# Applications of the Laplace differential transform method to the solution of linear and nonlinear reaction diffusion equations

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**Abstract-**
In this research, we investigate the Laplace-Differential Transform method (LDTM) for solving the non-linear Reaction-Diffusion equation and provide our findings for both exact and approximation solutions as well as numerical solutions. In the time domain, we use the Laplace transform, and in the spatial domain, we use the differential transform with initial and boundary conditions. Unlike conventional methods, which typically include integration, we discover that this approach only necessitates straightforward differentiation and a few elementary operations for the result. The computational domain can be considerably reduced with this strategy, and only a small number of iterations are needed to provide closed-form answers in the form of series expansions of certain known functions. Numerous examples are provided to illustrate the technique's usefulness and effectiveness. Conclusions are reliable, and the computational effort required was less than that of some prior investigations.

***Keywords:*** *Laplace differential transform method, Reaction diffusion equation*
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**1. Introduction-** The field of nonlinear sciences has had huge advancement lately, bringing about the development of various logical and mathematical strategies that have tracked down application in assorted designing and logical spaces. Various mathematical frameworks and logical methods have been introduced for the arrangement of differential conditions across many sorts. The direct response dissemination condition is a fractional differential condition used to portray the elements of an amount, like focus, temperature, or populace thickness. This condition represents both dissemination and a direct response in the framework. The straight response dispersion condition in one aspect can be communicated in a general structure as follows:

$$\frac{\partial y}{\partial t} = D\frac{\partial^2 y}{\partial x^2} + R(u)$$

Where $y$ is a dependent variable, $t$ is time, $x$ is spatial variable, $D$ is the diffusion coefficient and $R(u)$ is linear term.

In nonlinear reaction-diffusion equations, the reaction term does not directly proportionately change when the dependent variable changes. A one-dimensional nonlinear reaction-diffusion equation can be written in the form:

$$\frac{\partial y}{\partial t} = D\frac{\partial^2 y}{\partial x^2} + F(u)$$

Where $y$ is a dependent variable, $t$ is time, $x$ is spatial variable, $D$ is the diffusion coefficient and $F(u)$ is nonlinear term.

While tackling straight standard differential conditions, the Laplace Change is a significant numerical instrument. It has more trouble being utilized with nonlinear conditions. As a crossover of the Laplace transform and the Differential transform method (DTM), the Laplace differential transform method (LDTM) can be utilized to settle various halfway differential conditions, including response dissemination conditions, in both the direct and nonlinear systems.

To tackle non-homogeneous straight PDEs, Alquran et al. (2012) used a mixture variant of the Laplace transform method (LTM) and the differential transform method (DTM). Non-homogeneous straight incomplete differential conditions with a variable coefficient can be settled utilizing the blend method.Utilizing common and postpone differential conditions, Rihan and Rahman (2013) researched the elements between a malignant growth and the resistant arrangement of solid effector cells with regards to the human immunodeficiency infection.Polynomial differential quadrature procedure (PDQM) was utilized by Jiwari et al. (2014) to explore summed up FN conditions with time-subordinate coefficients in 1D.Utilizing the Laplace-Differential transform method (LDTM) and Pad'e estimation, Kumari and Gupta (2016) examined rough insightful arrangements of homogeneous direct PDEs with introductory circumstances.Numerous issues in designing and the applied sciences include waves, consequently Kumari et al. (2016) utilized the Differential transform method (DTM) related to the Laplace transform method to tackle wave conditions and wave-like conditions. The outcomes show that the proposed technique is compelling and gives quickly meeting series arrangements.By building the functional network of partial subordinate, Kumar et al. (2019) tackled the partial request non-straight response dispersion issue utilizing the collocation technique. They did this by approximating any capability by utilizing the essential meanings of fragmentary request subsidiaries, the Genocchi polynomial, and the properties of the Kronecker result of frameworks.Utilizing response dissemination conditions and two particular three-layered mathematical recreations, Jaroudi et al. (2020) researched a model of mind growth movement. To settle the perplexing conditions overseeing smooth motion in permeable media, Temitayo Sheriff and Ibukun Joel (2020) presented another strategy (a cross breed of the Laplace and Differential changes) that yields answers that are both straightforward and extremely exact. The obtained results approve the adequacy, proficiency, and common sense of the proposed technique.The novel Covid (Coronavirus) pandemic in China, Spain, and Italy enlivened Laib et al. (2021) to make a mathematical model for the conventional type of an arrangement of nonlinear Volterra delay integro-differential conditions.For Giesekus viscoelastic stream issues, Lee and Lee (2021) presented the least-squares limited component arrangement utilizing a versatile lattice.Li et al. (2021) utilized nonlinear incomplete differential conditions, the qualities strategy, and a blended limited volume component to show two-stage compressible removal issues.To rough Fisher's response dissemination condition, Tamsir and Huntul (2021) recommended a half breed mathematical methodology. Differential quadrature method (DQM) and cubic uniform algebraic trigonometric (CUAT),tension B-spline capabilities structure the premise of the method. Weighting coefficients are registered in DQM through the CUAT tension B-spline, decreasing Fisher's response dispersion condition to an arrangement of normal differential conditions.The competitive dynamics in biological populations were modeled by Gortsas et al. (2022) using the nonlinear Fisher-KPP diffusion-reaction equation. The Boundary element method (BEM) was also developed as a reliable numerical approach to solve diffusion problems.

