A NEW SUBCLASS OF CLOSE TO STAR ANALYTIC FUNCTIONS AND ITS COEFFICIENT BOUNDS

Dr.Paras Uchat, Assistant Professor, Indian Institute of Teacher Education Gandhinagar, GUJARAT-382016, parasu@iite.ac.in
Dr. Gurmeet Singh, Associate Professor,,GSSDGS Khalsa College, Patiala, Punjab, meetgur111@gmail.com

ABSTRACT:
We introduce some subclasses of analytic functions and obtain sharp upper bounds of the functional $|a_3 - \mu a_2^2|$ for the analytic function $f(z) = z + \sum_{n=2}^{\infty} a_n z^n, |z| < 1$ belonging to these classes.
KEYWORDS: Univalent functions, Starlike functions, Close to convex functions and bounded functions.
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1. INTRODUCTION : Let $\mathcal{A}$ denote the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1.1}$$

which are analytic in the unit disc $\mathbb{E} = \{z: |z| < 1|\}$. Let $\mathcal{S}$ be the class of functions of the form (1.1), which are analytic univalent in $\mathbb{E}$.

In 1916, Bieberbach[2] proved that $|a_2| \leq 2$ for the functions $f(z) \in \mathcal{S}$. In 1923, Löwner [3] proved that $|a_3| \leq 3$ for the functions $f(z) \in \mathcal{S}$..

With the known estimates $|a_2| \leq 2$and $|a_3| \leq 3$, it was natural to seek some relation between a_3 and ${a_2}^2$ for the class $\mathcal{S}$,Fekete and Szegö[5] used Löwner's method to prove the following well known result for the class $\mathcal{S}$.

Let $f(z) \in \mathcal{S}$, then

$$|a_3 - \mu a_2^2| \leq \begin{cases} 3 - 4\mu, if \mu \leq 0; \\ 1 + 2\exp\left(\frac{-2\mu}{1-\mu}\right), if\ 0 \leq \mu \leq 1; \\ 4\mu - 3, if \mu \geq 1. \end{cases} \tag{1.2}$$

The inequality (1.2) plays a very important role in determining estimates of higher coefficients for some sub classes of$\mathcal{S}$ (Chhichra[11], Babalola[1]).

Let us define some subclasses of $\mathcal{S}$.

We denote by S*, the class of univalent starlike functions

$g(z) = z + \sum_{n=2}^{\infty} b_n z^n \in \mathcal{A}$ and satisfying the condition

$$Re\left(\frac{zg'(z)}{g(z)}\right) > 0, z \in \mathbb{E}. \tag{1.3}$$

We denote by $\mathcal{K}$, the class of univalent convex functions $h(z) = z + \sum_{n=2}^{\infty} c_n z^n, z \in \mathcal{A}$ and satisfying the condition

$Re\frac{((zh'(z))'}{h'(z)} > 0, z \in \mathbb{E}.$ (1.4)

A function $f(z) \in \mathcal{A}$ is said to be close to star if there exists $g(z) \in S^*$ such that

$Re\left(\frac{f(z)}{g(z)}\right) > 0, z \in \mathbb{E}.$ (1.5)

The class of close to star functions is denoted by C^* and was introduced by Reade [7] and it was shown by him that all close to star functions are univalent.

A function $f(z) \in \mathcal{A}$ is said to belong to the class C_1^*, if there exists $h(z) \in \mathcal{K}$ such that

$Re\left(\frac{f(z)}{h(z)}\right) > 0, z \in \mathbb{E}.$ (1.6)

In this paper, we will also discuss $Fekete - Szegö$ inequality for the following subclass of Close to star functions:

$$C^*(A,B); f(z) \in \mathcal{A}\ with\ \frac{f(z)}{g(z)} \prec \frac{1+Az}{1+Bz}; g(z) \in S^*, -1 \le A \le B \le 1.$$

This class was introduced by Mehrok and it is to be noted that $C^*(-1,1) = C^*$.

