A Quantum Brain Model of Decision-Making Process Incorporated with Social Psychology

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ABSTRACT

In order to understand human mind and brain, the concepts of quantum theory have been applied so far. In this paper, we attempt to construct a quantum brain model of decision-making process, and determine the Hamiltonian, combining with social psychology in Synergetics. We adopt Weidlich model dealing with opinion formation of individuals on psychology. The fundamental idea of our modeling is that the transition process between two opinions in psychology is similar to the one in the quantum two-level system. Thus, we provide not only a new quantum model of brain, but also a new approach of modeling in this field.

Key Words: quantum brain, decision-making process, Hamiltonian, quantum two-level system, social psychology in Synergetics

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Introduction

The human brain is constructed by complex physical systems as well as general biological tissues (Schrödinger, 1944). Focusing on the conception that constituents of the brain are physical systems, we are never able to divide mind and matter. In order to describe the physical systems, broadly speaking, there are two ways on the approach by means of epistemology. One way is approach from classical physics such as Newtonian mechanics, the other is from quantum physics which is indispensable to describe microscopic systems. Nowadays, it has been recognized that the brain operates not only at classical level, but also at quantum level (Hameroff and Penrose, 2003). Interestingly, many phenomena of neuroscience are consistent with the quantum concepts (Tarlaci, 2010b). Especially, in the cognitive neuroscience, it has been considered that the quantum concepts are indispensable (Tarlaci, 2010a). Recently, a hybrid model of chaos theory with quantum physics has been reported to mathematically explain the development of creativity, which is one of the most attractive issues in psychology (Koyama and Niwase, 2017).

In social psychology, there are many mathematical models on the movement of human mind (Galam, 2012; Weidlich and Haag, 1983). One of these is the model which deals with opinion formation of individuals among many people (Carbone and Giannoccaro, 2015). Especially, a model of the decision-making process was established by Weidlich in terms of analogy with spin model of ferromagnetic substance (Weidlich, 1971; 1972). The Weidlich model belongs to "Synergetics" (Haken, 1978) which describes self-organization processes within human behavior (Weidlich, 1991) as well as natural phenomena.

In the present study, we attempt to construct a quantum model of the decision-making process by combining with Weidlich model. The fundamental idea of our modeling is that the transition process between two opinions in psychology is associated with the two-level system in quantum mechanics.

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Thus, we propose a new approach of quantum modeling of brain in terms of incorporating with social psychology in Synergetics.

Essence of the Opinion Formation Model in Social Psychology

As a preparation for the later discussion we recapitulate briefly the essential aspects of a psychological model of opinion decision-making process developed by Weidlich (1971; 1972).

In order to describe opinion in human behavior mathematically, there are many approaches in mathematical modeling in psychology. Focusing the stochastic process of changes of opinions, we can find an analogy between human decision-making mechanism and the ferromagnetic spin model in statistical physics. As a matter of fact, Weidlich combined the spin model with human opinion making processes.

The components of this model are represented by spin variable. Namely, two opposite human opinions are expressed by spin up ↑ or down ↓. Total number of components (population) is denoted by n:

$$n = n_{\uparrow} + n_{\downarrow},$$

where $n_{\uparrow}$ and $n_{\downarrow}$ are total number of spin ↑ and ↓ respectively. Although values of $n_{\uparrow}$ and $n_{\downarrow}$ can change, n does not change, so n is a conserved quantity. In order to reduce the number of variables in the model, new variable x is defined as follows:

$$x = \frac{n_{\uparrow} - n_{\downarrow}}{2n} \left(-\frac{1}{2} \leq x \leq +\frac{1}{2} \right).$$

In this model, the transition probabilities per unit time for an individual are set up to following equations by an analogy with statistical mechanics:

$$p_{\uparrow \Rightarrow \downarrow}(x) = \nu \exp\{-(kx+h)\},$$

$$p_{\downarrow \Rightarrow \uparrow}(x) = \nu \exp\{+(kx+h)\},$$

where parameter k is a measure of the strength of adaptation to neighbors; h is a preference parameter; ν is a measure for the frequency of the flipping process.

