Common Fixed Point Theorems For Compatible Mappings of Type (A) In Dislocated Metric Spaces

Naveen Jha[1*]

ABSTRACT

The study of common fixed point of mappings satisfying contractive type conditions has been a very active field of research activity. Jungck [9] initiated the concept of commuting maps and generalized it with the concept of compatible maps Jungck([10],[11]) and established some important fixed point results. Jungck, Murthy and Cho[10] initiated the concept of compatible mappings of type (A) in metric space. Recently Goyal [5] proved a common fixed point theorem for six mappings using the concept of compatible mapping of type (A). In this paper we have obtained common fixed point result for six mappings satisfying integral type contractive condition and using the concept of compatible mapping of type (A) in dislocated metric space. Our result generalizes the result of Panthi and Kumari [18] for integral type contraction in complete dislocated metric space for six mappings.

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Introduction

The notion of metric space, introduced by Frechet [2] in 1906, is one of the useful topics not only in mathematics but also in several quantitative sciences. Due to its importance and application potential, this notion has been extended, improved, and generalized in many ways. An incomplete list of such attempts are following: symmetric space, b-metric space, partial metric space, quasi-metric space, fuzzy metric space, dislocated metric space, dislocated quasi-metric space, right and left dislocated metric spaces etc.

In 1922, Banach proved a fixed point theorem for contraction mapping in a complete metric space. Banach contraction theorem is one of the pivotal results of functional analysis. It has many applications in various fields of mathematics such as differential equations, integral equations etc. After Banach contraction theorem number of fixed point theorems have been established by various authors and they made different generalizations of this theorem.

The concept of dislocated metric (d-metric) was introduced by Hitzler and Seda in ([6],[7]) which is very useful in Logic Programming Semantics. With the passage of time many papers have been published concerning fixed point and common fixed point theorems satisfying certain contractive conditions in dislocated metric space (see[8], [13]–[22],[24]).

Sessa [23], initiated the tradition of improving commutativity conditions in metrical common fixed point theorems. While doing so Sessa [23] introduced the notion of weak commutativity. Motivated by Sessa [23], Jungck [10] defined the concept of compatibility of two mappings, which includes weakly commuting mappings as a proper subclass. G. Jungck, P. P. Murthy and Y. J. Cho [12] initiated the concept of compatible mappings of type (A) in metric space. The purpose of this article is to establish a common fixed point theorem for three pairs of mappings of compatible type (A) in dislocated metric spaces which generalize and improve similar results of fixed point in the literature

2.Preliminaries

We begin by recalling some basic concepts of the theory of dislocated metric (*d-metric*) spaces.

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Definition 2.1 Let X be a non-empty set and let $d: X \times X \to [0, \infty)$ be a function satisfying the following conditions:
(i) $d(x, y) = d(y, x)$
(ii) $d(x, y) = d(y, x) = 0$ implies $x = y$
(iii) $d(x, y) \leq d(x, z) + d(z, y)$ for all $x, y, z \in X$
Then d is called dislocated metric (or simply d-metric) on X.
Definition 2.2 A sequence $\{x_n\}$ in a d-metric space (X, d) is called a Cauchy sequence if for given $\epsilon > 0$, there exists $n_0 \in N$ such that for all $m. n \geq n_0$,we have $d(x_m, x_n) < \epsilon$.
Definition 2.3 A sequence in d-metric space converges if there exists $x \in X$ such that $d(x_n, x) \to 0$ as $n \to \infty$.
Definition 2.4 A d-metric space (X, d) is called complete if every Cauchy sequence is convergent.
Definition 2.5 Let (X, d) be a d-metric space. A map $T: X \to X$ is called contraction if there exists a number λ with $0 \leq \lambda < 1$ such that $d(Tx, Ty) \leq \lambda d(x, y)$.
Definition 2.6 Let A and S be two self mappings on a set X. Mappings A and S are said to be commuting if $ASx = SAx \quad \forall x \in X$.
Definition 2.7. Let S and T be mappings of a metric space (X, d) into itself. Then (S, T) is said to be **weakly commuting** pair if

$$d(STx, TSx) \leq d(Tx, Sx) \text{ for all } x \in X.$$

Obviously, a commuting pair is weakly commuting but its converse need not be true as is evident from the following example.
Example 2.1. Consider the set X = [0, 1] with the usual metric. Let $Sx = \frac{x}{2}$ and $Tx = \frac{x}{2+x}$ for every x ∈ X. Then for all x ∈ X

$$STx = \frac{x}{4+2x},\ TSx = \frac{x}{4+x}.$$

Hence ST ≠ TS. Thus S and T do not commute.
Again

$$d(STx, TSx) = \left|\frac{x}{4+2x} - \frac{x}{4+x}\right| = \frac{x^2}{(4+x)(4+2x)}$$
$$\leq \frac{x^2}{(4+2x)} = \frac{x}{2} - \frac{x}{2+x} = d(Sx, Tx),$$

and so, S and T commute weakly.
Obviously, the class of weakly commuting is wider and includes commuting mappings as subclass.
Definition 2.8. Two self mappings S and T from a d-metric space (X, d) into itself are called compatible if and only if
$\lim_{n\to\infty} d(STx_n, TSx_n) = 0$, whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t \text{ for some } t \in X.$$

More recently, Jungck et. al. [12] introduced the concept of compatible mapping of type (A) which is stated as follows.
Definition 2.9. Let S and T be mappings from a metric space (X, d) into itself. The pair (S, T) is said to be **compatible of type (A)** on X if whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z \text{ in X, then}$$
$$d(STx_n, TTx_n) \to 0 \text{ and } d(TSx_n, SSx_n) \to 0 \text{ as } n \to \infty.$$