**2. Basic idea of LDTM:** If $u(x,t)$ is a function, then its differential transform in one variable is given by:

$$Y_k(t) = \frac{1}{k!}\left[\frac{\partial^k y}{\partial x^k}\right]_{x=a};\, k \geq 0 \qquad (1)$$

where $y(x,t)$ represents the unaltered function and $Y_k(t)$ represents the converted function. What is the definition of the inverse differential transform of $U_k(t)$

$$y(x,t) = \sum_{k=0}^{\infty} Y_k(t)(x-a)^k \qquad (2)$$

where $a$ represents the starting position in the provided setup. If so, we can express $y(x,t)$ as

$$y(x,t) = \sum_{k=0}^{\infty} Y_k(t)x^k \qquad (3)$$

We analyze the general form of linear and non-linear reaction diffusion equations in this study to demonstrate the principle behind the Laplace differential transform method.

$$\frac{\partial y}{\partial t} = D\frac{\partial^2 y}{\partial x^2} + f(x,t,y,y^2) \qquad (4)$$

$$y(x,0) = h_1(x), y(0,t) = h_2(t), y_x(0,t) = h_3(t) \qquad (5)$$

Applying the Laplace transformation with respect to time $(t)$ to equation (4) is the first step in the Laplace differential transform method, yielding the following:



$$sL\{y(x,t)\} - y(x,0) = L\left\{\frac{\partial^2 y}{\partial x^2} + f\right\}$$

$$sL\{y(x,t)\} = L\left\{\frac{\partial^2 y}{\partial x^2} + f\right\}$$

$$sL\{y(x,t)\} = h_1(x) + L\left\{\frac{\partial^2 y}{\partial x^2} + f\right\}$$

$$L\{y(x,t)\} = \frac{1}{s}h_1(x) + \frac{1}{s}L\left\{\frac{\partial^2 y}{\partial x^2} + f\right\} \qquad (6)$$

Now, if we take both sides of equation (6) and apply the inverse Laplace transformation with respect to $s$, we get:

$$y(x,t) = h_1(x)g(t) + L^{-1}\left[\frac{1}{s}L\left\{\frac{\partial^2 y}{\partial x^2} + f\right\}\right] \qquad (7)$$

After plugging equations (7) into a differential transform with respect to the $x$-axis, we get

$$Y_0(t) = h_2(t), Y_1(t) = h_3(t) \qquad (8)$$

$$Y_k(t) = g(t)DTM[h_1(x)] + L^{-1}\left[\frac{1}{s}L\{(k+1)(k+2)U_{k+2}(t) + DTM[f]\}\right]$$

$$L^{-1}\left[\frac{1}{s}L\{(k+1)(k+2)Y_{k+2}(t) + DTM[f]\}\right] = Y_k(t) - g(t)DTM\{h_1(x)\}$$

$$\left[\frac{1}{s}L\{(k+1)(k+2)U_{k+2}(t) + Y_k(t)\}\right] = L[Y_k(t) - g(t)DTM\{h_1(x)\}]$$

$$L\{(k+1)(k+2)Y_{k+2}(t)\} = sL[Y_k(t) - g(t)DTM\{h_1(x)\}] - L[Y_k(t)]$$

$$Y_{k+2}(t) = \frac{1}{(k+1)(k+2)}L^{-1}\big[sL[Y_k(t) - g(t)DTM\{h_1(x)\}] - L[Y_k(t)]\big] \qquad (9)$$

