Original Article

A Natural Quantum Neural-Like Network

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Abstract
A neural-net-like quantum information dynamics is presented. The model is based on Hopfield's and holographic neural nets which were successfully simulated on conventional computers. An analogous natural, i.e. relatively non-artificial, quantum information processing system is developed in this article. Neuro-quantum interaction can regulate the "collapse"-readout of quantum computation results. This paper is a comprehensive introduction into associative processing and memory-storage in quantum-physical framework.

Key Words: quantum, neural networks, information processing, pattern recognition, holography
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1. Introduction

The aim of this paper is to provide a general introduction and a short coherent overview of recent research of quantum Hopfield-like information processing — to show quantum information theorists where they have common issues with neural-net modellers.

In Perus (1996, 1997, 1998) it was systematically presented how and where the mathematical formalism of associative neural network models (Hopfield, 1982; Amit, 1989) and synergetic computers (Haken, 1991) is analogous to the mathematical formalism of quantum theory. In this paper some of these analogies will be used in an original presentation of some information processing capabilities of quantum systems in nature – i.e.,

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not in artificial devices, although the latter option is also open. In parallel to ours, two interesting models of artificial quantum neural networks have been independently developed by Bonnell and Papini (1997) and by Zak and Williams (1998).

This contribution attempts to describe one of the simplest, but fundamental, quantum Hopfield-like information processing "algorithms". We have chosen this "algorithm", simple but nevertheless effective, technically realizable and biophysically plausible enough (Pribram, 1991, 1993), as a convenient one for presenting neural-network-like quantum information dynamics. By saying Hopfield-like we mean a system that is based on the Hopfield model of neural nets or spin glass systems, respectively (Dotsenko, 1994; Geszti, 1990). The Hopfield "algorithm" has been extensively tested by our computer simulations using various concrete data sets (Perus, 2002; Perus and Ecimovic, 1998; Haykin, 1994). Among others, we have effectively realized parallel-distributed content-addressable memory, selective associative reconstruction or recognition of patterns memorized in a compressed form, and even some limited capability for predictions based on a learned data set. We have analyzed how the results are dependent on the correlation structure of a specific set of input patterns, and conditions for successful processing (Perus, 2002; see also Perus and Dey, 2000).

2. Associative neural net model

In the Hopfield associative network model, emergent collective computation or learning is globally regulated by minimization of the spin-glass-like Hamiltonian energy function (Amit, 1989; Dotsenko, 1994)

$$H = -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} J_{ij}\, q_i\, q_j \;=\; -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{P} v_i^k\, v_j^k\, q_i\, q_j \qquad (1)$$

q_i is the actual activity of the i^{th} neuron. There are N neurons in the network: $\vec{q} = (q_1,...,q_N)$.

v_i^k is the activity of the neuron *i* when taking part in encoding the k^{th} memory pattern (*k* is pattern's *superscript* index). The process of gradient-descent of energy function (1) is a result of interactions between the system of neurons, described by $\vec{q}$, and the system of "synaptic" connections, described by weights J_{ij} which are elements of the memory matrix $\mathbf{J}$ (Perus and Ecimovic, 1998). In an equivalent, local "interactional" description, the dynamical equation for neuronal activities

$$q_i(t_2 = t_1 + \Delta t) = \sum_{i=1}^{N} J_{ij}\, q_j\, (t_1) \qquad \text{or} \qquad \vec{q}(t_2) = \mathbf{J}\, \vec{q}(t_1) \qquad (2)$$

is coupled with the Hebb dynamical equation for "synaptic" connections

$$J_{ij} = \sum_{k=1}^{P} v_i^k\, v_j^k \qquad \text{or} \qquad \mathbf{J} = \sum_{k=1}^{P} \vec{v}^k (\vec{v}^k)^T \qquad (3)$$

This system of equations is enough for realizing parallel-distributed information processing of input data. It is the core of one of the simplest "algorithms" useful for theoretical brain modeling (Amit, 1989) as well as for automatic empirical modeling of concrete experimental data sets (Haykin, 1994).