We will also consider the following subclass of C_1^*:

$$C_1^*(A,B); f(z) \in \mathcal{A}\ with\ \frac{f(z)}{h(z)} \prec \frac{1+Az}{1+Bz}; h(z) \in \mathcal{K}, -1 \le A < B \le 1.$$

This class was introduced by Mehrok and it is to be noted that $C_1^*(-1,1) = C_1^*$. 2246

We will also discuss $Fekete - Szegö$ inequality for the following subclass of class of close to convex functions:

$C_1; f(z) \in \mathcal{A}; Re\left(\frac{zf'(z)}{h(z)}\right) > 0; h(z) \in \mathcal{K}, z \in E.$

Symbol $\prec$ stands for subordination, which we define as follows:

PRINCIPLE OF SUBORDINATION: Let $f(z)$ and $F(z)$ be two functions analytic in $\mathbb{E}$. Then $f(z)$ is called subordinate to *F(z)* in $\mathbb{E}$ if there exists a function $w(z)$ analytic in $\mathbb{E}$ satisfying the conditions $w(0) = 0$ and $|w(z)| < 1$ such that $f(z) = F(w(z))$; $z \in \mathbb{E}$ and we write $f(z) \prec F(z)$. (See [4])

By $\mathcal{U}$, we denote the class of analytic bounded functions of the form $w(z) = \sum_{n=1}^{\infty} d_n z^n, w(0) = 0, |w(z)| < 1.$ (1.7)

It is known that $|d_1| \le 1, |d_2| \le 1 - |d_1|^2$.

2. PRELIMINARY LEMMAS:

LEMMA2.1 (Nevanlinna,1921): If $f(z)$ is in the class S*,
then $|a_n| \le n$, for each $n \ge 2$(2.1)
Further, the sharpness of this inequality for each n can be viewed from Koebe function or one of its rotations.

COROLLARY: If f (z) is in the class $\mathcal{K}$, then
$|a_n| \le 1$, for each $n \ge 2$. (2.2)
Further, the sharpness can be seen from the function
$f(z) = \frac{z}{1-z}, z \in E$. (2.3)

LEMMA 2.2: Let $g(z) \in S^*$, then

$$|b_3 - \mu b_2^2| \le \begin{cases} 3 - 4\mu, if\ \mu \le \frac{1}{2}: \\ 1, if \frac{1}{2} \le \mu \le 1: \\ 4\mu - 3, if \mu \ge 1. \end{cases}$$

LEMMA 2.3: Let $g(z) \in \mathcal{K}$, then

$$|c_3 - \mu c_2^2| \leq \begin{cases} 1-\mu, if\ \mu \leq \frac{2}{3}: \\ 1, if \frac{2}{3} \leq \mu \leq \frac{4}{3}: \\ \mu - 1, if \mu \geq \frac{4}{3}. \end{cases}$$

LEMMA 2.4: :Let $h(z) \in \mathcal{K}$, then

$$|d_3 - \mu d_2^2| \leq \begin{cases} 1-\frac{3}{4}\mu, if\ \mu \leq \frac{8}{9}: \\ \frac{1}{3}, if \frac{8}{9} \leq \mu \leq \frac{16}{9}: \\ \frac{3}{4}\mu - 1, if \mu \geq \frac{16}{9}. \end{cases}$$

This result is a direct consequence of Lemma 2.3.

3. **MAIN RESULTS:**

THEOREM 3.1: If f(z) ∈ C_1, then

$$|a_3 - \mu a_2^2| \leq \begin{cases} \frac{5}{3} - \frac{9\mu}{4}, if\ \mu \leq \frac{2}{9}: & (3.1.1) \\ \frac{2}{3} - \frac{1}{9\mu}, if\ \frac{2}{9} \leq \mu \leq \frac{2}{3}: & (3.1.2) \\ 1 - \frac{\mu}{4} + \frac{(3\mu-2)^2}{12(4-3\mu)}, if\ \frac{2}{3} \leq \mu \leq \frac{8}{9}: & (3.1.3) \\ \frac{7}{9} + \frac{(3\mu-2)^2}{12(4-3\mu)}, if\ \frac{8}{9} \leq \mu \leq \frac{10}{9}: & (3.1.4) \\ 2\mu - \frac{11}{9}, if\ \frac{10}{9} \leq \mu \leq \frac{16}{9}: & (3.1.5) \\ \frac{9\mu}{4} - \frac{5}{3}, if\ \mu \geq \frac{16}{9}. & (3.1.6) \end{cases}$$

