Coefficient inequality for sub classes of a class of Analytic Functions involving integral operator

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Abstract:
We introduce a new class of analytic functions and its subclasses and obtain sharp upper bounds of the functional $|a_3 - \mu a_2^2|$ for the analytic function $f(z) = z + \sum_{n=2}^{\infty} a_n z^n, |z| < 1$ belonging to these classes.
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I. Introduction

Let $\mathcal{A}$ denote the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1.1}$$

which are analytic in the unit disc $\mathbb{E} = \{z: |z| < 1|\}$. Let $\mathcal{S}$ be the class of functions of the form (1.1), which are analytic univalent in $\mathbb{E}$.

In 1916, Bieber Bach ([2]) proved that $|a_2| \leq 2$ for the functions $f(z) \in \mathcal{S}$. In 1923, Löwner proved that $|a_3| \leq 3$ for the functions $f(z) \in \mathcal{S}$..

With the known estimates $|a_2| \leq 2$and $|a_3| \leq 3$, it was natural to seek some relation between a_3 and ${a_2}^2$ for the class $\mathcal{S}$, Fekete and Szegö [5] used Löwner's method to prove the following well known result for the class $\mathcal{S}$.
Let $f(z) \in \mathcal{S}$, then

$$|a_3 - \mu a_2^2| \leq \begin{cases} 3 - 4\mu, if\ \mu \leq 0; \\ 1 + 2\exp\left(\frac{-2\mu}{1-\mu}\right), if\ 0 \leq \mu \leq 1; \\ 4\mu - 3, if \mu \geq 1. \end{cases} \tag{1.2}$$

The inequality (1.2) plays a very important role in determining estimates of higher coefficients for some sub classes $\mathcal{S}$ (De Brangs [3], Duren[4], G. Singh et al. [7-9], [14-16], [22-54]).
Let us define some subclasses of $\mathcal{S}$.

We denote by S^*, the class of univalent starlike functions $g(z) = z + \sum_{n=2}^{\infty} b_n z^n \in \mathcal{A}$ satisfying the condition

$$Re\left(\frac{zg(z)}{g(z)}\right) > 0, z \in \mathbb{E}. \tag{1.3}$$

We denote by $\mathcal{K}$, the class of univalent convex functions $h(z) = z + \sum_{n=2}^{\infty} c_n z^n \in \mathcal{A}$ satisfying the condition

$$Re\frac{((zh'(z))}{h'(z)} > 0, z \in \mathbb{E}. \tag{1.4}$$

Gurmeet Singh, Saroa M. S. and Mehrok, B. S. [16] have introduced the class of Inverse Starlike functions as the functions $g(z) = z + \sum_{n=2}^{\infty} b_n z^n \in \mathcal{A}$ and satisfying the condition

$$Re\left(\frac{zf(z)}{2\int_0^z f(z)dz}\right) > 0, z \in \mathbb{E} \ i.e. \frac{zf(z)}{2\int_0^z f(z)dz} \prec \frac{1+z}{1-z} \quad (1.5)$$

And denoted this class by $(S^*)^{-1}$.

The subclass of $(S^*)^{-1}$ consisting of the functions $g(z) = z + \sum_{n=2}^{\infty} b_n z^n \in \mathcal{A}$ and satisfying the condition

$$\frac{zf(z)}{2\int_0^z f(z)dz} \prec \frac{1+Az}{1+Bz}; -1 \le B \le A \le 1 \quad (1.6)$$

is denoted by $(S^*)^{-1}[A,B]$.

Symbol $\prec$ stands for subordination, which we define as follows:

p-valent functions: A function $f(z) \in \mathcal{A}_p$ is said to be a p-valent function in E if it assumes no value more than p times in E.