It is shown in [9] that under certain conditions the compatible and compatible of type (A) mappings are equivalent for instance.
Proposition 1.1. Let S and T be continuous self mapping on X. Then the pair (S, T) is compatible on X, where as in (Jungck [10], Gajic [4]) demonstrated by suitable examples that if S and T are discontinuous then the two concepts are independent of each other.
The following examples also support this observation.
Example 2.2.
Let X = R with the usual metric we defineS, T: X → X as follows.

$$Sx = \begin{cases} 1/x^2 & x \neq 0 \\ 0 & x = 0 \end{cases} \text{ and } \quad Tx = \begin{cases} 1/x^3 & x \neq 0 \\ 0 & x = 0. \end{cases}$$

Both S and T are discontinuous at x = 0 and for any sequence$\{x_n\}$ in X, we have $d(STx_n, TSx_n) = 0$. Hence the pair (S, T) is compatible.
Now consider the sequence $x_n = n \in N$. Then, $Sx_n \to 0$ and $Tx_n \to 0$ as $n \to \infty$
and

$$d(STx_n, TTx_n) = |n^6 - n^9| \leq |n^6| + |n^9| \to \infty \text{ as } n \to \infty.$$

Hence the pair (S,T) is not compatible of type (A).

Example 2.3.
Let X = [0, 1] with the usual metric $d(x, y) = |x - y|$.
Define S, T : [0, 1] → [0, 1] by

$$Sx = \begin{cases} x, & x \in [0,1/2) \\ 1, & x \in [1/2, 1] \end{cases} \quad \text{and} \quad Tx = \begin{cases} 1-x, & x \in [0,1/2) \\ 1, & x \in [1/2, 1] \end{cases}$$

Then S and T are not continuous at $t = 1/2$. Now, we assert that S and T are not compatible but they are compatible of type (A). To see this, suppose that $\{x_n\} \subset [0,1]$ and that $Tx_n, Sx_n \to t$. By definition of S and T, $t \in \{1/2,1\}$. Since S and T agree on [1/2,1], we need only consider $t = 1/2$. So we can suppose that $x_n \to 1/2$ and that $x_n < 1/2$ for all n. Then $Tx_n = 1 - x_n \to 1/2$ from the right and $Sx_n = x_n \to \frac{1}{2}$ from the left. Thus, since $1 - x_n > 1/2$, for all n, $STx_n = S(1 - x_n) = 1$ and, since $x_n < 1/2$, $TSx_n = Tx_n = 1 - x_n \to 1/2$
Consequently,

$$d(STx_n, TSx_n) \to 1/2,$$

but

$$d(STx_n, TTx_n) = |STx_n - TTx_n| = |1 - T(1 - x_n)| = |1 - 1| \to 0$$

and

$$d(TSx_n, SSx_n) = |TSx_n - SSx_n| = |(1 - x_n) - 1| = |1 - 2x_n| \to 0,$$

as $x_n \to 1/2$. Therefore, S and T are compatible mappings of type (A) but they are not compatible.

Example 2.4.
Now we define

$$Sx = \begin{cases} \frac{1}{x^3}, & x > 1 \\ 1, & 0 \le x \le 1 \\ 0, & x < 0 \end{cases}$$

and

$$Tx = \begin{cases} -\frac{1}{x^3}, & x > 1 \\ 1, & 0 \le x \le 1 \\ 0, & x < 0. \end{cases}$$

Observe that the restriction of S and T on $(-\infty, 1]$ are equal.
Thus, we take a sequence $\{x_n\}$ in $(1, \infty)$. Then $\{Sx_n\} \subset (0, 1)$ and $\{Tx_n\} \subset (-1, 0)$.
Thus, for every n, $TTx_n = 0$, $TSx_n = 1$, $STx_n = 0$, $SSx_n = 1$. So that
$d(STx_n, TTx_n) = 0$, $d(TSx_n, TTx_n) = 0$ for every $n \in N$.
This shows that the pair (S, T) is compatible of type (A). Now let $x_n = n$, $n \in N$. Then $Tx_n \to 0$, $Sx_n \to 0$ as $n \to \infty$ and $STx_n = 0$, $TSx_n = 1$ for every $n \in N$ and so
$d(STx, TSx) \neq 0$ as $n \to \infty$. Hence the pair (S, T) is not compatible.

Proposition 2.1 Let S and T be mappings of compatible type (A) from a metric space (X, d) into itself. Suppose that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = x$ for some $x \in X$.
If S is continuous then $\lim_{n\to\infty} TSx_n = Sx$.