PROOF: By definition of C_1, we have $\frac{zf'(z)}{g(z)} = \frac{1+w(z)}{1-w(z)}; w(z) \in U$

Expanding the series, we get

$(1 + 2a_2z + 3a_3z^2 + - - -) = (1 + b_2z + b_3z^2 + - - -)(1 + 2c_1z + 2(c_2 + c_1^2)z^2 + - - -)$
(3.3.7)

Identifying the terms, we get,

$a_2 = \frac{b_2}{2} + c_1$ (3.1.8)

$a_3 = \frac{b_3}{3} + \frac{2}{3}b_2c_1 + \frac{2}{3}(c_2 + c_1^2)$ (3.1.9)

From (3.8) and (3.9) we get

$$a_3 - \mu a_2^2 = \frac{b_3}{3} + \frac{2}{3}b_2c_1 + \frac{2}{3}(c_2 + c_1^2) - \mu\frac{{b_2}^2}{4} - \mu{c_1}^2 - \mu b_2c_1$$
$$= \frac{2}{3}c_2 + \frac{1}{3}\left(b_3 - \frac{3}{4}\mu b_2^2\right) + \frac{1}{3}(2-3\mu)b_2c_1 + \frac{1}{3}(2-3\mu)c_1^2$$

Taking absolute value and using $|c_2| \leq 1 - |c_1|^2$, we get

$$\left|a_3-\mu a_2^2\right| \le \frac{2}{3}+\frac{1}{3}\left|b_3-\frac{3}{4}\mu b_2^2\right|+\frac{1}{3}|2-3\mu||b_2||c_1|+\frac{1}{3}|2-3\mu||c_1^2|$$

$\le \frac{2}{3}+\frac{1}{3}\left|b_3-\frac{3}{4}\mu b_2^2\right|+\frac{1}{3}|2-3\mu|xy+\frac{1}{3}(|2-3\mu|-2)x^2$ (3.1.10)

Where $x=|c_1|\le 1, y=|b_2|\le 1$.

Case I :$\mu \le \frac{8}{9}$.

Using lemma (2.4), we get

$$\left|a_3-\mu a_2^2\right| \le 1-\frac{\mu}{4}+\frac{1}{3}(2-3\mu)x-\mu x^2 = H_1(x)(say)$$

$$\text{Therefore, } H_1'(x)=\frac{1}{3}(2-3\mu)-2\mu x \text{ and } H_1''(x)=-2\mu$$

$$H_1'(x)=0 \Rightarrow x=\frac{2-3\mu}{6\mu}=x_1\ (say)$$

If $x_1>1$, we have $\frac{2-3\mu}{6\mu}>1 \Rightarrow 2-3\mu>6\mu \Rightarrow \mu<\frac{2}{9}$.

And $H_1'(x)>0$.

SubcaseI(a): $\mu \le \frac{2}{9}$.

Maximum value of $H_1(x)$ in $[0,1]$ is $H_1(1)=\frac{5}{3}-\frac{9\mu}{4}$ as $H_1(x)$ becomes increasing function in $[0,1]$ in this subcase.

Subcase I(b):$\frac{2}{9}\le\mu\le\frac{2}{3}$. In this subcase, $H_1(x)$ has maxima at $x=x_1$ in $[0,1]$.

$\therefore$ Maximum value of $H_1(x)$ in $[0,1]$ is

$$H_1(x_1)=1-\frac{\mu}{4}+\frac{1}{3}(2-3\mu)\frac{2-3\mu}{6\mu}-\mu\left(\frac{2-3\mu}{6\mu}\right)^2=\frac{2}{3}+\frac{1}{9\mu}$$

Subcase I(c):$\frac{2}{3}\le\mu\le\frac{8}{9}$. (3.3.10) can be written as

$$\left|a_3-\mu a_2^2\right| \le 1-\frac{\mu}{4}+\frac{1}{3}(3\mu-2)x-\frac{1}{3}(4-3\mu)x^2 = H_2(x)(say)$$

$$\text{Thus, } H_2'(x)=\frac{1}{3}(3\mu-2)-\frac{2}{3}(4-3\mu)x \text{ and } H_1''(x)=-\frac{2}{3}(4-3\mu)$$

$$H_2'(x)=0 \Rightarrow x=\frac{3\mu-2}{2(4-3\mu)}=x_2\ (say)$$

If $x_2<1$, we have $3\mu-2<2(4-3\mu) \Rightarrow 9\mu>10 \Rightarrow \mu<\frac{10}{9}$.