Input-data vectors $\vec{v}^k$ can be inserted into the system of neurons $\vec{q}$ iteratively, or can be put in the very beginning simultaneously into the Hebb matrix $\mathbf{J}$ which contains all "synaptic" weights J_{ij} (equation (3)). Let us rewrite the system (2) and (3) into continuous description of activities of neurons and synapses at position $\vec{r}$ and time t:

$$q(\vec{r}_2, t_2 = t_1 + \Delta t) = \int J(\vec{r}_1, \vec{r}_2)\, q(\vec{r}_1, t_1)\, d\vec{r}_1 \qquad (2b)$$

$$J(\vec{r}_1, \vec{r}_2) = \sum_{k=1}^{P} v^k(\vec{r}_1)\, v^k(\vec{r}_2) \qquad (3b).$$

The memory recall is done by $\vec{q}_{output} = \mathbf{J}\vec{q}_{input} = \mathbf{J}\vec{q}\,'$. Variable with a prime means that its quantitative value is close to the variable without prime, i.e. $q' \approx q$. This can be analyzed by

$$\begin{aligned}
q(\vec{r}_2, t_2) &= \int J(\vec{r}_1, \vec{r}_2)\, q'(\vec{r}_1, t_1)\, d\vec{r}_1 = \int \left(\sum_{k=1}^{P} v^k(\vec{r}_1)\, v^k(\vec{r}_2) \right) q'(\vec{r}_1, t_1)\, d\vec{r}_1 = \\
&= \left(\int v^1(\vec{r}_1)\; q'(\vec{r}_1, t_1)\, d\vec{r}_1 \right) v^1(\vec{r}_2) + \left(\int v^2(\vec{r}_1)\; q'(\vec{r}_1, t_1)\, d\vec{r}_1 \right) v^2(\vec{r}_2) + \ldots \\
&\quad + \left(\int v^P(\vec{r}_1)\; q'(\vec{r}_1, t_1)\, d\vec{r}_1 \right) v^P(\vec{r}_2) = \\
&= A v^1(\vec{r}_2) + B \qquad \textit{where} \quad A \approx 1 \;\; ('signal'), \;\; B \approx 0 \;\; ('noise')
\end{aligned} \qquad (4)$$

or in another description

$$\begin{aligned} q(\vec{r},t) &= \sum_{k=1}^{P} C'^{k}(t)\, v^{k}(\vec{r}) = \sum_{k=1}^{P} \Big(\int v^{k}(\vec{r})\, q'(\vec{r},t)\, d\vec{r} \Big) v^{k}(\vec{r}) = \\ &= \Big(\int v^{1}(\vec{r})\, q'(\vec{r},t)\, d\vec{r} \Big) v^{1}(\vec{r}) + \Big(\int v^{2}(\vec{r})\, q'(\vec{r},t)\, d\vec{r} \Big) v^{2}(\vec{r}) + \ldots \\ &\qquad + \Big(\int v^{P}(\vec{r})\, q'(\vec{r},t)\, d\vec{r} \Big) v^{P}(\vec{r}) = \\ &= A\, v^{1}(\vec{r}) + B \quad where \quad A \approx 1 \;\; ('signal'), \;\; B \approx 0 \;\; ('noise') \end{aligned} \tag{5}$$

The first row of equalities in eq. (5) follows, among others, from the neurosynergetic model by Haken (1991). In eq. (4) and eq. (5) we had to choose such an input $\vec{q}'$ that is *more similar* to $\vec{v}^1$, for example, than to any other $\vec{v}^k, k \neq 1$. At the same time, the input $\vec{q}'$ should be *almost orthogonal* to all the other $\vec{v}^k, k \neq 1$. In this case, $\vec{q}$ converges to the memory pattern-qua-attractor $\vec{v}^1$, as it is shown in the last row of eq. (4) and in the last row of eq. (5). Thus, $\vec{v}^1$ is recalled.