Proof: Since S is continuous, so

$$Sx_n \to x \Rightarrow SSx_n \to Sx \text{ and } Tx_n \to x \Rightarrow STx_n \to Sx.$$

Therefore,

$$d(TSx_n, Sx) \le d(TSx_n, SSx_n) + d(SSx_n, STx_n) + d(STx_n, Sx)$$

Now taking limit as $n \to \infty$ and using above relations, we get

$$\lim_{n\to\infty} d(TSx_n, Sx) = 0 \Rightarrow \lim_{n\to\infty} TSx_n = Sx.$$

Now we establish the following result

3.Main Results

Theorem 3.1 Let (X, d) be a complete dislocated metric space. Let $A, B, S, T, P, Q: X \to X$ satisfying the following conditions

(i) $AB(X) \subset Q(X)$ and $ST(X) \subset P(X)$ ... (1)
(ii) The pair (AB, P) and (ST, Q) are compatible of type (A) ... (2)
(iii) $\int_0^{d(ABx,STy)} \emptyset(t)dt \le \int_0^{M(x,y)} \emptyset(t)\, dt,$ ... (3)

where $\emptyset: R^+ \to R^+$ is a Lebesgue integrable mapping which is summable, non-negative and such that
$\int_0^{\epsilon} \emptyset(t)dt > 0$ for all $\epsilon > 0$ and

$$M(x,y) = \alpha\left[\frac{d(Qy,STy)d(Px,Qy)}{d(Qx,ABx)+d(STy,Qx)}\right]+\beta\left[\frac{d(ABx,Qy)d(ABy,Py)}{d(Qx,ABx)+d(STy,Qx)}\right]$$
$$+\gamma\left[\frac{d(Qx,STx)d(STy,Qy)}{d(Qx,ABx)+d(STy,Qx)}\right]+\kappa\left[\frac{d(Px,Qy)d(ABx,STy)}{d(Px,ABx)+d(Px,STy)}\right]$$
$$+\delta\left[\frac{d(Px,ABx)d(Qy,STy)}{d(Qx,ABy)+d(Px,ABy)}\right]+\left[\frac{d(Qx,STx)d(Py,ABy)}{d(Px,Qy)+d(Qx,ABy)}\right], \qquad \ldots (4)$$

for all $x, y \in X$ and $\alpha, \beta, \gamma, \kappa, \delta, \mu \geq 0$ such that $0 \leq \alpha+\beta+\gamma+\kappa+\delta+\mu < 1$. If any one of AB, ST, P and Q is continuous then AB, ST, P and Q have a unique common fixed point in X. Furthermore, if the pairs (A,B), (B,P), (S,T), (S,Q) and (T,Q) are commuting mappings, then A, B, S, T, P and Q have a unique common fixed point.

Proof: Let us define a sequence $\{y_n\} \in X$ such that

$$ABx_{2n+1} = y_{2n+2},\ Qx_{2n} = y_{2n},\ STx_{2n+1} = y_{2n+2},\ Px_{2n} = y_{2n} \text{ for } n = 1,2,3,\ldots \qquad \ldots (5)$$

Let us consider
Let us consider

$$\int_0^{d(y_{2n+1},y_{2n+2})}\emptyset(t)dt = \int_0^{d(ABx_{2n},STx_{2n+1})}\emptyset(t)dt \leq \int_0^{M(x_{2n},x_{2n+1})}\emptyset(t)dt$$

where,

$$M(x_{2n},x_{2n+1}) = \alpha\left[\frac{d(Qx_{2n+1},STx_{2n+1})d(Px_{2n},Qx_{2n+1})}{d(Qx_{2n},ABx_{2n})+d(STx_{2n+1},Qx_{2n})}\right]+\beta\left[\frac{d(ABx_{2n},Qx_{2n})d(ABx_{2n+1},Px_{2n+1})}{d(Qx_{2n},ABx_{2n})+d(STx_{2n+1},Qx_{2n})}\right]$$
$$+\gamma\left[\frac{d(Qx_{2n},STx_{2n})d(STx_{2n+1},Qx_{2n+1})}{d(Qx_{2n},ABx_{2n})+d(STx_{2n+1},Qx_{2n})}\right]+\kappa\left[\frac{d(Px_{2n},Qx_{2n+1})d(ABx_{2n},STx_{2n+1})}{d(Px_{2n},ABx_{2n})+d(Px_{2n},STx_{2n+1})}\right]$$
$$+\delta\left[\frac{d(Px_{2n},ABx_{2n})d(Qx_{2n+1},STx_{2n+1})}{d(Qx_{2n},ABx_{2n+1})+d(Px_{2n},ABx_{2n+1})}\right]+\mu\left[\frac{d(Qx_{2n},STx_{2n})d(Px_{2n+1},ABx_{2n+1})}{d(Px_{2n},Qx_{2n+1})+d(Qx_{2n},ABx_{2n+1})}\right]$$
$$= \alpha\left[\frac{d(y_{2n+1},y_{2n+2})d(y_{2n},y_{2n+1})}{d(y_{2n},y_{2n+1})+d(y_{2n+2},y_{2n})}\right]+\beta\left[\frac{d(y_{2n+1},y_{2n})d(y_{2n+2},y_{2n+1})}{d(y_{2n},y_{2n+1})+d(y_{2n+2},y_{2n})}\right]$$
$$+\gamma\left[\frac{d(y_{2n},y_{2n+1})d(y_{2n+2},y_{2n+1})}{d(y_{2n},y_{2n+1})+d(y_{2n+2},y_{2n})}\right]+\kappa\left[\frac{d(y_{2n},y_{2n+1})d(y_{2n+1},y_{2n+2})}{d(y_{2n},y_{2n+1})+d(y_{2n},y_{2n+2})}\right]$$
$$+\delta\left[\frac{d(y_{2n},y_{2n+1})d(y_{2n+1},y_{2n+2})}{d(y_{2n},y_{2n+1})+d(y_{2n},y_{2n+2})}\right]+\mu\left[\frac{d(y_{2n},y_{2n+1})d(y_{2n+1},y_{2n+2})}{d(y_{2n},y_{2n+1})+d(y_{2n},y_{2n+2})}\right]$$
$$= (\alpha+\beta+\gamma+\kappa+\delta+\mu)d(y_{2n},y_{2n+1})$$