In the case of $n \gg 1$, x is regarded as a continuous variable, and the probability distribution $f(x;t)$ is determined by the Fokker-Planck equation:

$$\frac{f(x;t)}{\partial t} = \frac{\partial}{\partial x}\{K_1(x) f(x;t)\} + \frac{1}{2}\frac{\partial^2}{\partial x^2}\{K_2(x) f(x;t)\},$$

where t is time parameter. The specific coefficients of this model are given by

$$K_1(x) = \left(\frac{1}{2} - x\right) p_{\downarrow \Rightarrow \uparrow}(x) - \left(\frac{1}{2} + x\right) p_{\uparrow \Rightarrow \downarrow}(x)$$

$$= \nu\{\sinh(kx+h) - 2x\cosh(kx+h)\},$$

$$K_2(x) = \left(\frac{1}{n}\right)\left\{\left(\frac{1}{2} - x\right) p_{\downarrow \Rightarrow \uparrow}(x) - \left(\frac{1}{2} + x\right) p_{\uparrow \Rightarrow \downarrow}(x)\right\}$$

$$= \left(\frac{\nu}{n}\right)\{\cosh(kx+h) - 2x\sinh(kx+h)\}$$

The stationary solution $f_{st}(x)$ of the Fokker-Planck equation is calculated, giving:

$$f_{st}(x) = cK_2^{-1}(x)\exp\left\{2\int_{-\frac{1}{2}}^{x} \frac{K_1(y)}{K_2(y)} dy\right\}.$$

where c is normalization constant. In the factor inside the exponential of $f_{st}(x)$, ν is not included in the result of $K_1(y)/K_2(y)$, hence important parameters for stationary states are k and h.

In order to determine the model parameters from observation data, we can utilize two equations in probability theory as follows:

$$\mu_x = \int_{-\frac{1}{2}}^{+\frac{1}{2}} x f_{st}(x;k,h)dx,$$

$$\sigma_x^2 = \int_{-\frac{1}{2}}^{+\frac{1}{2}} x^2 f_{st}(x;k,h)dx - \left[\int_{-\frac{1}{2}}^{+\frac{1}{2}} x f_{st}(x;k,h)dx,\right]^2,$$

Where μ_x and σ_x^2 are the mean value of x and variance in measurements respectively. By comparing the results of numerical calculations with the observed values μ_x and σ_x^2, k and h can be determined. Here we emphasize that parameters k and h can be obtained by the statistical numerical calculations on the target human group, hence these parameters should reflect the state of the group.

A Quantum Brain Model of Decision-Making Process

In this section, we build a quantum model incorporating with the Weidlich model described in Section 2. We consider that the transition process between two opinions is similar to the one in the quantum two-level system such as ammonia molecule, so that:

$$(\downarrow) \leftrightarrow |1\rangle,\ (\uparrow) \leftrightarrow |2\rangle$$

where, the terms on the left-hand side mean human opinion states in Weidlich model, and the ones on the right-hand side mean quantum stationary states. Here we describe a construction of simple model to deal with pure states in quantum mechanics. We treat quantum states of macroscopic systems around the brain cells as wave functions of many-body system in ordinary quantum mechanics (Fukuda, 1991). As a candidate of the quantum two-level states in our model, we can adopt two wave modes $\omega_\pm$ in many-body system which has undergone spontaneous symmetry braking in the quantum brain dynamics (Appendix).

Thus, we construct a model based on the following hypothesis of decision-making process:

When the target brain cell receives stimuli of necessary information, the state of cell becomes the superposition of $|1\rangle$ and $|2\rangle$, then "reduction of state" (von Neumann, 1932) occurs by an observation of other cells in the brain. (Fig. 1)

Now, we treat a simple model, of which Hamiltonian does not depend on time. The Hamiltonian of total system is $\hat{H} = \hat{H}^{(0)} + \hat{V}$, where $\hat{V}$ is the potential of external action, and $\hat{H}^{(0)}$ is the energy operator of stationary states. Then, $|i\rangle$ satisfies following eigenvalue problems:

$\hat{H}^{(0)}|1\rangle = E_1^{(0)}|1\rangle$, $\hat{H}^{(0)}|2\rangle = E_2^{(0)}|2\rangle$, ($E_1^{(0)} < E_2^{(0)}$)

Plausible situation is that external influences contribute to the transition of states only, so we can express the matrix form of Hamiltonian:

$$\langle i|\hat{H}|j\rangle = \begin{pmatrix} E_1^{(0)} & v \\ v^* & E_2^{(0)} \end{pmatrix}, \quad (i,j = 1,2).$$

Non-diagonal components of Hamiltonian are restricted to v, v^* because of the Hermitean character of Hamiltonian, where v^* means the complex conjugate of v.

According to the superposition principle (Dirac, 1958), the general state is a linear combination of stationary states $|1\rangle$ and $|2\rangle$ before observation:

$$|\psi(t)\rangle = a_1(t)|1\rangle + a_2(t)|2\rangle$$

Coefficient $a_i(t)$ has a meaning that $|a_i(t)|^2$ equals to probability of finding state $|i\rangle$.