And $H_2'(x)>0$.

Therefore, Maximum value of $H_2(x)$ occur at $x=x_2$, which is

$$H_2(x_2)=1-\frac{\mu}{4}+\frac{1}{3}(3\mu-2)\frac{3\mu-2}{2(4-3\mu)}-\frac{1}{3}(4-3\mu)\left(\frac{3\mu-2}{2(4-3\mu)}\right)^2=1-\frac{\mu}{4}+\frac{(3\mu-2)^2}{12(4-3\mu)}$$

Case II:$\frac{8}{9}\le\mu\le\frac{16}{9}$.

Using Lemma (2.4), we get

$$\left|a_3-\mu a_2^2\right| \le \frac{7}{9}+\frac{1}{3}(3\mu-2)x-\frac{1}{3}(4-3\mu)x^2 = H_3(x)(say)$$

$$\text{Thus, } H_3'(x)=\frac{1}{3}(3\mu-2)-\frac{2}{3}(4-3\mu)x \text{ and } H_3''(x)=-\frac{2}{3}(4-3\mu)$$

$$H_3'(x)=0 \Rightarrow x=\frac{3\mu-2}{2(4-3\mu)}=x_2$$

If $x_2<1$, we have $\mu<\frac{10}{9}$. and $H_2'(x)<0$.

Subcase II(b):$\frac{8}{9}\le\mu\le\frac{10}{9}$

In this subcase, maximum value of $H_3(x)$ in $[0,1]$ is

$$H_3(x_2)=\frac{7}{9}+\frac{(3\mu-2)^2}{12(4-3\mu)}$$

Subcase II (b):$\frac{10}{9}\le\mu\le\frac{4}{3}$.

If $x_2 > 1$, we have $\mu > \frac{10}{9}$. and $H_3'(x) > 0$.
Therefore, $H_3(x)$is increasing and maximum value of $H_3(x)$ in [0,1] is

$$H_3(1) = \frac{7}{9} + \frac{1}{3}(3\mu - 2) - \frac{1}{3}(4 - 3\mu) = 2\mu + \frac{11}{9}$$

Subcase II (c):$\frac{4}{3} \leq \mu \leq \frac{16}{9}$.
(3.3.10) becomes

$$|a_3 - \mu a_2^2| \leq \frac{7}{9} + \frac{1}{3}(3\mu - 2)x + \frac{1}{3}(3\mu - 4)x^2 = H_4(x)(say)$$

$$\therefore H_4'(x) = \frac{1}{3}(3\mu - 2) + \frac{2}{3}(3\mu - 4)x > 0$$

Which means $H_4(x)$is increasing in [0,1]
Therefore, Maximum value of $H_4(x)$in [0,1] is

$$H_3(1) = \frac{7}{9} + \frac{1}{3}(3\mu - 2) - \frac{1}{3}(4 - 3\mu) = 2\mu + \frac{11}{9}$$

Combining subcases II (b) and II (c), we get

$$|a_3 - \mu a_2^2| \leq 2\mu + \frac{11}{9} \; if \; \frac{10}{9} \leq \mu \leq \frac{16}{9}$$

Case III: $\mu \geq \frac{16}{9}$.
Using Lemma (2.4), we get

$$|a_3 - \mu a_2^2| \leq \frac{1}{3} + \frac{\mu}{4} + \frac{1}{3}(3\mu - 2)x + \frac{1}{3}(3\mu - 4)x^2 = H_5(x)(say)$$

$$\text{Therefore}, H_5'(x) = \frac{1}{3}(3\mu - 2) + \frac{2}{3}(3\mu - 4)x > 0$$ 2249

Which means $H_5(x)$ is increasing in [0,1]
Therefore, maximum value of $H_5(x)$in [0,1] is

$$H_5(1) = \frac{9\mu}{4} - \frac{5}{3}$$

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