Let $h = (\alpha+\beta+\gamma+\kappa+\delta+\mu) < 1$

Then

$$\int_0^{d(y_{2n+1},y_{2n+2})}\emptyset(t)dt \leq h\int_0^{d(y_{2n},y_{2n+1})}\emptyset(t)dt$$

Similarly, we can obtain

$$\int_0^{d(y_{2n},y_{2n+1})}\emptyset(t)dt \leq h\int_0^{d(y_{2n-1},y_{2n})}\emptyset(t)dt$$

Hence

$$\int_0^{d(y_{2n+1},y_{2n+2})}\emptyset(t)dt \leq h\int_0^{d(y_{2n},y_{2n+1})}\emptyset(t)dt \leq h^2\int_0^{d(y_{2n-1},y_{2n})}\emptyset(t)dt \leq \cdots \leq h^n\int_0^{d(y_0,y_1)}\emptyset(t)dt$$

This shows that

$$\int_0^{d(y_{n+1},y_n)}\emptyset(t)dt \leq h\int_0^{d(y_n,y_{n-1})}\emptyset(t)dt \leq h^2\int_0^{d(y_{n-1},y_{n-2})}\emptyset(t)dt \leq \cdots \leq h^n\int_0^{d(y_1,y_0)}\emptyset(t)dt$$

For every integer $q > 0$ we have

$$\int_0^{d(y_{n+q},y_n)}\emptyset(t)dt \leq \int_0^{d(y_{n+q},y_{n+q-1})}\emptyset(t)dt+\cdots+\int_0^{d(y_{n+2},y_{n+1})}\emptyset(t)dt+\int_0^{d(y_{n+1},y_n)}\emptyset(t)dt$$
$$\leq (h^{q-1}+\cdots+h^2+h+1)\int_0^{d(y_{n+1},y_n)}\emptyset(t)dt$$
$$\leq (h^{q-1}+\cdots+h^2+h+1)h^n\int_0^{d(y_1,y_0)}\emptyset(t)dt$$

Hence $h < 1$, so $h^n \to 0$ as $n \to \infty$.

Therefore, $\int_0^{d(y_{n+q},y_n)}\emptyset(t)dt \to 0 \Rightarrow d(y_{n+q},y_n) \to 0$, as $n \to \infty$.

This implies that $\{y_n\}$ is a Cauchy sequence.

Since X is complete, so there exists a point $z \in X$ such that $\{y_n\} \to z$. Consequently subsequences

$$\{Qx_{2n}\} = \{Px_{2n}\} \to z \text{ and } \{STx_{2n+1}\} = \{ABx_{2n+1}\} \to z \qquad \ldots (6)$$

Since the pair (AB, P) is compatible of type (A) and let AB is continuous, then by proposition (1) we have

$$ABABx_{2n} \to ABz \text{ and } PABx_{2n} \to ABz \text{ as } n \to \infty \qquad \ldots (7)$$

Now, we show that z is the fixe d point of AB i.e. $ABz = z$. For this,

$$\int_0^{d(ABABx_{2n},STx_{2n+1})}\emptyset(t)dt \leq \int_0^{M(ABx_{2n},x_{2n+1})}\emptyset(t)dt,$$

where,

$$M(ABx_{2n}, x_{2n+1}) = \alpha\left[\frac{d(Qx_{2n+1},STx_{2n+1})d(PABx_{2n},Qx_{2n+1})}{d(QABx_{2n},ABABx_{2n})+d(STx_{2n+1},QABx_{2n})}\right] + \beta\left[\frac{d(ABABx_{2n},QABx_{2n})d(ABx_{2n+1},Px_{2n+1})}{d(QABx_{2n},ABABx_{2n})+d(STx_{2n+1},QABx_{2n})}\right]$$
$$+\gamma\left[\frac{d(QABx_{2n},STABx_{2n})d(STx_{2n+1},Qx_{2n+1})}{d(QABx_{2n},ABABx_{2n})+d(STx_{2n+1},QABx_{2n})}\right] + \kappa\left[\frac{d(PABx_{2n},Qx_{2n+1})d(ABABx_{2n},STx_{2n+1})}{d(PABx_{2n},ABABx_{2n})+d(PABx_{2n},STx_{2n+1})}\right]$$
$$+\delta\left[\frac{d(PABx_{2n},ABABx_{2n})d(Qx_{2n+1},STx_{2n+1})}{d(QABx_{2n},Qx_{2n})+d(PABx_{2n},ABx_{2n+1})}\right]+\mu\left[\frac{d(QABx_{2n},STABx_{2n})d(Px_{2n+1},ABx_{2n+1})}{d(PABx_{2n},Qx_{2n+1})+d(QABx_{2n},ABx_{2n+1})}\right]$$