Using equation (9), we get the values of $Y_0(t), Y_1(t), Y_2(t), ..$ and putting these values in the following equation

$$y(x,t) = \sum_{k=0}^{\infty} Y_k(t)\,x^k = Y_0(t) + Y_1(t)x + Y_2(t)x^2 + Y_3(t)x^3 + Y_4(t)x^4 + Y_5(t)x^5 + \cdots$$

**3. Application of LDTM:**As an example of LDTM's versatility, we have used it to solve linear and nonlinear response diffusion equations. The ability to use two different approaches to find both accurate and approximate solutions is a major strength of this strategy.

**Test Problem 3.1:**$\frac{\partial y}{\partial t} = D\frac{\partial^2 y}{\partial x^2} + y + e^t sinx; 0 \leq x \leq 1, 0 \leq t \leq 1$ and $D = 1$ (10)

with these parameters as the starting point and endpoint:

$y(x,0) = sinx, y(0,t) = 0, y_x(0,t) = e^t$ (11)

Applying the Laplace transformation with regard to time to equation (10) and plugging in the initial condition from equation (11) yields the result expected from the Laplace differential transform approach.

$$sL\{y(x,t)\} - y(x,0) = sinx\frac{1}{s-1} + L\left\{\frac{\partial^2 y}{\partial x^2} + y\right\}$$

$$sL\{y(x,t)\} - sinx = sinx\frac{1}{s-1} + L\left\{\frac{\partial^2 y}{\partial x^2} + y\right\}$$

$L\{y(x,t)\} = sinx\frac{1}{s-1} + \frac{1}{s}L\left\{\frac{\partial^2 y}{\partial x^2} + y\right\}$ (12)

Taking both sides of (12) and applying the Inverse Laplace transformation with regard to $s$, we get:

$y(x,t) = e^t sinx + L^{-1}\left[\frac{1}{s}L\left\{\frac{\partial^2 y}{\partial x^2} + y\right\}\right]$ (13)



When we use the Differential transformation technique to solve equations (13) with respect to the spatial variable $(x)$, we get

$$Y_0(t) = 0, Y_1(t) = e^t$$

$Y_k(t) = e^t\frac{1}{k!}sin\frac{k\pi}{2} + L^{-1}\left[\frac{1}{s}L\{(k+1)(k+2)Y_{k+2}(t) + Y_k(t)\}\right]$

$Y_{k+2}(t) = \frac{1}{(k+1)(k+2)}L^{-1}\left[sL\left\{Y_k(t) - e^t\frac{1}{k!}sin\frac{k\pi}{2}\right\} - L\{Y_k(t)\}\right]$ (14)

Putting $k = 0, k = 1, k = 2, \ldots$in equation (14), we get

$$Y_2(t) = 0, Y_3(t) = -\frac{e^t}{3!}, Y_4(t) = 0, Y_5(t) = \frac{e^t}{5!}\ldots$$

Putting the value of $Y_0(t), Y_1(t), Y_2(t), ..$ in the following equation

$$y(x,t) = \sum_{k=0}^{\infty} Y_k(t)\,x^k = Y_0 + Y_1x + Y_2x^2 + Y_3x^3 + Y_4x^4 + Y_5x^5 + \cdots$$

$y(x,t) = \sum_{k=0}^{\infty} Y_k(t)\,x^k = e^t x - \frac{e^t}{3!}x^3 + \frac{e^t}{5!}x^5 + \cdots = e^t\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots\right) = e^t sinx$ (15)

**Test Problem 3.2:**$\frac{\partial y}{\partial t} = D\frac{\partial^2 y}{\partial x^2} - 0.5y;\ 0 \leq x \leq 1, 0 \leq t \leq 1$ and $D = 1$

with these parameters as the starting point and endpoint:

