A MATHEMATICAL MODEL OF SNOWBALLMELTING

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ABSTRACT

Mathematical modelling is an essential tool for understanding the world. In olden days the Chinese, Egyptians, Indians, Babylonians and Greeks indulged in understanding and predicting the natural phenomena through their knowledge of mathematics. The architects, artisans and craftsmen based many of their works of art on geometric prinicples. Suppose a surveyor wants to measure the height of a tower. It is physically very difficult to measure the height using the measuring tape. So, the other option is to find out the factors that are useful to find the height. From his knowledge of trigonometry, he knows that if he has an angle of elevation and the distance of the foot of the tower to the point where he is standing, then he can calculate the height of the tower.

Keywords **:** *Mathematical modelling,Architects, Artisans and Craftsmen etc.*

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Introduction

Mathematics helps us to organize and systemize our ideas about patterns. By doing so not only can we admire and enjoy these patterns but also we can use them to infer some of the underlying principles that govern the world of nature. Galileo Galilei stated that the Universe “cannot be read until we have learnt the language and become familiar with the characters in which it is written. It is written in mathematical language. A mathematical model is a description of a system using mathematical concepts and language. The process of developing a mathematical model is termed *mathematical modelling*. The aim of mathematical modeling is practical application of mathematics to help unravel the underlying mechanisms involved in,for *e.g.*: economic, physical, biological, or other systems andprocesses.

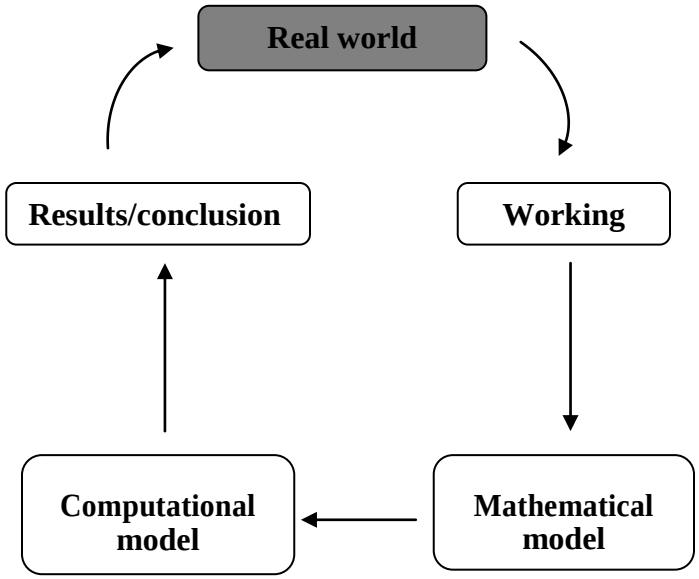

Fig. : 1.1 Mathematical Modelling

InthederivationtofollowitwillbeassumedthattheEarthisflatandthedensityofwater isconstant,thattheCoriolisforceisnegligible,thatthedensityofaircanbeneglectedcompared tothedensityofwater,thatthebodyofthewaterisofinfiniteextentandcompletelycoveredby waves,andthatviscosityandsurfacetensioncanbeneglected.Moreovertosimplifytheproblem it is assumed that there is no variation in the properties of waves in the ydirection.

Some plausible assumptions are follows:

- Assume that the snowball is a sphere of radius *r*(*t*) at alltimes.
- Assume that the density of the snow/ice mixture is constant throughout the snowball, so there are no difference in snow packing.
- This may be reasonablefor smallsnowballs(handsizedones),butlargeonesformedbyrollingwillprobably become more denselypacked as their weight increases. A major advantage of the constant density assumption is that the mass (and weight) of the snowball is then directly proportional to itsvolume.
- Assumethatthemassofthesnowballdecreasesatarateproportionaltoitssurface area and only this. It appears to make sense because it is the outside surface only that make contact with the warmer air that inducesmelting.
- Assume that no external factors change during the lifetime of the snowball like humidity of the air, angle of incidence, intensity of sunlight and the external temperature.

