure 2,

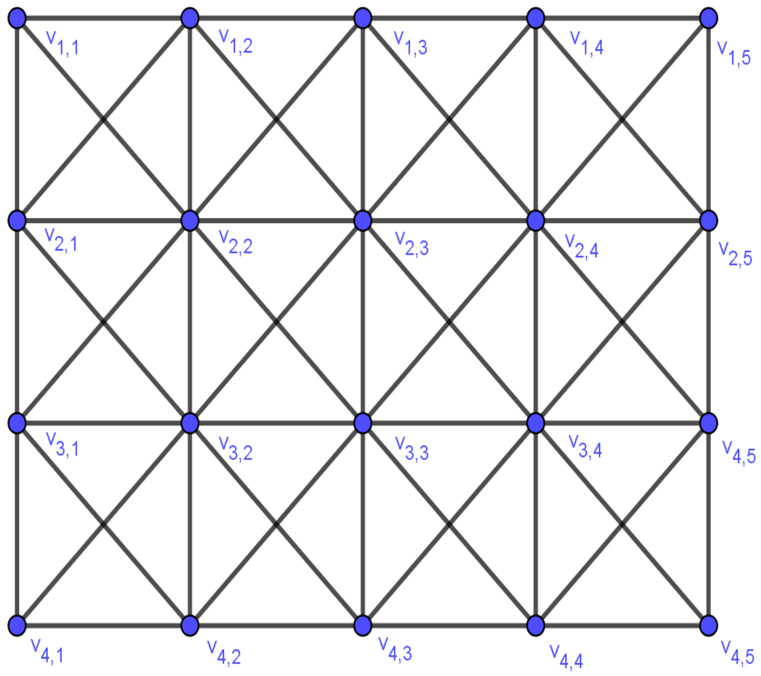

Figure 2: $(P_4 \otimes P_5)$

Here $S = \{v_{1,1}, v_{1,4}, v_{4,1}, v_{4,4}\}$ is a maximal Substantial independent set with maximum cardinality and therefore $\beta_s(P_4 \otimes P_5) = 4$. Also $< V - S >$ is triple connected. Hence S is a maximal Complementary triple connected Substantial independent set.

Therefore, $\beta_{ctcs}(P_4 \otimes P_5) = 4 = \left\lceil \frac{4}{3} \right\rceil \left\lceil \frac{5}{3} \right\rceil$.

Theorem : 2.5

If C_m is a cycle with m vertices, P_n is a path with n vertices and $m, n \geq 2$ then

$\beta_{ctcs}(C_m \otimes P_n) = \left\lfloor \frac{m}{3} \right\rfloor \left\lceil \frac{n}{3} \right\rceil$.

Proof:

The graph $C_m \otimes P_n$ includes m rows and n columns.

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To form a maximal complementary triple connected substantial independent set, we can select vertices $v_{1,1}$, $v_{1,4}$, $v_{1,7}$, . . . from the first row of vertices.

Then we cannot select any vertices from the second row and the n^{th} row of vertices, since, the first row of vertices is adjacent to the second and the n^{th} row of vertices.

Also, we can select vertices $v_{4,1}$, $v_{4,4}$, $v_{4,7}$, . . . from the fourth row of vertices.

Then we cannot select any vertices from the third and the fifth row of vertices, because the fourth row of vertices is adjacent to the third and the fifth row of vertices.

Then we can select the vertices from the seventh row and so on.

Hence the Substantial independent set S is { $v_{1,1}$, $v_{1,4}$, $v_{1,7}$, . . . , $v_{4,1}$, $v_{4,4}$, $v_{4,7}$, . . . }

Also < V - S > is triple connected.

Thus the maximal Complementary triple connected substantial independent set is of the form

$$S = \left\{ v_{1+3i,}v_{1+3j} : i = 0,1,2,.\ .\ .,\left(\left\lceil\frac{m}{3}\right\rceil - 1\right), j = 0,1,2,\ldots,\left(\left\lceil\frac{n}{3}\right\rceil - 1\right) \right\}$$

Consider the set S' = S $\cup$ {$v_{i,j}$} where $v_{i,j} \epsilon$ V - S is never a Complementary triple connected Substantial independent set, because the vertex $v_{i,j} \epsilon$ V - S is either join by an edge to a vertex in S or join by an edge to neighbor vertex of S or both.