3. Holographic neural net model

If we introduce the *oscillatory* time-evolution of neuronal activities, we get more biologically plausible dynamics (Baird, 1990; Schempp, 1994; Haken, 1991; Kapelko and Linkevich, 1996). Neural net variables then become complex-valued. This is more akin to quantum dynamics (cf.: Acebron & Spigler, 2000). However, in quantum theory, complex-valued formalism is essential, but in the network of coupled oscillatory neurons complex-valued formalism is just mathematically more convenient.

Simulated holographic neural system by Sutherland (1990) realizes effcient Hopfield-like information processing which incorporates oscillatory activities. A Hebb-like "interference" of sequences of input–output pairs (indexed by *k*), each combining an input vector $\vec{s}^k = (s_1^k e^{iq_1^k}, \ldots, s_N^k e^{iq_N^k}),\ i = \sqrt{-1}$, and an output vector $\vec{o}^k = (o_1^k e^{ij_1^k}, \ldots, o_M^k e^{ij_M^k})$, analogous to eq. (3), is made:

$$\mathbf{J} = \sum_{k=1}^{P} \vec{o}^k (\vec{s}^k)^{\dagger} \Leftrightarrow J_{hj} = \sum_{k=1}^{P} s_h^k o_j^k e^{i(j_j^k - q_h^k)} \tag{6}$$

Every J_{hj} encodes a sequence of local input–output amplitude correlations ($s_h o_j$) and corresponding phase differences ($\varphi_j^k - \theta_h^k$) in local input and output oscillatory dynamics. Thus, the phase differences appear in the elements of the memory matrix $\mathbf{J}$ which represents the "hologram". If $\vec{s}^{\,k}$ and $\vec{o}^{\,k}$ are defined as two parts of a concatenated vector $\vec{v}^{\,k}$ (occupying its "upper" and "lower" set of components), then we have the Hebbian "self-interference", like in eq. (3). Usually, during learning, inputs and output (indexed by k = 1, …, P) are "interfered" in a time sequence – each pair $\vec{s}^{\,k}$ and $\vec{o}^{\,k}$ corresponds to a discrete time t_k.

A pattern can be reconstructed (analogously to equations (4) or (5)) from the "neural hologram" using a recall-key $\vec{s}\,'$:

$$\vec{o}\,' = \mathbf{J}\,\vec{s}\,' \Leftrightarrow \sum_{h=1}^{N} J_{hj} s'_h e^{i\theta'_h} = \sum_{h=1}^{N}\sum_{k=1}^{P} s_h^k o_j^k e^{i(\varphi_j^k - \theta_h^k)} s'_h e^{i\theta'_h} \approx o_j^1 e^{i\varphi_j^1} \qquad (7).$$

This is valid if we choose a recall-key that is *similar* to one of the learned inputs, say $\vec{s}^{\,1}$. Thus, $s'_h \approx s_h^1$ and $\theta'_h \approx \theta_h^1$, for all h. In such a case, $s'_h s_h^1 \approx 1$ (if they are normalized) and $e^{i\theta'_h} e^{-i\theta_h^1} \approx 1$. Other terms (with $k \neq 1$) are relatively very small ('noise'). (See Sutherland (1990) for comprehensive presentation.)

Holographic associative memories are implemented optically, acoustically, quantum-electronically and quantum-biologically (Psaltis *et al.*, 1990; Psaltis and Mok, 1995; Nobili, 1985; Schempp, 1994). This supports our view that Hopfield-like associative dynamics with Hebbian learning can be implemented in various (bio)physical (see Jibu *et al.*, 1996) and especially quantum systems (Bonnell and Papini, 1997; Zak and Williams, 1998; Nishimori and Nonomura, 1996; Ma *et al.*, 1993), including coherent states, quantum dots, quantum information channels (Alicki, 1989; Ohya, 1989). Quantum holography (e.g., Leichtle *et al.*, 1998; Abouraddy *et al.*, 2001) offers the most natural implementation.

