Article

A New Three Dimensional Bivalent Hypercube Description, Analysis, and Prospects for Research

Jeremy Horne

ABSTRACT
A three dimensional hypercube representing all of the 4,096 dyadic computations in bi-valent systems has been created. There are 16 functions arrayed in a table of functional completeness that can compute a dyadic relationship, each component of the dyad, as well as its operator, being a function. Each function in the hypercube has been color keyed to a frequency on the light spectrum to reflect a discussion in August Stern's work *Matrix Logic* and *The Quantum Brain* that purports the ability of binary spaces to reveal quantum mechanical relationships. At the minimum, the hypercube is a "multiplication table" or table of computations and values that shorten the time to do operations that normally would take longer using conventional truth table methods. Already, various hypercubes consisting of binary spaces are used to compute optimal communications paths, as with "hamming distances". In chaos theory, the hypercube seems to show the origins of patterns generated by basins of attraction in binary spaces. Patterns don't emerge from randomness but from deep innate structures in the universe. While specialty areas, such as telecommunications and quantum mechanics are not within the purview of his expertise, the author thinks that there may be significant relevance of what has been developed here to those fields. Finally, it is suggested that research proceed on recursion within each of the 4,096 cubes to finally determine the nature of refined binary space,.

Key Words:3-D hypercube, logic, binary systems, innate order

NeuroQuantology 2012; 1: 38-48

Introduction

A three dimensional hypercube representing all of the 4,096 dyadic computations in bi-valent systems has been created. There are 16 functions that can compute a dyadic relationship, each component of the dyad, as well as its operator, being a function. These functions are arrayed in a table of functional completeness that reflects a binary counting from 0000 to 1111. Each function in the hypercube has been color keyed to a frequency on the light spectrum. This has been done as a reflection of a discussion in August Stern's work *Matrix Logic* and *The Quantum Brain* that purports the ability of binary spaces to reveal quantum mechanical relationships. The hypercube is not a substitute for his work but is new canonization of the binary space upon which he builds his work. At the minimum, the hypercube and the canonization underlying it serve as a "multiplication table" or table of computations and values that shorten the time to do operations that normally would take longer using conventional truth table methods. Yet, there are other uses. Already, various hypercubes consisting of binary spaces are used to compute optimal communications paths. Those concerned with "hamming distances" may find whatis described in this paper to be of assistance. In the field of chaos theory, the hypercube seems to show the origins of patterns generated by basins of attraction in binary spaces. Patterns don't emerge from randomness but from deep innate structures in the universe. At this point, the hypercube is being presented for research purposes. The background of the author is in logic, philosophy, and consciousness studies, and various specialty areas, such as telecommunications and quantum mechanics are not within the

Corresponding author: Jeremy Horne
Address: Jeremy Horne, AvenidaMoreras 131, San Felipe, Baja California C.P. 21850, USA
Phone: + (480) 299-9980 (cell)
mindsurgeon@hotmail.com
ReceivedNov 2, 2010. RevisedFeb 24, 2011. AcceptedJan 18, 2012.

purview of his expertise. Yet, he is familiar there may be significant relevance of what has been developed here to those fields.

Functional notation

To understand the hypercube some background to the notations and canonizations used here is necessary, as they vary from standard textbook presentations of binary logic. It will be found that this new system is shorter, faster, and more intuitive.

The two-valued, variable, or binary system is foundational in deduction, as it uses the lowest number of variables possible to construct a system of relations. This observation stems from an extreme halving of any object in the universe; ultimately, it will be reduced to the smallest of the smallest, or Planck area in terms of not-Planck area, or vacuum space. Of course, this sub-quantum world must be apprehended in terms of the whole in order to place matters in proper perspective. This world is syntactically binary. The two placeholders can have any semantics: *on-off, up-down, true-false, 0 – 1, information-no information,* and so forth. Constructing logical space is straight forward, as it is based upon permutations of zeros and ones and permutations of those permutations, as well as a hierarchy of increasing quantity. We will present binary logic using 0s and 1s (as opposed to the Ts and Fs used by logicians), as the mathematics is easier to follow.

Logical systems are about relations between two variables, as well as relations of those relations. Binary systems minimally have two placeholders, one for each variable, and these are designated by letters, usually lower case ones. We will start by asserting "p" and "q" to designate those place holders. The simplest system is two-placed, as something cannot have a relationship with itself, unless one considers a property called "reflexivity" a relationship. Usually, however, reflexivity means something bears itself to itself as a relationship, and this normally is not considered a system (However, it will be shown later that basic operations and their results are reflexive through recursion). A system must have at least two components to have a relationship, and the simplest, as we have said, is binary. The two variables p and q and each contain one of two values. Technically, these values have a meaning, or enough with the literature to ascertain that semantics. As indicated above, the meaning can be up-down, red-black, or any two different things (objects or processes). In our case, we use 0 or 1, as these are numerals designating numbers. The simplest logical space, where a variable can assume one of two values, is:

p
0
1

Table 1. Primary Logical Space

We have in the following table showing the four permutations of 0s and 1 for the two variables. This is the second most complex logical space.

p	q
0	0
0	1
1	0
1	1

Table 2. Secondary Logical Space

In keeping with an ontological approach to logic, ascertaining that which exists, the 0 and 1 are existents and the only things existing in our system. Naturally, there are definitions, processes, letters designating values, and so forth, but the bare bones structures simply consist of 0s and 1s. An example of this is machine code displayed on a monitor; the code is a broken down version of higher level programs or applications. It is the simplest rendition of whatever the computer is conveying or doing – programs, graphics, text, and so forth. *Open Office* is a program written in a high level language, such as C++ and then rendered into assembler, but this can be produced ultimately in binary form, albeit the length of the program would be enormous.