Now taking limit as $n \to \infty$ and using relations (5) and (6) we have

$$\int_0^{d(ABz,z)} \emptyset(t)dt \le \kappa \int_0^{\left[\frac{d(ABz,z)d(ABz,z)}{d(ABz,ABz)+d(ABz,z)}\right]} \emptyset(t)dt \le \frac{\kappa}{3}\int_0^{d(ABz,z)} \emptyset(t)dt,$$

which is a contradiction, hence
$d(ABz, z) = 0 \Rightarrow ABz = z$.
Again, since the pair (AB, P) is compatible of type (A) and P is continuous then
$PPx_{2n} \to Pz$ and $ABPx_{2n} \to Pz$ ...(8) 302
We show that z is the fixed point of P i.e. $Pz = z$.
For this put $x = Px_{2n}$ and $y = x_{2n+1}$ in the relation (3), we have

$$\int_0^{d(ABPx_{2n},STx_{2n+1})} \emptyset(t)dt \le \int_0^{M(Px_{2n},x_{2n+1})} \emptyset(t)dt$$

where,

$$M(Px_{2n}, x_{2n+1}) = \alpha\left[\frac{d(Qx_{2n+1},STx_{2n+1})d(PPx_{2n},Qx_{2n+1})}{d(QPx_{2n},ABPx_{2n})+d(STx_{2n+1},QPx_{2n})}\right] + \beta\left[\frac{d(ABPx_{2n},QPx_{2n})d(ABx_{2n+1},Px_{2n+1})}{d(QPx_{2n},ABPx_{2n})+d(STx_{2n+1},QPx_{2n})}\right]$$
$$+\gamma\left[\frac{d(QPx_{2n},STPx_{2n})d(STx_{2n+1},Qx_{2n+1})}{d(QPx_{2n},ABPx_{2n})+d(STx_{2n+1},QPx_{2n})}\right] + \kappa\left[\frac{d(PPx_{2n},Qx_{2n+1})d(ABPx_{2n},STx_{2n+1})}{d(PPx_{2n},ABPx_{2n})+d(PPx_{2n},STx_{2n+1})}\right]$$
$$+\delta\left[\frac{d(PPx_{2n},ABPx_{2n})d(Qx_{2n+1},STx_{2n+1})}{d(QPx_{2n},Qx_{2n})+d(PPx_{2n},ABx_{2n+1})}\right]+\mu\left[\frac{d(QPx_{2n},STPx_{2n})d(Px_{2n+1},ABx_{2n+1})}{d(PPx_{2n},Qx_{2n+1})+d(QPx_{2n},ABx_{2n+1})}\right]$$

Now taking limit as $n \to \infty$ and using relations (5) and (7) we have

$$\int_0^{d(Pz,z)} \emptyset(t)dt \le \kappa \int_0^{\left[\frac{d(Pz,z)d(Pz,z)}{d(Pz,Pz)+d(Pz,z)}\right]} \emptyset(t)dt \le \frac{\kappa}{3}\int_0^{d(Pz,z)} \emptyset(t)dt,$$

which is a contradiction, hence $d(Pz, z) = 0 \Rightarrow Pz = z$.
Again, since the pair (ST, Q) is compatible of type (A) and if ST is continuous then by proposition (1) we have $STSTx_{2n} \to STz$ and $QSTx_{2n} \to STz$
If we consider $x = x_{2n+1}$ and $y = STx_{2n}$ in relation (3) and proceed as in above cases, then we obtain $STz = z$.
Similarly one can show that if the pair (ST, Q) is compatible of type (A) and Q is continuous then, $Qz = z$.
Therefore, from above relations, we have $Qz = Pz = STz \ne= ABz = z$.
Thus z is the common fixed point of the mappings AB, ST, P and Q.

Uniqueness :

Let $z \neq w$ are two common fixed points of the mappings AB, ST, P and Q. Then

$$\int_0^{d(z,w)} \emptyset(t)dt = \int_0^{d(ABz,STw)} \emptyset(t)dt = \int_0^{M(z,w)} \emptyset(t)dt,$$

where,

$$M(z,w) = \alpha\left[\frac{d(Qw,STw)d(Pz,Qw)}{d(Qz,ABz)+d(STw,Qz)}\right]+\beta\left[\frac{d(ABz,Qz)d(ABw,Pw)}{d(Qz,ABz)+d(STw,Qz)}\right] + \gamma\left[\frac{d(Qz,STz)d(STw,Qw)}{d(Qz,ABz)+d(STw,Qz)}\right]$$
$$+\kappa\left[\frac{d(Pz,Qw)d(ABz,STw)}{d(Pz,ABz)+d(Pz,STw)}\right]+\delta\left[\frac{d(Pz,ABz)d(Qw,STw)}{d(Qz,ABw)+d(Pz,ABw)}\right] +\mu\left[\frac{d(Qz,STz)d(Pw,ABw)}{d(Pz,Qw)+d(Qz,ABw)}\right]$$
$$= \alpha\left[\frac{d(w,w)d(z,w)}{d(z,z)+d(w,z)}\right]+\beta\left[\frac{d(z,z)d(w,w)}{d(z,z)+d(w,z)}\right]+\gamma\left[\frac{d(z,z)d(w,w)}{d(z,z)+d(w,z)}\right]$$
$$+\kappa\left[\frac{d(z,w)d(z,w)}{d(z,z)+d(z,w)}\right]+\delta\left[\frac{d(z,z)d(w,w)}{d(z,w)+d(z,w)}\right]+\mu\left[\frac{d(z,z)d(w,w)}{d(z,w)+d(z,w)}\right]$$
$$\le \left(\frac{2\alpha}{3} + \frac{4(\beta+\gamma)}{3} + 2(\delta+\mu) + \frac{\kappa}{3}\right) d(z,w)$$

Hence,

$$\int_0^{d(z,w)} \emptyset(t)dt \le \left(\frac{2\alpha}{3} + \frac{4(\beta+\gamma)}{3} + 2(\delta+\mu) + \frac{\kappa}{3}\right)\int_0^{d(z,w)} \emptyset(t)dt,$$

which is a contradiction. Therefore $d(z,w) = 0 \Rightarrow z = w$.
Hence AB, ST, P and Q have a unique common fixed point.
Finally, we prove that z is also a common fixed point of A, B, S, T, P and Q.
Let both the pairs (AB, P) and (ST, Q) have a unique common fixed point z.
Then,

$$Az = A(ABz) = A(BAz) = AB(Az),\quad Az = A(Pz) = P(Az)$$
$$\text{and } Bz = B(ABz) = B(A(Bz)) = BA(Bz) = AB(Bz),\quad Bz = B(Pz) = P(Bz),$$

which implies that (AB, P) has common fixed points which are Az and Bz.
We get thereby $Az = z = Bz = Pz = ABz$.