4. Quantum associative net model

So far, we have discussed the Hopfield-like neural net or spin glass model. Now we will present its quantum relative. From the very beginning, let us put attention to the following correspondence scheme between the neural (left) and quantum variables (right) ("$\Leftrightarrow$" means "corresponds to"):

$$q \Leftrightarrow \Psi, \quad q' \Leftrightarrow \Psi', \quad v^k \Leftrightarrow y^{\,k}, \quad J \Leftrightarrow G \quad \text{or} \quad \mathbf{J} \Leftrightarrow \mathbf{G}, \quad C^k \Leftrightarrow c^k$$

$$\textit{equation}\,(2b) \Leftrightarrow \textit{equation}\,(8), \quad (3b) \Leftrightarrow (9), \quad (4) \Leftrightarrow (10), \quad (5) \Leftrightarrow (11)$$

The phase j is hidden in the exponent of the wave-function Ψ which describes the state of the quantum system: $\Psi(\vec{r},t) = A(\vec{r},\mathrm{t})\exp(\frac{i}{\mathbf{h}} j(\vec{r},t))$; A is the amplitude; $\mathbf{h}$ is the Planck constant (divided by $2p$).

The equations in pairs are *mathematically equivalent*, because the *collective* dynamics in neural and quantum complex systems are similar, in spite of different nature of neurons and their connections on one hand, and quantum "points" $\Psi(\vec{r})$ and their "interactions" described by $G(\vec{r}_1,\vec{r}_2)$ on the other.

The quantum Hopfield-like network model combines the dynamical equation for the quantum state (Feynman and Hibbs, 1965; Derbes, 1996)

$$\Psi(\vec{r}_2,t_2) = \iint G(\vec{r}_1,t_1,\vec{r}_2,t_2)\ \Psi(\vec{r}_1,t_1)\ d\vec{r}_1\, dt_1 \quad \textit{or} \quad \Psi(t_2) = \mathbf{G}\ \Psi(t_1) \tag{8}$$

and the expression for the parallel-distributed interactive transformation of the quantum system (Feynman and Hibbs, 1965)

$$\begin{gathered} G(\vec{r}_1,t_1,\vec{r}_2,t_2) = \sum_{k=1}^{P} y^{\,k}(\vec{r}_1,t_1)^* y^{\,k}(\vec{r}_2,t_2) \\ \textit{or} \\ G(\vec{r}_1,\vec{r}_2) = \sum_{k=1}^{P} y^{\,k}(\vec{r}_1)^* y^{\,k}(\vec{r}_2) \end{gathered} \tag{9}$$

Note that expression (9), i.e. for $G(y^{\,k}(\vec{r}_1,t_1), y^{\,k}(\vec{r}_2,t_2))$, presents the kernel of eq. (8) (cf., Vapnik, 1998). The system (8) and (9) is just the usual Schrödinger propagation reinterpreted for associative processing and measurement-like readout.

We want to *encode some information* in eigenfunctions $y^{\,k}$. Then $y^{\,k}$ would become quantum codes of patterns, although not necessarily geometrically isomorphic to some external patterns. It may not be possible to encode information in wave-functions Ψ, or y^k, in the same sense (by the same directly decodable way, respectively) that information is encoded in neural-net state- vectors $\vec{q}$, or $\vec{v}^k$, for two reasons. First, any natural (non-model) network may initially be in some "natural" state, i.e. eigenstate, of its own. Second, $\vec{q}$, or $\vec{v}^k$, are in principle directly observable, but Ψ, or y^k, are (in general) not – not even in principle, as long as they remain quantum. Therefore, we act as follows.