From the p and q permutation table, above, a functional process emerges. Binary-valued logic (binary logic, for short) relates two existents, and the result is another existent. If two simple existents are being related, a third simple existent will emerge. For example, a 0 may be related to a 1, as in 0 * 1, where "*" is the relationship, and there may result a 1, so we have 0 * 1 = 1. There also is the possibility of 0 * 1 = 0, as well. We may have 1 related to 0, or 1 * 0. The two possibilities here are 1 * 0 = 0 and 1 * 0 = 1. Since there are two placeholders in our system, as it is binary, we see also that 0 can be related to 0 and 1 to 1. We now are on the way of constructing a more complex logical space of functions. A function is a process that determines the outcome of relating two existents to each other.

The four rows of permutations of existent relationships yield a 16-column space, known, for our purposes, as the Table of Functional Completeness (TFC). It is called "complete", as all possibilities, or permutations, of 0s and 1s appear for the placeholders p and q. This is generated by the same method as with the above tables - serially, successively, and in ascending order (binary counting) in the same manner as the previous tables and in this case from 0000 to 1111, each column being vertically read. Columns are headed by an "f" with a subscripts ranging from 0 through 15, with each designating a particular function. While the table includes the p and q generators, or placeholders, they could be omitted, leaving the functions. It is important to notice that each function is an operator, as well an object of computation. (Notice, also, that in keeping with our ontological commitment of having only two existents, 0 and 1, that stripped of the letters and the function designators, all that remains are 0s and 1s). We will see more of this shortly, where a function is an operator, as well as a result of a computation. This makes the logical space an entirely closed and complete space. One function is always the result of two other functions being computed via an operator function. The TFC showing all the permutations of relationships between existents as functions is the following:

p	q	f_0	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}	f_{12}	f_{13}	f_{14}	f_{15}
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
0	1	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	0	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1

Table 3. Table of Functional Completeness (TFC)

For example, referring to simple existents, row 3 for function 4, is read, "function 4 relates p=1 to q=0 for 0." From the TFC those teaching propositional logic create notational standards for expressing relationships between two existents. Four functions normally are taught – and (&), or (∨), equivalence (≡), and implication (⊃)- but there are 16 functions, and each can be given an operator symbol, viz:

$\mathbf{f_0}$	X - Contradiction
$\mathbf{f_1}$	&, and, conjunction
$\mathbf{f_2}$	>, p is greater than q
$\mathbf{f_3}$	1>, 1 precedes, or, simply "p"
$\mathbf{f_4}$	<, q is less than p
$\mathbf{f_5}$	>1, 1 follows (or simply "q")
$\mathbf{f_6}$	≠, p or q is true (1) but not both (XOR);exclusive "or"
$\mathbf{f_7}$	∨, p or q is true or both are true; inclusive "or", disjunction
$\mathbf{f_8}$	NOR, neither p nor q or both is/are true
$\mathbf{f_9}$	≡, p is equivalent to q in truth value
$\mathbf{f_{10}}$	>0, 0 follows (or simply "not q")
$\mathbf{f_{11}}$	⊂, q contains p
$\mathbf{f_{12}}$	0>, 0 precedes (or simply "not p")
$\mathbf{f_{13}}$	⊃, p contains q – **defines deduction**
$\mathbf{f_{14}}$	NAND, not both p and q are true
$\mathbf{f_{15}}$	T, tautology

Table 4. Functions, Symbols, and Their Names.

One should note that, although functional completeness is discussed, it is usually the case that names are not given to the operations, or functions. The above names for some functions for the least used functions are not standardized in the field of logic. From the above, f_9, equivalence, often is written as:

p	q	p ≡ q
0	0	1
0	1	0
1	0	0
1	1	1

Table 5. Truth Table for Equivalence

Notice in this table that p, read downward is 0011, corresponding to f_3, and q is 0101, or f_5 in the Table of Functional Completeness (TFC). The equivalence, (≡) corresponds to f_9. Functions relate functions to functions, as we see with f_3, and f_5. In this case, we may say:

$$f_9(f_3,f_5) \rightarrow f_9.$$

Any function operating over f_3, and f_5, or the four permutations of 0 and 1, will yield itself. That is, the function defines itself. Here, the canonization does not add much to the extant presentations of the propositional calculus, as the truth table rendition above suffices quite well. However, the value starts to emerge with calculations like f_{10} over f_{12} and f_4, which is f_{11}. That is, $f_{10}(f_{12},f_4) \rightarrow f_{11}$).First, f_{10} means:

p	q	p >0q
0	0	1
0	1	0
1	0	1
1	1	0

Table 6. Truth Table for f_{10}.