Let the initial radius of the snowball be r(0) = R. ρ denotes the uniform density of the snowball.ItsmassbyM(t)andvolumebyV(t)andmeasuretimetinhours,thenthemassofthe snowball at any time tis

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$$M(t) = \frac{4}{3}\pi pr^3(t) \quad (1.1)$$

It follows that the instantaneous rate of change of mass or time derivative is

$$\frac{dM}{dt} = 4\pi pr^2 \frac{dr}{dt} \quad (1.2)$$

By assumption

$$\frac{dM}{dt} = -4\pi pr^2 k \quad (1.3)$$

Where k is a positive quantity and the negative sign shows that the mass is decreasing with time. By equating the last two expressions it follows that

$$\frac{dr}{dt} = -\frac{k}{p} = -\propto, say \quad (1.4)$$

Thus, according to this model, the radius of the snowball decreases uniformly with time.Upon integrating the above equation and applying initial conditions we obtain

$$t(t) = R - \propto t = R\left(1 - \frac{t}{t_m}\right) \quad where = \frac{R}{\propto} t_m \quad (1.5)$$

Where t_m is the time for the original snowball to melt, which occurs when its radius is zero. We don't know the value of . Assume that half the snowball melts in one hour. So from equation(1.5)
we get,

$$r(1) = R - \alpha$$

A graph of linear equation is given below. Its slope is α .

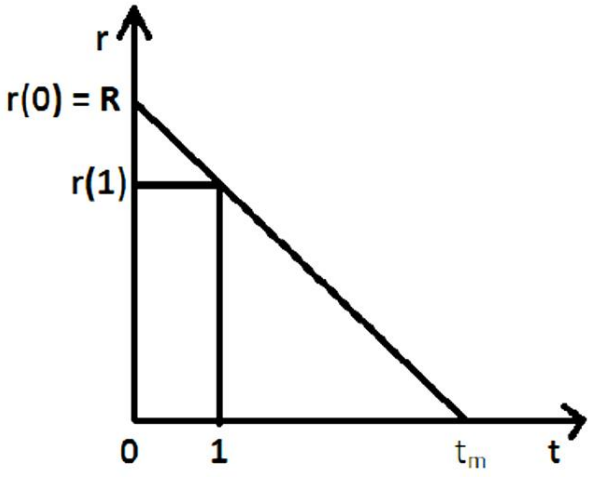

Fig. : 1.2 Time dependence of the radius of snowball

and furthermore

$$\frac{V(1)}{V(0)} = 2^{-\frac{1}{3}} R \approx 0.79R$$

Hence t_m = 4.8 hours, so according to this model the snowball will take a little less than 4 more hours to melt away completely. This is a rather long time and certainly the sun's position will have changed during that time, so in retrospect assumption is not really justified. A further implication of the equation is that both the volume and mass of the snowball (by assumption decreases like a cubic polynomial in *t*. i.e,

$$V(t) = (0)\left(1 - \frac{t}{t_m}\right)^3$$

The graph for the above expression is given below :

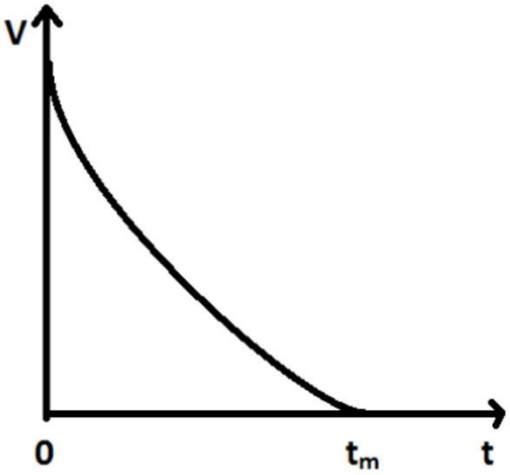

Fig. : 1.3 Time dependence of the volume of snowball

Note that $V'(t) < 0$ as required and $V'(t_m) = 0$. Since $V''(t) = 0$ it is clear that snowball melts more quickly at first when $|V'|$ is larger, than at later times as the value for t_m arrests. If we generalize the original problem by suggesting instead that "a fraction β of a snowball melts in h hours…..". The melting time is then found to be $t_m = \frac{h}{1 - \sqrt[3]{1-\beta}}$ which depends linearly on h and in a monotonically decreasing manner on β. The dependence on h is not surprising. If a given fraction β melts in half the time, the total melting time is also halved. For a given valueof h, the dependence of β is also plausible: the larger the fraction that melts in time h, the shorter the meltingtime.

The final point is the existence and uniqueness of the solution to this mathematical model. There is a theorem behind this which states that there exists a unique solution to the ordinarydifferential equation.

$$\frac{dy}{dx} = f\left(x, y\right)$$

satisfying the initial condition $y\left(x_o\right) = y_o$ provided that both the function $f\left(x, y\right)$ and its partial derivative $\frac{\partial f}{\partial y}$ are continuous functions of x and y in a neighborhood of the initial point $\left(x_o, y_o\right)$. From equation

$$\frac{dr}{dx} = f\left(r,t\right) = -\alpha, \qquad r\left(o\right) = r_o = R$$

Where ∞ is a positive constant. Since constants and their derivatives are continuous everywhere the theorem applies and hence a unique solution exists for the problem as posed.

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