By a classical interaction or perturbation on an appropriate quantum system, we force the quantum network into a state Ψ which "implicitly reflects" our external influences (inputs), i.e. it is input-modulated. As soon as such a state Ψ stabilizes, becoming an eigenstate, y^k $(k=1)$, we can continue to "insert" (simultaneously or sequentially) other information-encoding states (k = 2, ..., p). All these eigenstates y^k interfere as prescribed by equation (9) and get thus stored in $\mathbf{G}$. *Quantum holography* (e.g., Leichtle *et al*., 1998; Abouraddy *et al*., 2001) is an example which demonstrates how this could be realized, with plane waves $\Psi = Ae^{\frac{i}{\hbar}j}$ or wavelets (Schempp, 1994), without extensive artificial efforts. Moreover, fast-developing specially-designed encoding/decoding (measurement) devices (e.g., Weinacht *et al*., 1999) enable enormous additional possibilities.

Now we want to *retrieve a pattern from memory*. In our quantum-net model we are not interested in Ψ, like we are not interested in $\vec{q}$ in our neural-net model. Our final result will directly be a single "post-measurement" information-encoding eigenstate y^k (say k = 1), or $\vec{v}^k$, respectively. We cannot observe Ψ (except in net-simulations by stopping the program) and we *need not* observe Ψ (or $\vec{q}$) , but we wait until the "measurement" (i.e., pattern-recall which is equivalent to the wave-function "collapse") is triggered by our final new input Ψ' or $\vec{q}'$. Then, the standard quantum *observables O* (corresponding to: $\hat{\mathbf{O}}y = l_O y$), e.g. spin states, can reveal the reconstructed pattern-encoding eigenstate y^k (k = 1). Namely, since the output is similar to the input (that we know!), the eigenvalue (l_O) information can be sufficient for knowing the output. Moreover, if the final $\Psi = y^k$ (k = 1) is / becomes *classical* (like the inputs may well be), then obtaining complete knowledge about the output pattern, encoded in y^k (k = 1) , is at least in some cases (e.g., optical) *relatively straight-forward* (e.g., like *seeing* the image reconstructed from a hologram). Beside quantum holography, an alternative fast-developing technique is quantum tomography for reconstruction of eigenstates ψ^k (D'Ariano *et al*., 2000). So, our information-processing result *can be extracted* from ψ^k using new quantum-optical (and computer-aided) techniques for measurement of observables or for quantum-holographic-(like) wavefront reconstruction.

Let's *analyze the memory and how a pattern is retrieved from it*. Quantum holography is our primary suggestion. If eigenfunctions ψ^k implicitly encode patterns presented to the net, then matrix $\mathbf{G}$ describes the *quantum memory*. The propagator expression *G* in eq. (9), which acts as a projector during the pattern-recall (measurement) process, is related to the usually-used Green function $\tilde{G}$ (e.g., Bjorken and Drell, 1964/65) by $G = -i\tilde{G}$.

If we, in eq. (9) which looks Hebbian, expose the phases φ explicitly, using $\Psi = A\exp(\frac{i}{\hbar}\varphi)$, we get an expression which is the quantum phase-Hebb learning rule:

$$G(\vec{r}_1,t_1,\vec{r}_2,t_2) = \sum_{k=1}^{P} A^k(\vec{r}_1,t_1)^* A^k(\vec{r}_2,t_2)\, e^{-\frac{i}{\hbar}(\varphi(\vec{r}_2,t_2)-\varphi(\vec{r}_1,t_1))} \qquad (9b).$$

This describes the memory encoding which is two-fold: it is both in amplitude-correlations $\sum_{k=1}^{P} A_k(\vec{r}_1,t_1)\, A_k(\vec{r}_2,t_2)$ (Hebb rule) and in phase-differences $\delta\varphi_k = \varphi_k(\vec{r}_2,t_2) - \varphi_k(\vec{r}_1,t_1)$.