Similarly,

$$Sz = S(STz) = S(TSz) = ST(Sz), \quad Sz = S(Qz) \quad = Q(Sz)$$

and $Tz = T(STz) = T(S(Tz)) = TS(Tz) = ST(Tz), Tz = T(Qz) = Q(Tz)$,

which implies that (AB, P) has common fixed points which are Az and Bz.

We get thereby $Sz = z = Tz = Qz = \ STz$.

Now, we need to show that $Az = Sz(Bz = Tz)$.

By using condition (3), we have

$$\int_0^{d(Az,Sz)} \emptyset(t)dt = \int_0^{d(A(ABz),S(STz))} \emptyset(t)dt = \int_0^{d(A(BAz),S(TSz))} \emptyset(t)dt = \int_0^{d(AB(Az),ST(Sz))} \emptyset(t)dt \leq \int_0^{M(Az,Sz)} \emptyset(t)dt$$

where ,

$$M(Az,Sz) = \alpha\left[\frac{d(QSz,STSz)d(PAz,QSz)}{d(QAz,ABAz)+d(STSz,QAz)}\right]+\beta\left[\frac{d(ABAz,QAz)d(ABSz,PSz)}{d(QAz,ABAz)+d(STSz,QAz)}\right]+\gamma\left[\frac{d(QAz,STAz)d(STSz,QSz)}{d(QAz,ABAz)+d(STSz,QAz)}\right]$$
$$+\kappa\left[\frac{d(PAz,QSz)d(ABAz,STSz)}{d(PAz,ABAz)+d(PAz,STSz)}\right]++\delta\left[\frac{d(PAz,ABAz)d(QSz,STSz)}{d(QAz,ABSz)+d(PAz,ABSz)}\right]+\mu\left[\frac{d(QAz,STAz)d(PSz,ABSz)}{d(PAz,QSz)+d(QAz,ABSz)}\right]$$
$$= \alpha\left[\frac{d(Sz,Sz)d(Az,Sz)}{d(Az,Az)+d(Sz,Az)}\right]+\beta\left[\frac{d(Az,Az)d(Sz,Sz)}{d(Az,Az)+d(Sz,Az)}\right]+\gamma\left[\frac{d(Az,Az)d(Sz,Sz)}{d(Az,Az)+d(Sz,Az)}\right]$$
$$+\kappa\left[\frac{d(Az,Sz)d(Az,Sz)}{d(Az,Az)+d(Az,Sz)}\right]+\delta\left[\frac{d(Az,Az)d(Sz,Sz)}{d(Az,Sz)+d(Az,Sz)}\right]+\mu\left[\frac{d(Az,Az)d(Sz,Sz)}{d(Az,Sz)+d(Az,Sz)}\right]$$
$$\leq \left(\frac{2\alpha}{3}+\frac{4(\beta+\gamma)}{3}+2(\delta+\mu)+\frac{\kappa}{3}\right)d(Az,Sz)$$

Hence,

$$\int_0^{d(Az,Sz)} \emptyset(t)dt \leq \left(\frac{2\alpha}{3}+\frac{4(\beta+\gamma)}{3}+2(\delta+\mu)+\frac{\kappa}{3}\right)\int_0^{d(Az,Sz)} \emptyset(t)dt$$

which is a contradiction.

Therefore $d(Az,Sz) = 0 \Rightarrow Az = Sz$. Similarly, $Bz = Tz$ can be shown.

Consequently, z is a unique common fixed point of A, B, S, T, P and Q i.e

$Az = Bz = Sz = Tz = Pz \quad = Qz = z$

Our result generalizes the result of Panthi and Kumari [18] for integral type contraction in complete dislocated metric space for six mappings.

Now we have following corollaries.

If we put AB = A, ST = B in Theorem (3.1), we get the following, which generalize the result of Panthi and Kumari [18] for integral type contraction.in complete dislocated metric space.

Corollary 3.1 Let (X, d) be a complete dislocated metric space. Let $A, B, P, Q: X \to X$ satisfying the following conditions

(i) $A(X) \subset Q(X)$ and $B(X) \subset P(X)$

(ii) The pair (A, P) and (B, Q) are compatible of type (A)