Or, the resultant value is 1 when 0 is the second value in the relationship. Otherwise, the resultant is 0. Now, replacing the f_{12} and f_4 values for the ones in p and q, p = 1100 and q = 0100, we have the following:

p	q	P>0q
1	0	1
1	1	0
0	0	1
0	0	1

Table 7. Truth Table for $f_{10}(f_{12},f_4) \rightarrow (f_{11})$.

Whenever q as 0 follows whatever value for p, the value is 1; if 1 follows, the value is 0. This 1011, f_{11}, now is a function that can be used to compute others; the result of a computation now can be an operator.

With the standard logic texts presenting only four operators, there would have to be a rather large expansion to express the above in terms of those operators. For example, 1100 might be a result of the AND operator over something like 1-1, 1-1, 0-1, and 1-0. These, in turn, might have resulted from other operators. The difference here is that we are using all 16 functions as operators. Of course, proofs are greatly shortened. The other extreme is using only one function and its negation, the disjunction (f_7) and conjunct (f_1) operators being the most frequently used to discuss minimal systems. A further word needs to be said about complete expression by use of all 16 functions. Even though it is not necessary to use them all, it is necessary to be aware that such can be done to appreciate the completeness of the logical space generated by the TFC.

As one can see, the notation $f_{10}(f_{12},f_4) \rightarrow f_{11}$ reduces the number of extra characters considerably. Note, also, that this canonization follows normal conventions in displaying functional computations; it just isn't standard in propositional logic, yet.

All operations in a binary system are dyadic, or two-place. All complex expressions ultimately are reduced to a final two-place computation. For example, a standard formula, such as (p v q) ⊃ (q & s) is written as $f_{13}(f_7(f_3,f_5),f_1(f_3,f_5))$. Since the f_3 and f_5 define the function, they are not necessary, so we have, instead, $f_{13}(f_7,f_1)$, a shortened version. In the first functional expression, the computation hasn't been carried out, but the value is f_9, or equivalence. That is $f_{13}(f_7,f_1) \rightarrow f_9$, a shorter way of rendering the result than with a full truth table display.

The syntax for dyadic computations is $f_n(f_x,f_y) \rightarrow f_{p,}$, where f_n represents a binary operator, such as f_7, or 0111. The f_x and f_y represent the operands, and the f_p is the result of the computation. N-adic computations take the form $f_q(f_n(f_x,f_y) \rightarrow f_{p,})) \rightarrow f_r$ … f_* … $(f_@((f_+, f_\#) \rightarrow …)$, q, n, *, and @ being subscripts designating operators, x, y, +, and # operands, and p and r being results of the computation. It cannot be emphasized enough that any function can be an operator, as well as an operand, depending upon its placement in the syntax. Such is one of the factors making binary logical space closed, each function being in dialectical relationship with the others (one in terms of the others), where it can serve in an opposite capacity – operator or operand. Process becomes object, and object becomes process.

Composition of the hypercube

The hypercube represents the 4,096 permutations of dyadic (two place) computations of the sixteen functions in the Table of Functional Completeness (TFC), i.e. $f_n(f_x,f_y)$, where the subscripts “n”, “x”, and “y” stand for “a selected function” as an operator, the first element in an ordered pair as operands, and the second element of the ordered pair, respectively. There are 16 plates, each corresponding to one of the 16 functions. Each plate displays a Cartesian coordinate form of a particular function operating over the 16 functions, including itself. That is, the plate shows the complete permutation of computations for a function. There are 16^2computations, in each plate or 256 results. In reading the hypercube one starts from the top left, reads downward and then across the top to arrive at an answer. Thus, in Plate f_6, for $f_6(f_9, f_{12})$ to get f_5, read down the left-hand most column to f_9 and then across to the column headed by f_{12} in the manner of a distance chart on a highway map to get the f_5. The same plate shows $f_6(f_8,f_{11}) = f_3$. Always read across and then down to get the result of the computation; a number of function pairs are not commutable, *i.e.,* yield the same result if the functions are switched. We can see that 16 plates times the 256 results for each plate yields the 4,096 as the total number of dyadic computations possible in binary space. This is the complete expression of computational completeness for all 16 functions in a dyadic relationship. The hypercube is to computational completeness for dyadic relationships as the TFC is to the permutations of 0s and 1s in a four place number. The first is three-dimensional, the second two-dimensional.

It should be borne in mind that the hypercube contains the smallest area that can be occupied in Euclidean space. This fact ensures that the resulting permutation space is optimal.