The class of functions $f(z) \in \mathcal{A}_p$ satisfying the condition

$$\frac{zf(z)}{(p+1)\int_0^z f(z)dz} \prec \frac{1+z}{1-z}$$

is denoted by $(S_p{}^*)^{-1}$.(See [4])

These functions were called p- valent inverse starlike functions. In this paper, We will deal with $(S_p{}^*)^{-1}[A,B]$, the subclass of $(S_p{}^*)^{-1}$ consisting of the functionsf$(z) \in \mathcal{A}_p$and satisfying the condition

$$\frac{zf(z)}{(p+1)\int_0^z f(z)dz} \prec \frac{1+Az}{1+Bz}; -1 \le B \le A \le 1.$$

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We will also deal with $(S_p{}^*)^{-1}[A,B;\delta]$, the subclass of $(S_p{}^*)^{-1}[A,B]$ consisting of the functionsf$(z) \in \mathcal{A}$and satisfying the condition

$$\frac{zf(z)}{(p+1)\int_0^z f(z)dz} \prec \left(\frac{1+Az}{1+Bz}\right)^{\delta}; -1 \le B \le A \le 1;\ \delta > 0.$$

We will establish Fekete-Szegö inequality for these classes.

Principle of subordination: Let $f(z)$ and $F(z)$ be two functions analytic in $\mathbb{E}$. Then $f(z)$ is called subordinate to $F(z)$ in $\mathbb{E}$ if there exists a function $w(z)$ analytic in $\mathbb{E}$ satisfying the conditions $w(0) = 0$ and $|w(z)| < 1$ such that $f(z) = F(w(z));\ z \in \mathbb{E}$ and we write $f(z) \prec F(z)$.

By $\mathcal{U}$, we denote the class of analytic bounded functions of the form

$w(z) = \sum_{n=1}^{\infty} d_n z^n, w(0) = 0, |w(z)| < 1.$ (1.7)

It is known that $|d_1| \le 1, |d_2| \le 1 - |d_1|^2$.

1. **Main Results**

Theorem 2.1: If $f(z) \in (S_p{}^*)^{-1}[A,B]$, then

$$\frac{1}{A-B}\left|a_{p+2} - \mu a_{p+1}^2\right| \le \begin{cases} (p+2)^2(A-B)\left(\dfrac{(p+3)\{(p+1)(A-B)-B\}}{2(p+2)^2(A-B)} - \mu\right), \\ \quad if\ \mu \le \dfrac{(p+3)\{(p+1)(A-B)-B\}}{2(p+2)^2(A-B)} \quad (2.1) \\ \dfrac{p+3}{2}, \\ \quad if\ \dfrac{(p+3)[\{(p+1)(A-B)-B\}-1]}{2(p+2)^2(A-B)} \le \mu \le \dfrac{p+3}{2}\dfrac{[\{(p+1)(A-B)-B\}+1]}{(p+2)^2(A-B)} \quad (2.2) \\ (p+2)^2(A-B)\left(\mu - \dfrac{(p+3)\{(p+1)(A-B)-B\}}{2(p+2)^2(A-B)}\right), \\ \quad if\ \mu \ge \dfrac{p+3}{2}\dfrac{[\{(p+1)(A-B)-B\}+1]}{(p+2)^2(A-B)} \quad (2.3) \end{cases}$$

The results are sharp.

Proof: By definition of $\left(S_p{}^*\right)^{-1}[A,B]$, we have

$$\frac{zf(z)}{(p+1)\int_0^z f(z)dz} = \left(\frac{1+Aw(z)}{1+Bw(z)}\right); -1 \le B \le A \le 1, \tag{2.4}$$

Expanding (2.4), we have

$\left(1 + a_{p+1}z + a_{p+2}z^2 + - - -\right) = \left(1 + \frac{p+1}{p+2}a_{p+1}z + \frac{p+1}{p+3}a_{p+2}z^2 + - - -\right)(1 + (A-B)c_1 z + (A-B)(c_2 - Bc_1^2)z^2 + - - -)$ (2.5)