In the case of the transition ↓⇒↑, we set the initial condition: $|\psi(0)\rangle = |1\rangle$

By solving Schrödinger equation in this condition, we obtain the probability of finding in each state:

$$|a_1(t)|^2 = 1 - \frac{4|v|^2}{(\hbar\varpi)^2}\sin^2\left(\frac{\varpi t}{2}\right), \quad |a_2(t)|^2 = \frac{4|v|^2}{(\hbar\varpi)^2}\sin^2\left(\frac{\varpi t}{2}\right),$$

where $\varpi = \sqrt{\omega_0^2 + 4|v|^2/\hbar^2}$, $\omega_0 = \left(E_2^{(0)} - E_1^{(0)}\right)/\hbar$, $\hbar$ is the Dirac constant (equal to the Planck constant divided by 2π). Average of $|a_i(t)|^2$ per one period is calculated by

$$\overline{|a_i(t)|^2} = \frac{1}{\tau}\int_0^\tau |a_i(t)|^2\, dt,$$

where τ is the time interval of one period. Performing this integration, we obtain

$$\overline{|a_1(t)|^2} = 1 - \frac{2|v|^2}{(\hbar\varpi)^2}, \quad \overline{|a_2(t)|^2} = \frac{2|v|^2}{(\hbar\varpi)^2}.$$

In Weidlich model, on the other hand, the transition probability is $p_{\downarrow\Rightarrow\uparrow}(x) = v\exp\{+(kx+h)\}$. Here, we associate $|a_2(t)|^2$ with $p_{\downarrow\Rightarrow\uparrow}$. When only one transition occurs, the transition probability is $\exp\{+(kx+h)\}$ because v means frequency of changing opinions. Therefore, we can find following correspondence:

$$\overline{|a_2(t)|^2} = \frac{2|v|^2}{(\hbar\varpi)^2} \leftrightarrow \exp\{+(kx+h)\}.$$

So, we obtain following result:

$$|v|^2 = \frac{1}{2}\cdot\frac{\left(E_2^{(0)} - E_1^{(0)}\right)^2}{\exp\{-(kx+h)\} - 2}.$$

By the same procedure, we can calculate the case of ↑⇒↓. The initial state should be set $|\psi(0)\rangle = |2\rangle$, and non-diagonal components of Hamiltonian are different to the case ↓⇒↑, because corresponding

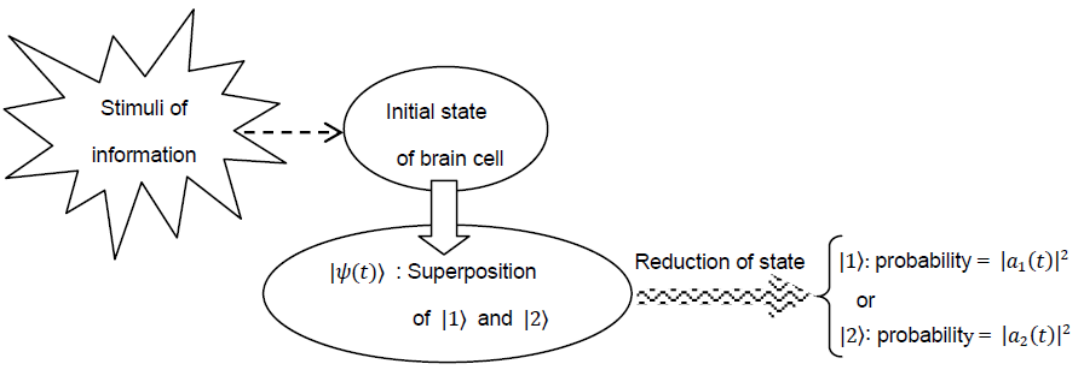

Figure 1. Schematic explanation of reduction of state in the quantum brain.

psychological transition probability is . is p↑⇒↓(x) = v exp{-(kx + h)}. So, we obtain

$$|v|^2 = \frac{1}{2} \cdot \frac{\left(E_2^{(0)} - E_1^{(0)}\right)^2}{\exp\{+(kx+h)\} - 2}.$$

Thus, there is a difference of non-diagonal components of Hamiltonian in the case of ↓⇒↑ and ↑⇒↓. So, these transitions should be regarded as distinct processes in our quantum brain model.

If the value of $E_2^{(0)} - E_1^{(0)}$ of brain cell is evaluated in molecular biology (Jibu *et al.*, 1994), the result of the model becomes remarkable as follows. The values of *k* and *h* are determined by numerical calculations based on experimental data, as mentioned in the previous section. Then we can express |*v*| as the function of *x*. Consequently, the magnitude of non-diagonal components of Hamiltonian |*v*| depends on the population *x*. Thus, |*v*| has the feature of the human group by the property of parameters *x*, *k* and *h*.

In the present model, we adapted the quantum pure states in the brain cells. One of the expansions of this model includes quantum statistical mixed states around the brain cells. In this way, the model should be constructed by the density matrix formalism. These arguments are devoted to the future research.