Structural significance of the hypercube

The hypercube acts as a multiplication table for doing dyadic computations in binary space. Rather than displaying a full truth table one can do a chained calculation, such as $f_{13}((f_7(f_4,f_8),f_1(f_{12},f_3)),f_9)$ simply by starting with the innermost parentheses, as in standard logical calculations, and working to the outermost function, f_{13}. In this case, using the hypercube, the result is:

$f_7(f_4,f_8) \rightarrow f_{12}$ f_7 plate – first innermost parentheses

$f_1(f_{12},f_3) \rightarrow f_0$ f_1 plate – second innermost parentheses

$f_7(f_{12}, f_0) \rightarrow f_{12}$ f_7 plate – f_7 function operating over the results of the previous two calculations

$f_{13}(f_{12},f_9) \rightarrow f_{11}$ f_{13} plate – final calculation

Imagine the extent that truth tables would be; the time to find the results using the hypercube is minimal as well, compared to the longhand way of finding the result. Additionally, the hypercube contains the complete three-dimensional logical space; all dyadic computations possible are included. In the traditional logic, it should be kept in mind that one is using only four plates, plus the negation operator.

Philosophical significance of the binary structures

This author's assertion is that logic is a language that describes innate order in the three dimensional universe and that it is the basis upon which mathematics rests. Logic is discovered, rather than invented: *"...a machinery for the combination of yes-no or true-false elements does not have to be invented. It already exists"* (Misner*et al.,* 1973; p.1209). Jean Piaget asserts,

> There exist outline structures that are precursors of logical structures. It is not inconceivable that a general theory of structures will be worked out, which will permit the comparative analysis of structures characterizing the outline structures to the logical structures characteristic of the higher stages of development. The use of the logical calculus in the description of neural networks on the one hand, and in cybernetic models on the other, shows that such a programmed is not out of the question (emphasis included) (Piaget, 1958; p.48).

Going further, the observer becomes the observed through what s/he is observing. Rapoport, the world renowned nuclear physicist, states that:

> "...object, subject, space and time, quantum physics and multivalued logics have a common ground, which is that of paradox and self-reference, which the latter have not been incorporated in the current studies of quantum physics nor in quantum computation in which a matrix representation of logic is the core mathematical element" (Rapoport, 2009; p.14).

Indeed, perforce, each of the 16 functions is not only recursive, where the outputs forward-fed as input reproduce the function, but all of space, which can be reduced to binary form in this way, also is recursive. Further, each function is an object of computation, as well as an operator; operator is operand and vice versa. This demonstrates Rapoport's assertion about self-reference.

The unfolding structure of the most basic logic, binary relationships, comes from natural ordering (Horne, 2000). Logical space is generated in an ascending fashion and ultimately contains all the relationships possible in this world. Recall, everything is reducible to Planck area and non-Planck area, a duality. From the singular and planar logical spaces comes the three dimensional hypercube. This hypercube is foundational to, but not a substitute for the space written about in August Stern's *The Quantum Brain*, where we he states:

> We show that logic theory can be formulated in the quantum mechanical language of the Pauli spin operators and that the evaluation of logic functions can be achieved via ordering of creation and annihilation operators modeled on the normal ordering of operators in quantum field theory. The creation and annihilation operators appear to be as fundamental for logic theory as they are for fundamental physics (Stern, 1992; p.4).
>
> If we were to replace the wave functions and a differentiation versus truth value by a differentiation versus time, instead of the logic state space, we would find ourselves in the Hilbert Space of quantum mechanics (Ibid. p. 5).

This paper contains no claims to discovering actual quantum mechanical descriptions within the hypercube, the TFC, or any of the spaces that they may generate. The cube, itself is a only display of all the dyadic computations in ordinary binary space and serves as a basis upon which a quantum vector calculus may rest, where the four dimensional may be described in terms of the three dimensional, analogous to a cube being represented in a plane. Given the indications that bivalent spaces seem to have been described by people such as Stern and Rapoport as containing such, this new canonization of functions and the spaces they generate may provide viable grounds for researching from a different perspective those relationships. There are other philosophical issues that the hypercube raises, as well.

In chaos dynamics AndrewWuenche suggests that a random concatenation of binary operations will result in patterns (basins of attraction). He states that these patterns emerge from chaos (Wuensche, 1993). However, his methodology is poorly explained, and there is no real thesis about the meaning of the patterns. This is true with similar authors, such as Kauffman (Kauffman). There is this “wow” factor, coupled with a very questionable metaphysics. Things don't come from nothing in this dimension, and emergence is predicated upon a “seed”, something inherent in an overall structure. By analyzing the hypercube, one may find such seeds. Indeed, what may emerging from the research is the existence of attractors in hypercubes and closed spaces, the nature of which needs to be examined further. This is important, if one is to accept the argument binary hyperspace is reflective of innate order in the universe and that all spaces can be reduced to binary ones. Again, further research is required to see if the seeds of patterns emerging from chaos are in the hypercube.