(iii) $\int_0^{d(Ax,By)} \emptyset(t)dt \leq \int_0^{M(x,y)} \emptyset(t)\,dt$,

where $\emptyset: R^+ \to R^+$ is a Lebesgue integrable mapping which is summable, non-negative and such that $\int_0^{\epsilon} \emptyset(t)dt > 0$ for all $\epsilon > 0$ and

$$M(x,y) = \alpha\left[\frac{d(Qy,By)d(Px,Qy)}{d(Qx,Ax)+d(By,Qx)}\right]+\beta\left[\frac{d(Ax,Qy)d(Ay,Py)}{d(Qx,Ax)+d(By,Qx)}\right]+\gamma\left[\frac{d(Qx,Bx)d(By,Qy)}{d(Qx,Ax)+d(By,Qx)}\right]$$
$$+\kappa\left[\frac{d(Px,Qy)d(Ax,By)}{d(Px,Ax)+d(Px,By)}\right]+\delta\left[\frac{d(Px,Ax)d(Qy,By)}{d(Qx,Ay)+d(Px,Ay)}\right]+\mu\left[\frac{d(Qx,Bx)d(Py,Ay)}{d(Px,Qy)+d(Qx,Ay)}\right],$$

for all $x, y \in X$ and $\alpha, \beta, \gamma, \kappa, \delta, \mu \geq 0$ such that $0 \leq \alpha+\beta+\gamma+\kappa+\delta+\mu < 1$. If any one of A, B, P and Q is continuous then A, B, P and Q have a unique common fixed point in X.

If we put $P = Q$ in Corollary 3.1, then it is reduced to following corollary.

Corollary 3.2 Let (X, d) be a complete dislocated metric space. Let $A, B, P: X \to X$ satisfying the following conditions

(i) $A(X) \subset P(X)$ and $B(X) \subset P(X)$

(ii) The pair (A, P) and (B, P) are compatible of type (A)

(iii) $\int_0^{d(Ax,By)} \emptyset(t)dt \leq \int_0^{M(x,y)} \emptyset(t)\,dt$,

where $\emptyset: R^+ \to R^+$ is a Lebesgue integrable mapping which is summable, non-negative and such that $\int_0^{\epsilon} \emptyset(t)dt > 0$ for all $\epsilon > 0$ and

$$M(x,y) = \alpha\left[\frac{d(Py,By)d(Px,Py)}{d(Px,Ax)+d(By,Px)}\right]+\beta\left[\frac{d(Ax,Py)d(Ay,Py)}{d(Px,Ax)+d(By,Px)}\right]+\gamma\left[\frac{d(Px,Bx)d(By,Py)}{d(Px,Ax)+d(By,Px)}\right]$$
$$+\kappa\left[\frac{d(Px,Py)d(Ax,By)}{d(Px,Ax)+d(Px,By)}\right]+\delta\left[\frac{d(Px,Ax)d(Py,By)}{d(Px,Ay)+d(Px,Ay)}\right]+\mu\left[\frac{d(Px,Bx)d(Py,Ay)}{d(Px,Py)+d(Px,Ay)}\right]$$

for all $x, y \in X$ and $\alpha, \beta, \gamma, \kappa, \delta, \mu \geq 0$ such that $0 \leq \alpha+\beta+\gamma+\kappa+\delta+\mu < 1$. If any one of A, B and P is continuous then A, B and P have a unique common fixed point in X.

If we put $P = Q$ and $B = A$,then the above Corollary 3.1 is reduced to the following corollary.

Corollary 3.3 Let (X, d) be a complete dislocated metric space. Let $A, P: X \to X$ satisfying the following conditions

(i) $A(X) \subset P(X)$

(ii)The pair (A, P) are compatible of type (A)

(iii) $\int_0^{d(Ax,Ay)} \emptyset(t)dt \le \int_0^{M(x,y)} \emptyset(t)\, dt,$

where $\emptyset: R^+ \to R^+$ is a Lebesgue integrable mapping which is summable, non-negative and such that

$\int_0^{\epsilon} \emptyset(t)dt > 0$ for all $\epsilon > 0$ and

$$M(x,y) = \alpha \left[\frac{d(Py,Ay)d(Px,Py)}{d(Px,Ax)+d(Ay,Px)}\right] + \beta \left[\frac{d(Ax,Py)d(Ay,Py)}{d(Px,Ax)+d(Ay,Px)}\right] + \gamma \left[\frac{d(Px,Ax)d(Ay,Py)}{d(Px,Ax)+d(Ay,Px)}\right]$$
$$+\kappa \left[\frac{d(Px,Py)d(Ax,Ay)}{d(Px,Ax)+d(Px,Ay)}\right] + \delta \left[\frac{d(Px,Ax)d(Py,Ay)}{d(Px,Ay)+d(Px,Ay)}\right] + \mu \left[\frac{d(Px,Ax)d(Py,Ay)}{d(Px,Py)+d(Px,Ay)}\right]$$

for all $x, y \in X$ and $\alpha, \beta, \gamma, \kappa, \delta, \mu \ge 0$ such that $0 \le \alpha + \beta + \gamma + \kappa + \delta + \mu < 1$. If any one of A and P is continuous then A and P have a unique common fixed point in X.

If we put $P = Q = I$ and $A = B$ then the above Corollary 3.1 then we obtain following corollary.

Corollary 3.4 Let (X, d) be a complete dislocated metric space. Let $A, I: X \to X$ satisfying the following conditions

(i) $A(X) \subset I(X)$

(ii) The pair (A, I) are compatible of type(A)

(iii) $\int_0^{d(Ax,Ay)} \emptyset(t)dt \le \int_0^{M(x,y)} \emptyset(t)\, dt,$

where $\emptyset: R^+ \to R^+$ is a Lebesgue integrable mapping which is summable, non-negative and such that

$\int_0^{\epsilon} \emptyset(t)dt > 0$ for all $\epsilon > 0$ and

$$M(x,y) = \alpha \left[\frac{d(y,Ay)d(x,y)}{d(x,Ax)+d(Ay,x)}\right] + \beta \left[\frac{d(Ax,y)d(Ay,y)}{d(Px,Ax)+d(Ay,Px)}\right] + \gamma \left[\frac{d(x,Ax)d(Ay,y)}{d(x,Ax)+d(Ay,x)}\right]$$
$$+\kappa \left[\frac{d(x,y)d(Ax,Ay)}{d(x,Ax)+d(x,Ay)}\right] + \delta \left[\frac{d(x,Ax)d(y,Ay)}{d(x,Ay)+d(x,Ay)}\right] + \mu \left[\frac{d(x,Ax)d(y,Ay)}{d(x,y)+d(x,Ay)}\right]$$

for all $x, y \in X$ and $\alpha, \beta, \gamma, \kappa, \delta, \mu \ge 0$ such that $0 \le \alpha + \beta + \gamma + \kappa + \delta + \mu < 1$. If A is continuous then A has a unique common fixed point in X.