Conclusions

In this paper, we attempted to construct a quantum brain model, and reduced a candidate of the Hamiltonian by means of incorporating with Weidlich model which describes the opinion formation of individuals in social psychology. In particular, we concentrated on a simple phenomenon that one opinion is decided between two opinions in human mind. In order to determine the form of Hamiltonian, we assumed a simple process on the propagation of information in the brain.

In summary, we proposed a new approach on the determination of the Hamiltonian as combining psychological model and quantum brain model. We will show the application and expansion of present model elsewhere soon.

Appendix. Review: A Prototype of Quantum Brain Dynamics

– Takahashi Model –

Quantum brain dynamics (QBD) is constructed with some assumptions: each elements of brain are expressed by spin variables; these elements are capable of emitting and absorbing a boson; two elements interact through the exchange of the boson. A prototype of Hamiltonian of QBD was proposed by Takahashi (Stuart *et al.*, 1979) as follows:

$$H_{QBD} = \frac{1}{2}\sum_k \{p_k^{\dagger} p_k + K_k^2 q_k^{\dagger} q_k\}$$

$$+\sum_{j,k} \frac{f}{2\sqrt{\hbar\Omega}} \left\{\tau_1^{(j)} q_k \exp(ik\cdot x_j) - \tau_2^{(j)} \frac{p_k}{K_k} \exp(-ik\cdot x_j)\right\},$$

where p_k, q_k are canonical variables for the boson field with frequency *k*, $\tau_i^{(j)}$ (I = 1,2,3) , are spin variables in energy spin space, *j* is suffix of constituents, x_j is position of *j*, and *f* is a coupling constant of interaction between a spin variable and boson field. K_k and *Ω* are the energy of the boson and the normalization volume respectively.

Nowadays, it is considered that the identification of physiological entities of spin variables and boson field are electric dipoles of water molecules and electromagnetic field respectively (Del Giudice *et al.*, 1988). The Hamiltonian H_{QBD} is derived within the fundamental framework of quantum electrodynamics applied to the interaction between water and electromagnetic field inside and outside the cell membranes in the brain (Jibu and Yasue, 1995).

The Hamiltonian H_{QBD} is invariant under the transformation:

$$K_k q_k' = K_k q_k \cos\theta - p_k \sin\theta,$$

$$p_{-k}' = K_k q_k \sin\theta + p_{-k} \cos\theta,$$

$$\tau_1^{(j)'} = \tau_1^{(j)} \cos\theta + \tau_2^{(j)} \sin\theta,$$

$$\tau_2^{(j)'} = -\tau_1^{(j)} \sin\theta + \tau_2^{(j)} \cos\theta$$

$$\tau_3^{(j)'} = \tau_3^{(j)},$$

for a continuous parameter *θ*. This transformation corresponds to a continuous rotation around the third axis in energy spin space (Jibu *et al.*, 1996).

The time-independent solutions of Heisenberg equations in this model are calculated:

$$\langle \tau_1^{(j)} \rangle = u(\neq 0),$$

$$\langle p_k \rangle = \langle \tau_2^{(j)} \rangle = \langle \tau_3^{(j)} \rangle = 0,$$

$$\langle q_k \rangle = -\frac{u}{K_k^2} \sum_j \frac{f}{2\sqrt{\hbar\Omega}} \exp(-ik\cdot x_j) \equiv q_k^{(0)}$$

where ⟨…⟩ express time-independent solutions. These solutions mean the ordered state, so that all the elements are pointing along one and the same direction. This situation is the "Spontaneous Symmetry Breaking (SSB)" (Del Giudice *et al*, 1986).

In this model, when the system undergoes SSB, two distinct wave modes are generated as following results. The angular frequency of the normal modes is calculated, and two solutions are obtained:

$$\omega_-^2 = \frac{1}{2}\Big[\left\{K^2(k^2) + f^2V^2\right\} - \sqrt{\left\{K^2(k^2) + f^2V^2\right\} - f^2V^2\left\{K^2(k^2) - K^2(0)\right\}}\Big],$$

$$\omega_+^2 = \frac{1}{2}\Big[\left\{K^2(k^2) + f^2V^2\right\} + \sqrt{\left\{K^2(k^2) + f^2V^2\right\} - f^2V^2\left\{K^2(k^2) - K^2(0)\right\}}\Big],$$

where

$$V \equiv \frac{fu}{2\hbar\Omega}\sum_k\sum_j \frac{1}{K_k^2}\exp[ik\cdot(x_i - x_j)].$$

These solutions show that there are two distinct modes around the brain cell which has undergone SSB. The energy of ω_- mode is gapless, which means that the energy of excited states is continuous starting from zero energy in the ordered ground state. Namely, this mode is the Nambu-Goldstone mode, and Nambu-Goldstone boson is created when quantized. On the other hand, ω_+ mode starts from non-zero energy, and continuously rises.

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