Aside from any reference to order emerging from chaos in the binary world or whether there are attractors, there are already in each of the sixteen plates emerging patterns of color, as with Plate f_7, where, with f_7, there is a string of f_7s, followed by eight f_{15}s, both vertically and horizontally, suggesting boundary conditions of some type. The diagonals in each plate show a functional counting, one diagonal ascending, and the other descending. By coupling all the plates, a three-dimensional view may reveal more. Already can be found emerging grouping, or clustering, of functions, as in Plate f_4, where f_2 seems to congregate in groups of three at various places along the top of the diagonal. The figure on page 11 of this paper showing basins of attraction may be compared to various plates in the hypercube, where clustering and “holes”, or isolated coloring, occurs.

Extensive pattern analysis of the hypercube may reveal deeper and extended patterns. Suffice it to say that if Lorentz attractors are found in the hypercube, this may be a significant step in demonstrating that there are seeds in inherent structures found in what otherwise would appear to be chaotic phenomenon. It is not this author's thinking that patterns emerge from nowhere. Structures are innate in the universe, and logic can describe those patterns. The three-dimensional hypercube not only displays the 4,096 possible dyadic computations but also it appears to contain the seeds of patterns generated in larger binary spaces. Each of the sixteen functions is recursive, but it remains for research to determine how recursion occurs using more than one function (Horne, 2000). The section below on “Future

Directions in Research" discusses how work based on the hypercube might proceed.

Frequencies/Wavelengths as functions

Taking after August Stern's *Matrix Logic,* functions here have been correlated with frequencies of light in the visible spectrum, technically colors produced by a single wavelength in that spectrum. Each function has been assigned a color in descending order of frequency, starting with f_0 corresponding to the shortest wavelength and highest frequency to f_{15} being the lowest frequency and longest wavelength. The colors used are only approximate but reflect the intended ordering of correlations of functions with frequencies/wavelengths.

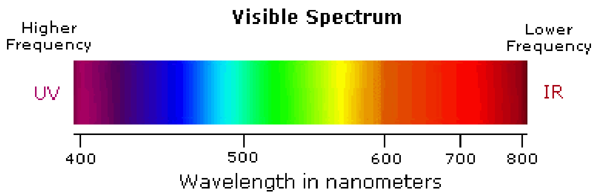

Figure 1. Visible Light Spectrum.

It must be born in mind that the functional correspondence is discrete, but each function could correspond to a range, one range being adjacent to the next. One may read the hypercube as display of the relationship of relative wave densities and a wave's ability to penetrate (At ultra-high frequencies, such as with gamma rays, it is the photon, rather than the wave, itself that penetrates). Several features manifest themselves immediately. The "end plates", f_0 and f_{15} are solid, while f_3 and f_5 the columns used to generate the 16 permutations of 0s and 1s constituting the logical space, show a rainbow configuration, ranging from the highest to the lowest frequency, *i.e.,* the shorter to the longer wavelengths. However the functions are perpendicular to each other, with the rainbow in $f_3$being displayed vertically and the rainbow in f_5 being displayed horizontally. The two plates juxtaposed to each other act as polarizers.

A way of reading the plates is analogous to reading a topographic map, where radically different frequencies, such as blue – 450 nm being next to a red - 600 nm block, reveal a radical differential. See the figure on page 11 for how contours are portrayed in a maxima-minima problem involving basins of attraction. In the hypercube, we have a map of a landscape where binary functions have been mapped to visible light frequencies, all in keeping with Stern's view that binary space contains quantum mechanical relationships. Contour maps seem to be a viable way of analyzing the color patterns.

The patterns of symmetry act as a correction mechanism for functional calculation. Mis-typed or calculated results will show up immediately as defilers of that symmetry, and by inspection - "completing the color", as it were, the correct function can beinserted readily.

Canonization and possible later correction

The manner in reading the plates is in keeping with the canonization used to generate the TFC, with the 0s being at the top and ascending to the 1's, from the least information to the most. Further, logicians construct truth tables using the same order of reading truth values matched to variable – across and then down. However, many of them place "T"s on the top row. It makes more sense, though, to place the Fs (or "F") on the top row, as information (designated as "1") is generated in the same manner as a truth table of basic space is read – top left to lower right. This method does not follow the canonization found in reading mathematical graphs, where one read along the x-axis to a point and then up on the y-axis. All of this will not change the patterns, albeit they may be rendered in a 90° rotation to conform to the standard for graphs.

The f_0 represents the shortest wavelength (dark magenta) and is on the left-hand most side of the scale of the visible spectrum, and it is in the hypercube, as well. It may be said that f_0 is the least "dense" function in logic, as it represents no informational content or impact, and thus correspond to the frequency having the lowest penetrability of a substance, i.e., the shortest waves. There does not appear to be a standardized way of displaying the visible light spectrum, as one will just as readily find the lower wavelengths displayed on the right-hand side of the page. The color scheme easily could be reversed, with f_0 representing the lowest frequency and longest wavelength. Yet, penetrability, itself could be considered as having the most impact on a substance insofar as being able to probe its inner nature. Again, a change can be made to the hypercube, but such would not alter the overall patterns of color, hence relationships of the frequencies corresponding to the functions.