But it is a contradiction to the definition of the Complementary triple connected substantial independent set, so it is never possible.

Hence S is 1-maximal, as Complementary triple connected substantial independent is hereditary. 3284

Therefore, we may conclude that, S is a maximal Complementary triple connected substantial independent set.

Therefore, $\beta_{ctcs}(C_m \otimes P_n) = |S| = \left\lfloor\frac{m}{3}\right\rfloor\left\lceil\frac{n}{3}\right\rceil$.

Illustration: 2.6

Consider the graph ($C_6 \otimes P_9$) in figure 3,

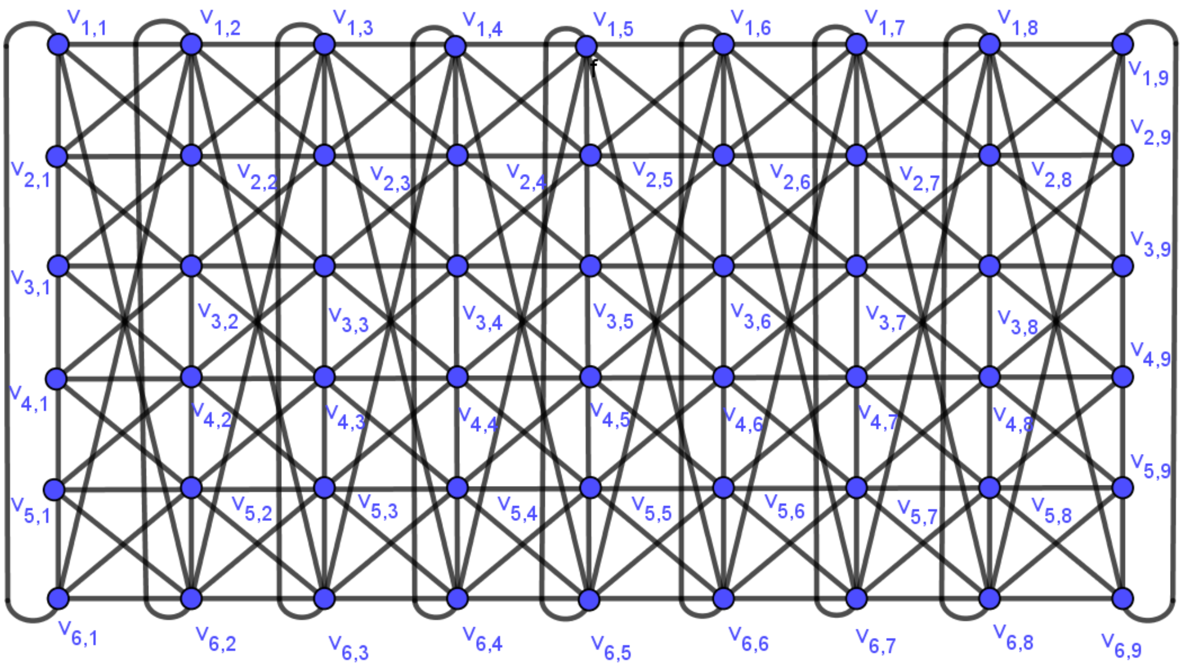

Figure 3: ($C_6 \otimes P_9$)

Here S = {$v_{1,1}$, $v_{1,4}$, $v_{4,1}$, $v_{4,4}$, $v_{7,1}$, $v_{7,4}$ } is a maximal Substantial independent set with maximum cardinality and therefore $\beta_s(C_6 \otimes P_9) = 6$. Also < V - S > is triple connected. Hence S is a maximal Complementary triple connected Substantial independent set.

Therefore, $\beta_{ctcs}(C_6 \otimes P_9) = 6 = \left\lfloor \frac{6}{3} \right\rfloor \left\lceil \frac{9}{3} \right\rceil$.

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