Identifying terms in (2.5), we get

$a_{p+1} = (p+2)(A-B)c_1$ and

$a_{p+2} = \frac{(p+3)(A-B)}{2}[c_2 + \{(p+1)(A-B) - B\}c_1^2]$ (2.6)

Using (2.5) and (2.6), we obtain

$$a_{p+2} - \mu a_{p+1}^2 = \frac{(p+3)(A-B)}{2}c_2 + (p+2)^2(A-B)^2\left[\frac{(p+3)\{(p+1)(A-B)-B\}}{2(p+2)^2(A-B)} - \mu\right]c_1^2$$

$$\frac{1}{A-B}\left(a_{p+2} - \mu a_{p+1}^2\right) = \frac{(p+3)}{2}c_2 + (p+2)^2(A-B)\left[\frac{(p+3)\{(p+1)(A-B)-B\}}{2(p+2)^2(A-B)} - \mu\right]c_1^2$$

This leads to

$$\frac{1}{A-B}\left|a_{p+2} - \mu a_{p+1}^2\right| \le \frac{p+3}{2}|c_2| + (+2)^2(-B)\left|\frac{(p+3)\{(p+1)(A-B)-B\}}{2(p+2)^2(A-B)} - \mu\right||c_1|^2$$

$$\le \frac{p+3}{2} + (p+2)^2(A-B)\left[\left|\frac{(p+3)\{(p+1)(A-B)-B\}}{2(p+2)^2(A-B)} - \mu\right| - \frac{p+3}{2(p+2)^2(A-B)}\right]|c_1|^2 \tag{2.7}$$ 3973

Case I: $\mu \le \frac{(p+3)\{(p+1)(A-B)-B\}}{2(p+2)^2(A-B)}$, we get from (2.7)

$$\frac{1}{A-B}\left|a_{p+2} - \mu a_{p+1}^2\right| \le \frac{p+3}{2} + (p+2)^2(A-B)\left[\frac{p+3}{2}\frac{[\{(p+1)(A-B)-B\}-1]}{(p+2)^2(A-B)} - \mu\right]|c_1|^2 \tag{2.8}$$

Subcase I(a): $\mu \le \frac{(p+3)[\{(p+1)(A-B)-B\}-1]}{2(p+2)^2(A-B)}$. From equation (2.8), we get

$$\frac{1}{A-B}\left|a_{p+2} - \mu a_{p+1}^2\right| \le (p+2)^2(A-B)\left(\frac{(p+3)\{(p+1)(A-B)-B\}}{2(p+2)^2(A-B)} - \mu\right). \tag{2.9}$$

Subcase I(b): $\mu \ge \frac{(p+3)[\{(p+1)(A-B)-B\}-1]}{2(p+2)^2(A-B)}$.From equation (2.8), we get

$$\frac{1}{A-B}\left|a_{p+2} - \mu a_{p+1}^2\right| \le \frac{p+3}{2}. \tag{2.10}$$

Case II: $\mu \ge \frac{(p+3)\{(p+1)(A-B)-B\}}{2(p+2)^2(A-B)}$, we get from (2.7)

$$\frac{1}{A-B}\left|a_{p+2} - \mu a_{p+1}^2\right| \le \frac{p+3}{2} + (p+2)^2(A-B)\left[\mu - \frac{p+3}{2}\frac{[\{(p+1)(A-B)-B\}+1]}{(p+2)^2(A-B)}\right]|c_1|^2 \tag{2.11}$$

Subcase II(a): $\mu \le \frac{p+3}{2}\frac{[\{(p+1)(A-B)-B\}+1]}{(p+2)^2(A-B)}$. From equation (2.11), we get

$$\frac{1}{A-B}\left|a_{p+2} - \mu a_{p+1}^2\right| \le \frac{p+3}{2}. \tag{2.12}$$