The 3-D hypercube was created using *Open Office 3.2*, and the color scheme was selected according to the following table:

f_0	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}	f_{12}	f_{13}	f_{14}	f_{15}
c9	m7	m8	b3	b5	b7	t2	g6	g8	y1	y2	y4	o2	r4	c11	c5

Figure 2. Color Key for Hypercube. Where:c = chart, m = magenta, b = blue, t = turquoise, g = green, y = yellow, o = orange, r = red.

Colors and their values, such as "magenta 8" are revealed when the mouse hovers over the chart crated from the drop down menu displayed by clicking on *Table>Table Properties*.

Future direction in research

My background is not in quantum mechanics; it is in logic. Yet, I can see that others have referred to binary systems in explaining order (Misner, 1973), discerning order from chaos (Wuensche, 1993; Kaufman, 1993; Misner, 1973), and making sense of quantum mechanics. Dialectically, something is apprehended in terms of what it is not, such requires a subdivision, and the most reductionist form of logic leads to the binary world in which is contained the innate order in the universe.

It is known that visual representations can reveal information that can be missed by straight symbolic manipulations on paper. Stern's *Matrix Logic* was the impetus for color coding the functions. Inasmuch as color is a manifestation of light waveforms and quantum mechanical processes render such, the hypercube may reflect some of Stern's findings. Where this will go depends upon the research, some suggestions of which are now discussed.

Aside from lofty considerations of the quantum world, the hypercube has more prosaic applications but from which may be developed algebra of spaces. Consider commutativity as error checking device. Commutatively is symmetric, where $f_C(fp,fq)=f_C(fq,fp)$. This goes for C=0, 1, 6, 7, 8, 9, 14, and 15. Functional computation may be expressed in algebraic form, such as the simple example in the f_5 plate, where $f_5(fn, fp) \rightarrow fp$, and where fn and fp represent any two distinct functions. Similarly, f_3(fn,fp)->fn exists for the f_3 plate. For plate f_8, x,y = f_0. Numerous and more complicated relations may be developed, but such is work for further research.

In searching for inference rules one displays a set of relations in horizontal form, constructs a truth table, and determines whether there is any row containing the conjunct of all the formula as being true and the conclusion being false. The addition of variables beyond two simply multiplies the logical space to 2^n rows. This is simply a set of multiple of the basic space generated by the TFC and is analyzable by dyadic computations. The object for research here is to see whether the new canonization with the functional notation can result in algebra of functions to generate inference and equivalence rules. For example, *modus ponens* is $p \supset q, p \therefore q$. In our canonization, this is $f_{13}(f_{13},f_3) \rightarrow f_{15}$. Research might produce a computer program to generate not only acceptable rules, but also these might be used to help produce algebra of spaces.

In everyday telecommunications, the hypercube may offer itself to applications. Hamming distances are the number of changed bits in a binary word of fixed length as a measure of error. In addition, hamming distances involve changing in the least number of steps one sets of symbols to another. For example, changing "hopper" to "mapped" involves "h" to "m", "o" to "a", and "r" to "d", or 3 as the hamming distance. Three dimensional hypercubes are used to represent the paths of change, but it may be asked whether the hypercube developed above may be of use. Reflecting on the previous, path analysis, in general might be correlated to how rules are generated and validated.

Dynamic pattern analysis is ideally suited for the hypercube. Logical spaces are commonly viewed as static entities, but operations within those spaces are dynamic. One may liken the hypercube to a keyboard on which can be played or type the various units of expression. Techniques exist about performing logical demonstrations, where the results of a functional computation are forward fed as inputs to a succeeding computation and this is repeated until the function reappears (Horne, 2000). A project also presents itself where one can start with a "blank" hypercube and trace the development of such computations in visual form to see what patterns may emerge. Andrew Wuensche's material centers on two-dimensional Basins of Attraction based on random generation, but what would emerge with a systematic generation, such as a demonstration or proof. Basins of attraction are in two dimensional form, but what of three dimensional ones? Perhaps by converting the problem into a maxima-minima problem, such basins might emerge. A "Team of Students" in Dr. Mark Parker's presented for the Mathematical Association of America the "Four Humped Camel" (Parker, 2006) in which is a maxima-minima analysis of basins of attraction. The following formula:

$$f(x_1,x_2) = (4-2.1x_1+\frac{x_1^4}{3})x_1^2 + x_1x_2 + (-4+4x_2^2)x_2^2$$

yields the following "basins of attraction", the basis of which is referred to in the presentation as a "contour plot":

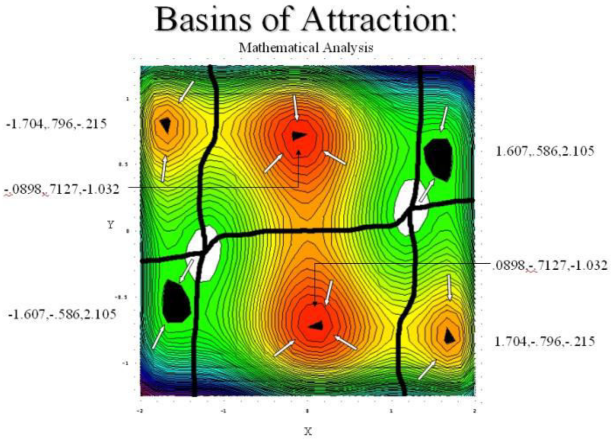

Figure 3. Example of Contour Plot of a Basin of Attraction.