The difference between the rule (9b) and a non-quantum phase-Hebb rule is that in eq. (9b) phases φ are quantum phases — i.e., Planck's constant $\hbar$ is present in the exponent.

The *quantum memory retrieval* ($\Psi_{output} = \mathbf{G}\Psi'$) is most-directly realized by the input-triggered, non-unitary *wave-function "collapse":*

$$\Psi(\vec{r}_2,t_2=t_1+\delta t) = \int G(\vec{r}_1,\vec{r}_2)\,\Psi'(\vec{r}_1,t_1)\,d\vec{r}_1 = \int\left(\sum_{k=1}^{P}\psi^k(\vec{r}_1)^*\psi^k(\vec{r}_2)\right)\Psi'(\vec{r}_1,t_1)\,d\vec{r}_1 =$$
$$= \left(\int\psi^1(\vec{r}_1)^*\Psi'(\vec{r}_1,t_1)\,d\vec{r}_1\right)\psi^1(\vec{r}_2) + \left(\int\psi^2(\vec{r}_1)^*\Psi'(\vec{r}_1,t_1)\,d\vec{r}_1\right)\psi^2(\vec{r}_2) + \ldots$$
$$+\left(\int\psi^P(\vec{r}_1)^*\Psi'(\vec{r}_1,t_1)\,d\vec{r}_1\right)\psi^P(\vec{r}_2) =$$
$$= A\psi^1(\vec{r}_2) + B \qquad where\ A \approx 1\ ('signal'),\ B \approx 0\ ('noise') \qquad (10)$$

or in another description

$$
\begin{aligned}
\Psi(\mathbf{r},t) &= \sum_{k=1}^{P} c'^{k}(t)\, y^{k}(\mathbf{r}) = \sum_{k=1}^{P} \Big(\int y^{k}(\mathbf{r})^{*}\, \Psi'(\mathbf{r},t)\, d\mathbf{r} \Big) y^{k}(\mathbf{r}) = \\
&= \Big(\int y^{1}(\mathbf{r})^{*} \Psi'(\mathbf{r},t)\, d\mathbf{r} \Big) y^{1}(\mathbf{r}) + \Big(\int y^{2}(\mathbf{r})^{*} \Psi'(\mathbf{r},t)\, d\mathbf{r} \Big) y^{2}(\mathbf{r}) + \ldots \\
&\qquad + \Big(\int y^{P}(\mathbf{r})^{*} \Psi'(\mathbf{r},t)\, d\mathbf{r} \Big) y^{P}(\mathbf{r}) = \\
&= A y^{1}(\mathbf{r}) + B \quad \textit{where} \quad A \approx 1 \ \ (\textit{'signal'}), \quad B \approx 0 \ \ (\text{'noise'})
\end{aligned} \tag{11}
$$

In eq. (10) and eq. (11) we had to choose such an "input" Ψ' that is *more similar* to y^{1}, for example, than to any other $y^{k}, \mathrm{k} \neq 1$. At the same time, the "input" Ψ' should be *almost orthogonal* to all the other $y^{k}, \mathrm{k} \neq 1$. In this case, Ψ converges to the quantum "pattern qua- attractor" y^{1}, as it is shown in the last row of eq. (10) and in the last row of eq. (11). Thus, the memory pattern y^{1} is recalled (measured). If the condition, well known from the Hopfield model simulations, that "input" must be similar to one stored pattern (at least more than to other stored patterns) is not satisfied, then there is no single-pattern recall.

5. Discussion

The system of quantum equations (8) and (9) is *similar, according to their mathematical structure and coupling,* to the system of neural-net equations (2) and (3). Because we are certain that the neural system (2) and (3) realizes efficient information processing, we have taken the similar system of equations from the quantum formalism in order to discover quantum Hopfield-like associative information dynamics.