$y(x,0) = sin\pi x, y(0,t) = 0, y_x(0,t) = \pi e^{(-0.5-\pi^2)t}$ (16)

Laplace's method of differential transformation states that we should first apply the Laplace transformation with regard to time, t, to equation (16), which yields

$$sL\{y(x,t)\} - y(x,0) = L\left\{\frac{\partial^2 y}{\partial x^2} - 0.5y\right\}$$

$sL\{y(x,t)\} - sin\pi x = L\left\{\frac{\partial^2 y}{\partial x^2} - 0.5y\right\}$

Applying the Inverse Laplace transformation with respect to $s$ to both sides of Equation (16) yields the following:

$y(x,t) = sin\pi x + L^{-1}\left[\frac{1}{s}L\left\{\frac{\partial^2 y}{\partial x^2} - 0.5y\right\}\right]$ (17)

Equations (17) are then transformed using the differential approach with respect to the spatial variable $x$, yielding

$$Y_0(t) = 0, Y_1(t) = \pi e^{(-0.5-\pi^2)t} = \pi e^{at}$$

$$Y_k(t) = \frac{\pi^k}{k!} sin\frac{k\pi}{2} + L^{-1}\left[\frac{1}{s}L\{(k+1)(k+2)Y_{k+2}(t) - 0.5Y_k(t)\}\right]$$

$$(k+1)(k+2)Y_{k+2}(t) = L^{-1}\left[sL\left\{Y_k(t) - \frac{\pi^k}{k!}sin\frac{k\pi}{2}\right\} + 0.5L\{Y_k(t)\}\right]$$

$$Y_{k+2}(t) = \frac{1}{(k+1)(k+2)}L^{-1}\left[sL\left\{Y_k(t) - \frac{\pi^k}{k!}sin\frac{k\pi}{2}\right\} + 0.5L\{Y_k(t)\}\right] \quad (18)$$

Putting $k = 0, k = 1, k = 2, \ldots$in equation (18), we get

$$Y_2(t) = 0, Y_3(t) = -\frac{\pi^3}{3!}e^{at}, Y_4(t) = 0, Y_5(t) = \frac{\pi^5}{5!}e^{at}, ..$$

Putting the value of $Y_0(t), Y_1(t), Y_2(t), ..$ in the following equation

$$y(x,t) = \sum_{k=0}^{\infty} Y_k(t)\, x^k = Y_0 + Y_1x + Y_2x^2 + Y_3x^3 + Y_4x^4 + Y_5x^5 + \cdots$$

$$y(x,t) = \sum_{k=0}^{\infty} Y_k(t)\, x^k = e^{at}\left(\pi x - \frac{\pi^3x^3}{3!} + \frac{\pi^5x^5}{5!} + \cdots\right) = e^{at}\,\mathrm{sin\pi x} = e^{(-0.5-\pi^2)t} sin\pi x \quad (19)$$

**Test Problem 3.3:**The equation for non-homogeneous non-linear reaction diffusion

$$\frac{\partial y}{\partial t} = D\frac{\partial^2 y}{\partial x^2} - y^2 + 2x^2t - 2xt^2 + x^5t^5 \quad (20)$$

when beginning and boundary conditions are present

$$y(x,0) = 0, y(0,t) = 0, y_x(0,t) = 0 \quad (21)$$

Using the beginning conditions in (21), we obtain the Laplace transformation with respect to $t$ on equation (20) by first performing the Laplace transformation.

$$sL\{y(x,t)\} - y(x,0) = \frac{2x^2}{s^2} - \frac{4x}{s^3} + \frac{120x^5}{s^6} + L\left\{\frac{\partial^2 y}{\partial x^2} - y^2\right\}$$

$$L\{y(x,t)\} = \frac{2x^2}{s^3} - \frac{4x}{s^4} + \frac{120x^5}{s^7} + \frac{1}{s}L\left\{\frac{\partial^2 y}{\partial x^2} - y^2\right\} \quad (22)$$

Assuming this for the time being, we do the Inverse Laplace transformation with respect to $s$ on both sides of equation (22).