The centers of the red areas are the minima and the white areas are the maxima. The coloring appears similar to the style of a topographic (contour) map, as well as in the hypercube in terms of color placement. This is not to show that the formula in the hypercube and this project are similar but that a resulting pattern similar to this one may be an indicator of a basin of attraction, which is indicative of a pattern generating aspect of the cube. For example in the plate for f_8, one sees what might be considered a basin of attraction around f_5.

It was noted above that each of the 16 functions is recursive, *i.e.,* the outputs forward fed as inputs into the function cause the function to reappear. Thus, each function acts as a self-maintaining, or homeostatic automaton. Of course, all binary spaces are composed of one or more of these functions, or partial functions. Starting with a set of set of formulas demarcating an initial space,it would be interesting to see how that space evolves until it repeats itself. No entity at whatever level is static, so tracing the dynamism of an initial state of binary functions would give an insight to pattern generation and possibly shed light on how basins of attraction form.

As a longer term project, one may map each function to a sound or color to see what patterns may emerge. Newton, following an idea by the ancient Greeks, suggested that there may be a correlation between color and sound (*Opticks, passim* for *"sound"*). Correlating sound to color is not novel these days. In various computer programs designed to play CDs, such as Windows Media, one can view colored patterns emerge when playing music.

Consciousness studies is a field that can be explored with processes applied to binary spaces. According to Tononi, consciousness is integrated information, "...the amount of information generated by a complex of elements,above and beyond the information generated by its parts" (Tononi, 2008; p.1). Consciousness arises from the condition of neural systems, and these can be represented in a binary manner, i.e., on-off switches, or as Tononi refers says, "photodiodes" (Ibid., p. 7).

Since effective information is implicitly specified once a mechanismand state are specified, it can be considered to be an "intrinsic"property of a system. To calculate it explicitly, from an extrinsicperspective, one can perturb the system in all possible ways(*i.e.,* try out all possible input states, corresponding to themaximum entropy distribution or potential repertoire) to obtainthe forward repertoire of output states given the system's mechanism.Finally, one can calculate, using

Bayes' rule, the actualrepertoire given the system's state (Ibid., p. 8).

From a probability distribution among 16 system states in binary space, Tononi generates a mathematical representation of intuition, or qualia. Binary representation is based on whether a neuron fire or not. This, for Toni is the basis for consciousness, but, "is not an all-or-none property,but is graded: specifically, it increases in proportion to asystem's repertoire of discriminable states" (Tononi, 2008; p.31). The density of interconnectivity of the 16 system states displays the degree of information produced from the system states; the greater the entropy there is, the greater the information. The number and type of interconnections determine the qualia (Ibid., p. 16). These interconnections are represented by a binary hypercube, but the shape of the quale (the figure representing an aspect of qualia) is influenced by the interdependence, or entanglement, of connections of nodes (Ibid, p. 19). Overall, different areas of the brain manage different functions, but "...hard-to-articulate phenomenologicaldifferences correspond to different basic sub-shapes in Q, suchas 2^n dimensional *grid-like* structures and *pyramid-like* structures,which emerge naturally from the underlying neuroanatomy"(Ibid., p. 23, Tononi's emphasis). It is worthy to note as a sidebar that Tononi regards consciousness as intrinsic, "'knowing' from the inside", much in the same manner as described by Rapoport, above (Ibid., p. 28)."It is solipsistic" (Ibid., p. 36). Hence, a working system representing anything near a conscious entity could start be designed without considering environmental dependencies? Of course, to represent anything approaching what people think is consciousness would involves enormous complexity, as Tononi admits, but his serves a model for research. As to the hypercube, it too is binary, and the canonization of functions and their relationships to each other could serve as a foundation for mapping Tononi's model. That consciousness is dynamic returns the discussion to binary automata and the patterns they may generate. This world of research is just beginning.

Conclusion

A basic binary logical space is generated from a single square to two squares one containing a value, and the other a second value. Two values is all it takes to construct binary space. The permutations of this two-squared space yield four permutations of the two values. These, in turn, produce the sixteen basic functions displayed in the Table of Functional Completeness (TFC). From the TFC is developed the three-dimensional hypercube. It is color coded according to specific frequency or frequency ranges in the visible light spectrum in order to make such mapping consistent with a further application described by August Stern in *Matrix Logic* and *The Quantum Brain*, where three-dimensional binary hyperspace is said to contain quantum mechanical relations and where colors are outgrowths of quantum processes.