If we put $A = B = I$ then the above Corollary 3.1 then we obtain following corollary.

Corollary 3.5 Let (X, d) be a complete dislocated metric space. Let $A, B, I: X \to X$ satisfying the following conditions

(i) $A(X) \subset I(X)$ and $B(X) \subset I(X)$

(ii) The pair (A, I) and (B, I) are compatible of type (A)

(iii) $\int_0^{d(Ax,By)} \emptyset(t)dt \le \int_0^{M(x,y)} \emptyset(t)\, dt,$

where $\emptyset: R^+ \to R^+$ is a Lebesgue integrable mapping which is summable, non-negative and such that

$\int_0^{\epsilon} \emptyset(t)dt > 0$ for all $\epsilon > 0$ and

$$M(x,y) = \alpha \left[\frac{d(y,By)d(x,y)}{d(x,Ax)+d(By,x)}\right] + \beta \left[\frac{d(Ax,y)d(Ay,y)}{d(Px,Ax)+d(By,Px)}\right] + \gamma \left[\frac{d(x,Bx)d(By,y)}{d(x,Ax)+d(By,x)}\right]$$
$$+\kappa \left[\frac{d(x,y)d(Ax,By)}{d(x,Ax)+d(x,By)}\right] + \delta \left[\frac{d(x,Ax)d(y,By)}{d(x,Ay)+d(x,Ay)}\right] + \mu \left[\frac{d(x,Bx)d(y,Ay)}{d(x,y)+d(x,Ay)}\right]$$

for all $x, y \in X$ and $\alpha, \beta, \gamma, \kappa, \delta, \mu \ge 0$ such that $0 \le \alpha + \beta + \gamma + \kappa + \delta + \mu < 1$. If any one of A and B is continuous then A and B have a unique common fixed point in X.

If we put $B = A$ in the above Corollary 3.1 then we obtain-

Corollary 3.6 Let (X, d) be a complete dislocated metric space. Let $A, P, Q: X \to X$ satisfying the following conditions

(i) $A(X) \subset Q(X)$ and $A(X) \subset P(X)$

(ii)The pair (A, P) and (A, Q) are compatible of type (A)

(iii) $\int_0^{d(Ax,Ay)} \emptyset(t)dt \le \int_0^{M(x,y)} \emptyset(t)\, dt,$

where $\emptyset: R^+ \to R^+$ is a Lebesgue integrable mapping which is summable, non-negative and such that

$\int_0^{\epsilon} \emptyset(t)dt > 0$ for all $\epsilon > 0$ and

$$M(x,y) = \alpha \left[\frac{d(Qy,Ay)d(Px,Qy)}{d(Qx,Ax)+d(Ay,Qx)}\right] + \beta \left[\frac{d(Ax,Qy)d(Ay,Py)}{d(Qx,Ax)+d(Ay,Qx)}\right] + \gamma \left[\frac{d(Qx,Ax)d(Ay,Qy)}{d(Qx,Ax)+d(Ay,Qx)}\right]$$
$$+\kappa \left[\frac{d(Px,Qy)d(Ax,Ay)}{d(Px,Ax)+d(Px,Ay)}\right] + \delta \left[\frac{d(Px,Ax)d(Qy,Ay)}{d(Qx,Ay)+d(Px,Ay)}\right] + \mu \left[\frac{d(Qx,Ax)d(Py,Ay)}{d(Px,Qy)+d(Qx,Ay)}\right]$$

for all $x, y \in X$ and $\alpha, \beta, \gamma, \kappa, \delta, \mu \ge 0$ such that $0 \le \alpha + \beta + \gamma + \kappa + \delta + \mu < 1$. If any one of A, P and Q is continuous then A, P and Q have a unique common fixed point in X.

4. CONCLUSION

Fixed point theory is a rich, interesting and exciting branch of mathematics. It is relatively young but fully developed area of research. Study of the existence of fixed points falls within several domains such as functional analysis, operator theory, general topology. Fixed points and fixed point theorems have always been a major theoretical tool in fields as widely apart as topology, mathematical economics, game theory, approximation theory and initial and boundary value problems in ordinary and partial differential equations. Moreover, recently, the usefulness of this concept for applications increased enormously by the development of accurate and efficient techniques for computing fixed points, making fixed point methods a major tool in the arsenal of mathematics. Jungck, Murthy and Cho [12] initiated the concept of compatible mappings of type (A) in metric space. Recently Goyal [5] proved a common fixed point theorem for six mappings using the concept of compatible mapping of type (A)in complete metric space. In this paper we have obtained common fixed point result for six mappings using the concept of compatible mapping of type (A) in dislocated metric space. Our result generalizes, improve and extend several known corresponding results.

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