Combining subcase I(a) and subcase II(b), we get

$$\frac{1}{A-B}\left|a_{p+2} - \mu a_{p+1}^2\right| \le \frac{p+3}{2}, if\ \frac{(p+3)[\{(p+1)(A-B)-B\}-1]}{2(p+2)^2(A-B)} \le \mu \le \frac{p+3}{2}\frac{[\{(p+1)(A-B)-B\}+1]}{(p+2)^2(A-B)} \tag{2.13}$$

Subcase II(b): $\mu \ge \frac{p+3}{2}\frac{[\{(p+1)(A-B)-B\}+1]}{(p+2)^2(A-B)}$. From equation (2.11), we get

$$\frac{1}{A-B}\left|a_{p+2} - \mu a_{p+1}^2\right| \le (p+2)^2(A-B)\left(\mu - \frac{(p+3)\{(p+1)(A-B)-B\}}{2(p+2)^2(A-B)}\right). \tag{2.14}$$

This completes the theorem. The results are sharp.
Extremal function for first and third inequality is

$$f_1(z) = (p+1)z^p(1+Az)(1+Bz)^{\frac{(p+1)(A-B)-B}{B}}$$

Extremal function for second inequality is

$$f_2(z) = (p+1)z^p(1+Az^2)(1+Bz^2)^{\frac{(p+1)(A-B)-2B}{2B}}$$

Corollary 2.2: Putting $\mathrm{A} = 1, \mathrm{B} = -1$ in Theorem 2.1, we get

$$\frac{1}{2}\left|a_{p+2} - \mu a_{p+1}^2\right| \leq \begin{cases} 2(p+2)^2\left(\frac{(p+3)(2p+3)}{4(p+2)^2} - \mu\right), if\ \mu \leq \frac{(p+3)(2p+3)}{4(p+2)^2}; \\ \frac{p+3}{2}, if\ \frac{(p+3)(p+1)}{2(p+2)^2} \leq \mu \leq \frac{p+3}{2(p+2)}; \\ 2(p+2)^2\left(\mu - \frac{(p+3)(2p+3)}{4(p+2)^2}\right), if\ \mu \geq \frac{p+3}{2(p+2)} \end{cases}$$

, which are the required results for the class $\left(\mathrm{S_p}^*\right)^{-1}$.

Corollary 2.3: Putting $\mathrm{p} = 1$ in Theorem 2.1, we get

$$\frac{1}{A-B}\left|a_3 - \mu a_2^2\right| \leq \begin{cases} 9(A-B)\left(\frac{2(2A-3B)}{9(A-B)} - \mu\right), if\ \mu \leq \frac{2(2A-3B)}{9(A-B)}; \\ 2, if\ \frac{2(2A-3B)}{9(A-B)} \leq \mu \leq \frac{2(2A-3B+1)}{9(A-B)}; \\ 9(A-B)\left(\mu - \frac{2(2A-3B)}{9(A-B)}\right), if\ \mu \geq \frac{2(2A-3B+1)}{9(A-B)} \end{cases}$$

, which are the required results for the class $(S^*)^{-1}[\mathrm{A}, \mathrm{B}]$.

Corollary 2.4: Putting $\mathrm{A} = 1, \mathrm{B} = -1, \mathrm{p} = 1$ in Theorem 2.1, we get

$$\frac{1}{4}\left|a_3 - \mu a_2^2\right| \leq \begin{cases} (5-9\mu), if\ \mu \leq \frac{4}{9}; \\ 1, if \frac{4}{9} \leq \mu \leq \frac{2}{3}; \\ (9\mu - 5), if\ \mu \geq \frac{2}{3}. \end{cases}$$

, which are the required results for the class $(S^*)^{-1}$.

Conclusion: A subclass of analytic functions which take a broad view of some well-known subclasses of analytic and univalent functions was demarcated. The better estimates for the Fekete-Szegö functional for the defined class were obtained along with extremal functions. The study combines existing results and attains new outcomes in geometric function theory. Forthcoming researches can be done to acquire the geometric properties.

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