The hypercube allows for more rapid and simplified dyadic computations in bi-valent space, but also may enable enhanced methods for computing hamming distances. There are indications that basins of attraction may exist within the hypercube, and these may indicate seeds from which order is generated from what was thought previously to be chaos. Basins of attraction are objects of study of chaos dynamics researchers, but this author's opinion is that something (a pattern) does not come from nothing. The universe has innate order, described by binary structures, the hypercube being one. There is no randomness in the universe, and chaos contains encoded order that must be untangled by logical analysis. Superimposed upon the basic binary logical structure, literally, are methods proposed by Stern and Rapoport to accomplish this. It is a matter for future research.

Numerous research areas stem from the development of the basic hypercube, all centering on pattern analysis of structures expressed in binary space. Once the original structure is mapped onto the binary one, the same analytical process of pattern recognition can be applied, thus leading to a uniform way of looking at reality in many of its diverse but reducible forms.

Outlook

It has been demonstrated that each of the 16 functions in the TFC is recursive, i.e., the outputs forward-fed as inputs causes the original function to reappear (Horne, Recursion, 2000), analogous to that of a repeating decimal. Research is ongoing to ascertain recursion in each of the 4,096 cubes of the hypercube. In the original recursion work, the starting computation was for each of the 16 functions ranging over f_3 and f_5, the permutation of possible values for dyadic relations in a bivalent world. There are other functions that would accomplish the same description of permutations, but these emerge from ascending ordering, following that of TFC generation (Ibid.). What has not been determined is the recursive behavior of any function ranging over any pair of functions, as in $f_{10}(f_3, f_{11})$. Each of the functions is recursive, with the number of available functions in each row being the maximum number of iterations. This can range from 1 (plates f_0 and f_{15} for all functions) up to 16. For example, $f_{12}(f_8, f_{11})$ can yield only f_7, while for f_5, a calculation of $f_5(f_x, f_y)$ can yield any of 16 functions, with a limit cycle of 16. Some recursions, focusing on the second operand, like $f_2(f_{14}, f_5)$ result in a stabilization at a cycle oscillating between f_4 and f_{10}, with a maximum cycle of eight. For a calculation like $f_4(f_{10}, f_y)$, the result is restricted to f_0, f_1, f_4, and f_5. The output must "land" on one of these. There can be a maximum of only four iterations before the cycle repeats. $F_{14}(f_{11}, f^{10})$ oscillates between f_5 and f_{14}. These observations apply as well to the first of the operands in a dyadic relationship. The result of any calculation is bounded horizontally by the values in the particular row of the operands. Incidentally, the $f_{10}(f_3, f_{11})$ oscillates between f_4 and f_{11}.

The implication of recursion inside each of the 4,096 cubes would be that the cube, itself, as representative of the closed space of natural binary deduction (without axioms) displays individual instances of the functions being closed, or homeostatic, thus giving a complete rendition, or proof, that any binary space is closed. That is to say, if a system can be determined as closed, its being able to be reduced to a binary state by successive division demonstrates that it is recursive, meaning that any closed system is recursive, hence, homeostatic. When color coding corresponding to the electromagnetic spectrum is overlaid, we may see an interesting display of dynamism as each of the cubes steps through the recursions. An added flair would be the correlation of sound with the colors, not unlike what Newton did.

Acknowledgement

The author wishes to thank Asia Flood, a professional artist located in Gold Canyon, AZ for her invaluable assistance in colorcoding the hypercube. Without her work, it would have been difficult, if not impossible to discern the colored patterns.

Please see supplementary document section for additional the three-dimensional hypercube plates.

References

Horne J. Logic as the Language of Innate Order in Consciousness.Informatica 1997; 21: 675-682.

Horne J. Recursion of Logical Operators and Regeneration of Discrete Binary Space.Informatica 2000;24:275-279.

Horne J. The General Theory and Method of Binary Logical Operations.Journal of Applied Science and Computations, 2000.

Kauffman S. The Origins of Order. New York: Oxford University Press, 1993.

Misner CW, Thorne KS, Wheeler JA. Gravitation. New York: W.H. Freeman and Company, 1973.

Newton I.Opticks or a Treatise of the Reflections, Refractions, Inflections, and Colours of Light. London: Printed for William Innys at the West-End of St. Paul's; 1703.

Parker M. 4 Hump Camel, class presentationMA 421: Mathematical Optimization, Applications, and Analysis class at Carroll College, Helena, MT; 2006.

Piaget J. Logic and Psychology. New York: Basic Books, Inc., 1958.

Rapoport D.Surmounting the Cartesian Cut Through Philosophy, Physics, Logic, Cybernetics, and Geometry:Self-reference, Torsion, the Klein Bottle, the TimeOperator, Multivalued Logics and Quantum Mechanics. Found Phys 2011; 41: 33-76.

Stern A. Matrix Logic and Mind..Elsevier: Amsterdam, 1992.

Stern A.Quantum Theoretic Machines. Elsevier: Amsterdam; 2000

Tononi GB. Consciousness as Integrated Information: a Provisional Manifesto 2008; 215**:** 216-242.

Wuensche A. The Ghost in the Machine - Basins of Attraction of Random Boolean Networks. Cognitive Science Research Papers (CSRP) 281: University of Sussex at Brighton, 1993.