$$y(x,t) = x^2t^2 - \frac{2xt^3}{3} + \frac{x^5t^6}{6} + L^{-1}\left[\frac{1}{s}L\left\{\frac{\partial^2 y}{\partial x^2} - y^2\right\}\right] \quad (23)$$

Differential transformations applied to equations (23) with respect to the spatial variable *'x'* yield

$$Y_0(t) = 0, Y_1(t) = 0$$

$$Y_k(t) = t^2\delta(k-2) - \frac{2t^3}{3}\delta(k-1) + \frac{t^6}{6}\delta(k-5) + L^{-1}\left[\frac{1}{s}L\left\{(k+1)(k+2)Y_{k+2}(t) - \sum_{i=0}^{k} Y_i(t)\, Y_{k-i}(t)\right\}\right]$$

$$(k+1)(k+2)Y_{k+2}(t) = \sum_{i=0}^{k} Y_i(t)\, Y_{k-i}(t) + L^{-1}\left[sL\left\{Y_k(t) - t^2\delta(k-2) + \frac{2t^3}{3}\delta(k-1) - \frac{t^6}{6}\delta(k-5)\right\}\right] \quad (24)$$

Putting $k = 0, k = 1, k = 2, \ldots$in equation (24), we get

$$Y_2(t) = 0, Y_3(t) = \frac{t^2}{3}, Y_4(t) = -\frac{t}{6}, Y_5(t) = \frac{t'}{30}, \ldots$$

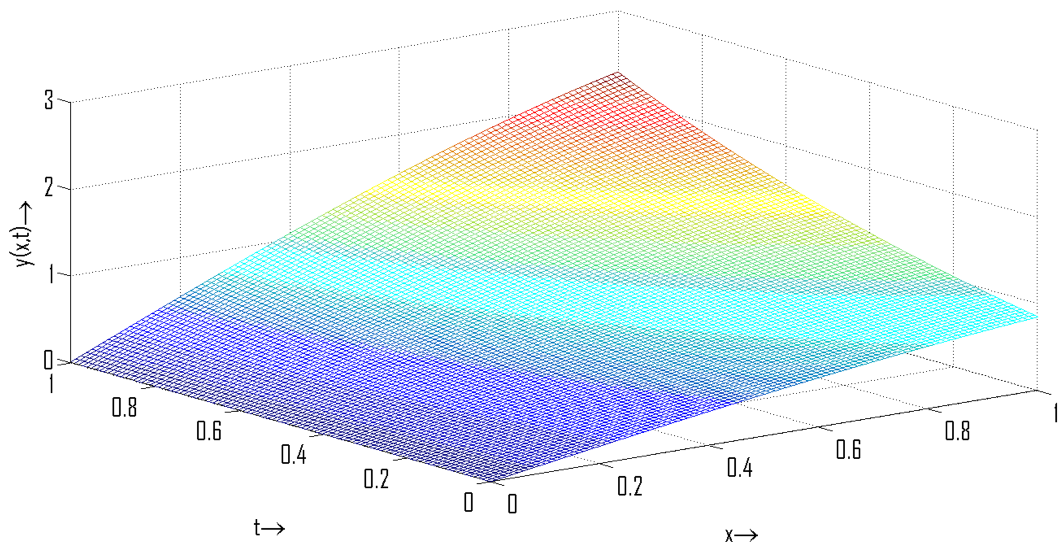
Putting the value of $Y_0(t), Y_1(t), Y_2(t), ..$ in the following equation

$$y(x,t) = \sum_{k=0}^{\infty} Y_k(t)\, x^k = Y_0 + Y_1x + Y_2x^2 + Y_3x^3 + Y_4x^4 + Y_5x^5 + \cdots$$