There is a difference between the neural "algorithm" (2)—(5) and the quantum "algorithm" (8)—(11): Neural variables like $\mathbf{q}$, $\mathbf{v}^{k}$ and $\mathbf{J}$ in equations (2)—(5) are real-valued, but quantum variables like Ψ, y^{k} and $\mathbf{G}$ are complex-valued. Important implications of this fact are discussed in detail in Perus (1996, 1997).

Anyway, (quantum) holography shows (Leichtle *et al*., 1998; Abouraddy *et al*., 2001; Psaltis *et al*., 1990, 1995) that the quantum neural-net-like information processing outlined here *is* realizable. Moreover, it seems that neural networks and quantum networks cannot necessarily be treated as complex systems with similar, but *independent*, collective dynamics. There are strong indications (Pribram, 1991, 1993) that biological neural networks essentially cooperate with quantum networks in the brain. Mediators are probably synapto-dendritic and microtubular nets. All these networks constitute a sort of fractal-like multi-level information processing (Perus, 1996, 1997). The wave-function collapse (a quantum sort of pattern recognition – remember eq. (10)) can be triggered by the system's interaction with environment (see Zurek, 1991; Brune *et al*., 1996). It seems that, in the brain, neural networks sensing the environment trigger the wave-function collapse and thus transform the quantum complex-valued, probabilistic dynamics into the neural (classical) real-valued, deterministic dynamics (Perus, 1996, 1997; Perus and Dey, 2000). A consequence of the wave-function collapse is that the quantum network becomes more neural-net-like, e.g. the observable "activities of a network of quantum points ('neurons')" are real-valued.

It is true that holographic (Sutherland, 1991) and other oscillatory neural networks (e.g., Baird, 1991), too, do not realize the deep essence of quantum dynamics, i.e. the EPR-manifested non-local interconnectedness and indivisibility, called entanglement. But these features are broken and dynamics is thus discretized during quantum measurements. The implicit parallel-distributed quantum dynamics is useful for computation, but it has to be collapsed during the readout (or memory recall) process in order to obtain results of computation. These essential quantum features, manifested in the complex-valued formalism, could be harnessed or partly eliminated in order to realize neural-net-like information processing. Although it is useful to harness quantum superpositional multiplicity for computational purposes (Steane, 1998; Scarani, 1998) as much as possible, it is practically unavoidable to collapse the quantum wholeness during the readout (measurement) of results. For this, neuro–quantum (classical–quantum) cooperation, as manifested in the collapse-readout, seems useful for the brain as well as for hypothetical neuro-quantum computers (Kak, 1995). The advantages of quantum information dynamics, i.e. miniaturization, speed, computational and memory capacity, are preserved in that case.

Detailed technical (or biological) realization of encoding of information into eigenfunctions y^k and the readout of results of quantum pattern-reconstruction is still an open and advancing field of research. For example, encoding and decoding (readout) methods using laser or NMR devices, developed for quantum computers (Ahn *et al*., 2000; Weinacht *et al*., 1999; Berman *et al*., 1998; Cirac and Zoller, 1995), could partially be used. Cf., e.g., Sanders *et al*., 2002; Biswas & Agarwal, 2004). There are numerous complex systems which are candidates for implementation of collective computation which ”follows algorithms” similar to the one described here (Perus, 1996, 1997; Pribram, 1991, 1993).

A fundamental quantum information processing ”algorithm” (8)—(11) was presented. It was constructed following the Hopfield and holographic neural network models which process information efficiently as tested by numerous computer simulations (e.g., Perus and Ecimovic, 1998; Amit, 1989; Sutherland, 1991).

It was shown how can the Hopfield-like associative information processing and content-addressable memory, in principle, be realized in ”natural” quantum systems, i.e. without need for special devices, except for encoding and decoding. This has important consequences for foundations of physics and informatics as well as for cognitive neuroscience. One of many possible applications, i.e. computer-simulated quantum image recognition, is presented in Loo, Perus and Bischof (2004). Concrete possibilities of quantum implementation are also discussed there.

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