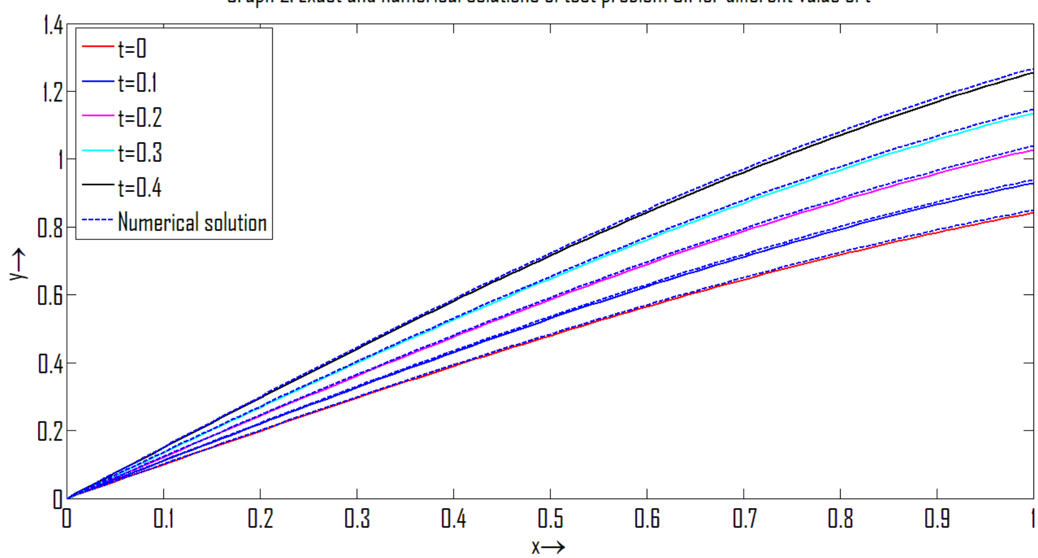
$$y(x,t) = \sum_{k=0}^{\infty} Y_k(t)\, x^k = \frac{t^2}{3}x^3 - \frac{t}{10}x^4 + \frac{t}{30}x^5 - \cdots \quad (25)$$

**4. Numerical Results:**

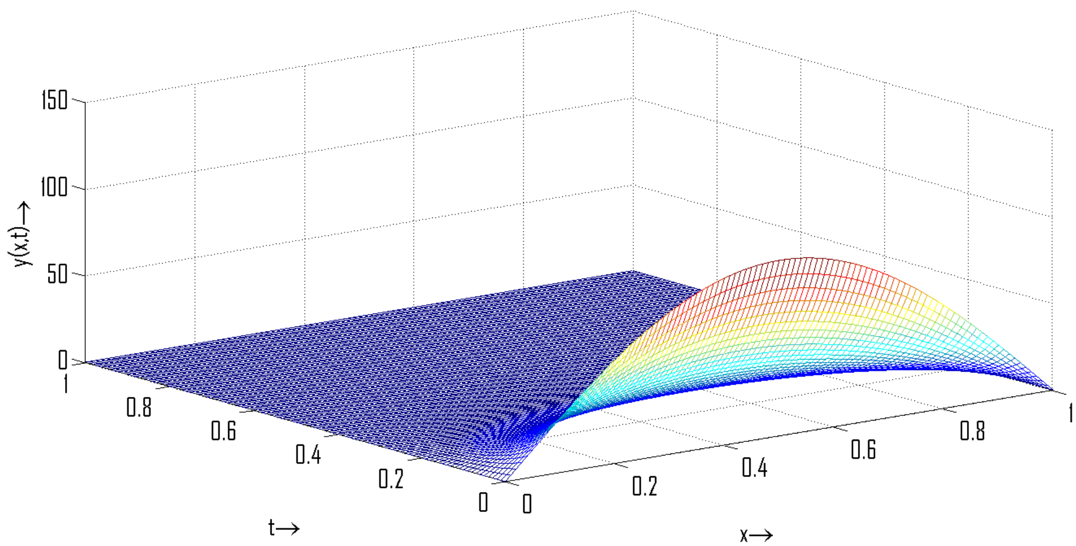
Graph 1: 3D Solution of test problem 3.1 by LDTM



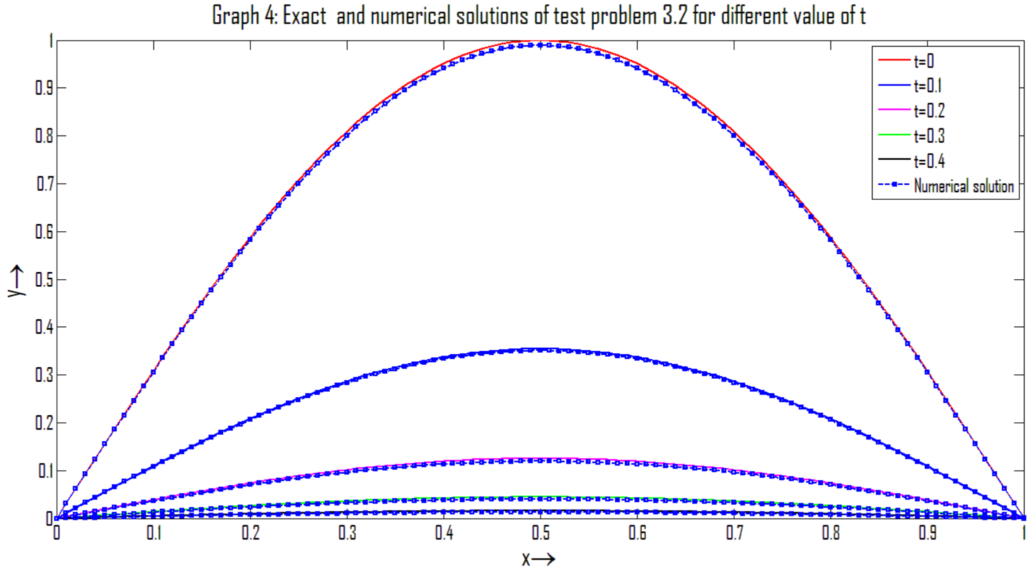

Graph 2: Exact and numerical solutions of test problem 3.1 for different value of t



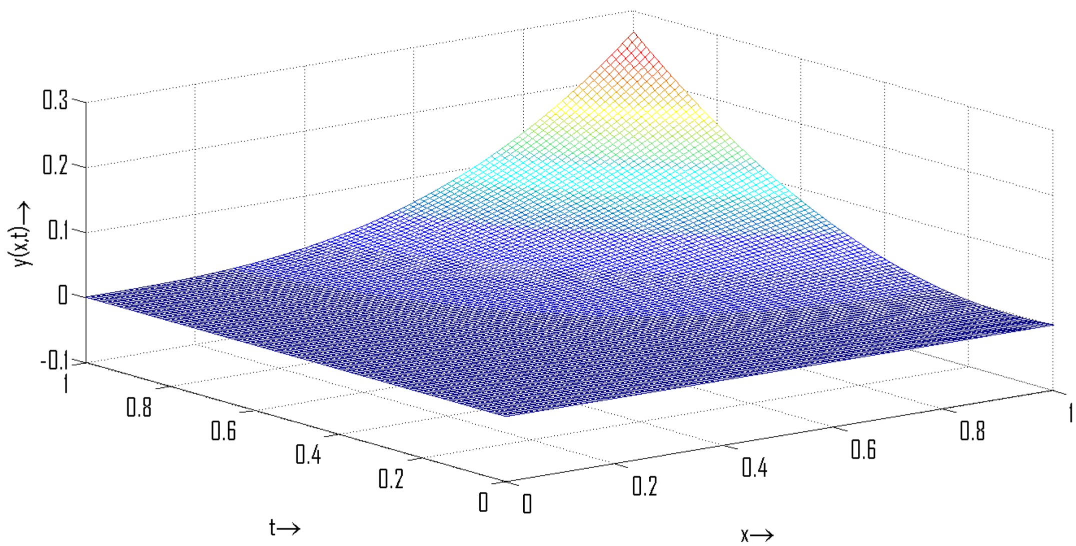
Graph 3: 3D Solution of test problem 3.2 by LDTM



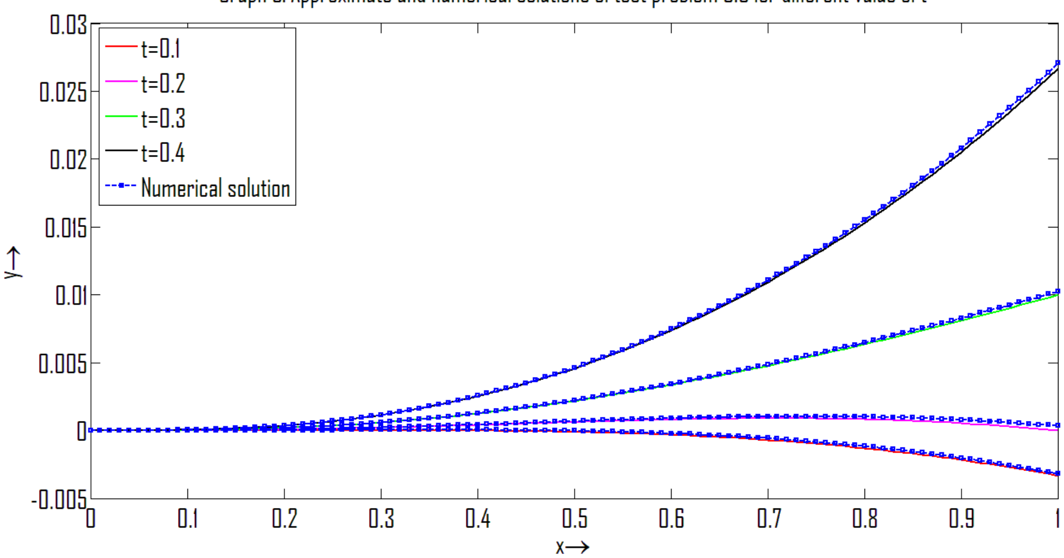

Graph 4: Exact and numerical solutions of test problem 3.2 for different value of t



Graph 5: 3D Solution of test problem 3.3 by LDTM




Graph 6: Approximate and numerical solutions of test problem 3.3 for different value of t






From $t = 0$ to $t = 1$ and $x = 0$ to $x = 1$, the LTDM 3D solution for (3.1) of equation (15) is shown in graph (1). The LTDM and MATLAB numerical solutions to (3.1) for $t = 0, 0.1, 0.2, 0.3, 0.4$ are depicted in graph (2).With the current selection of $t$ and $x$, the LDTM for the reaction diffusion equation is accurate within a manageable range, with absolute errors being quite minor. The graph (2) shows that there is no discernible difference between the two solutions, both of which were obtained using MATLAB.For the test issue (3.2) of equation (19), the LTDM 3D solution is shown in figure (3) from $t = 0$ to $t = 1$ and $x = 0$ to $x = 1$. The LTDM and MATLAB numerical solutions to (3.2) for $t = 0, 0.1, 0.2, 0.3, 0.4$ are depicted in graph (4).By adjusting $t$ and $x$, the LDTM for the reaction diffusion equation achieves a high degree of precision, with very small absolute errors. The graph (4) shows that there is no discernible difference between the two solutions, both of which were obtained using MATLAB.The LTDM-derived 3D solution for the test problem (3.3) from $= 0$ to $t = 1$ and $x = 0$ to $x = 1$ is depicted in Graph (5). The LTDM approximation and the MATLAB numerical solution to (3.3) for $t = 0.1,0.2,0.3,0.4$ are shown in Graph (6).Absolute errors are now quite modest with the current selection of $t$ and $x$, and the LDTM's accuracy for the reaction diffusion equation is under your control. Both sets of findings are obtained using MATLAB, and when compared using graph (6), it is clear that there is no discernible difference between the two approaches.

**5. Concluding Remarks:T**he reaction-diffusion equation is a useful model for many systems in engineering, science, and other disciplines. The differential transformation approach is investigated in this paper as a means of solving the reaction-diffusion equations. Three boundary value problems are solved using this technique. We solved every one of these problems using exact series computation in closed form. Approximations of better accuracy and closed form solutions, if they exist, can be obtained using the differential transformation approach, which is both powerful and efficient. It has been found that this strategy is a powerful and trustworthy resource for dealing with issues of this nature. Our example also shows that the effective method can be used as an alternative to traditional approaches to solving higher-order linear and non-linear starting and boundary value issues. The findings demonstrate that LDTM is an effective mathematical tool for solving linear and nonlinear partial differential equations, and so has broad applications in engineering. The LDTM series are calculated in MATLAB for this work. To better understand what factors must be present for solutions to non-linear Reaction-diffusion equations to converge, a new approach for the Laplace differential transform method is described here. The procedure described here can be used to resolve a problem with either beginning or boundary values. If closed form solutions exist, the LDTM provides approximations with improved precision and quantitatively reliable findings.

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