Comparative Analysis of Transverse and Axial Displacement for Simply Supported Laminated Beam with Thermal Load

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Abstract

The current study combines a solution for the displacement examination of fundamentally simply supported one, trilogy, and quadruple coat laminated layers with uniformly distributed load using a hyperbolic shear deformation theory that considers thermal impact. Higher shear deformation theory with dislodging field over the beam cross-area yields these beam mathematical results. The thickness of a laminated beam is used to determine the course of action of a beam, which is handled by a rule of virtual work procedure. The standard of imaginary work is utilized to administer divergent circumstances along with cut-off states for beams. In light of shear distortion, verbalizations for cross over removal of beams are obtained, as well as responsibility. MATLAB coding was used to manage the mathematical findings for aspect rations and thickness extents. To confirm the precision of the result, the current exploration's results are compared to those of other shear deformation theories. 293

Keywords: Displacement, Laminated beam, MATLAB Coding, Temperature effect, Hyperbolic Shear Deformation Theory

1. Introduction

A compound is a substance made up of two phases, typically a reinforced material supported in an ideal organisation, gathered in specific amounts to accomplish specific physical and manufactured features. As a result, a composite material is defined as any substance that combines two or more phases. Due to its enormous and respectably light weight, composite materials are quite probably the most widely employed building materials in such as mechanical design. Structure and strength materials make up the composite materials. Boay and Wee [1] proposed a closed design verbalization for determining a covered composite beam's viable flexural modulus. The bowing, fastening, as well as at liberty shaking reaction of usually overlaid composite horizontal members with varying breaking point holds are all affected by this high flexural modulus. The Euler-Bernoulli beam and the old-style cover hypothesis were combined to create the verbalization. Furthermore, the results of a comprehensive restricted part assessment are worn to support the indicative design. The relationship between's the wise outcomes, the restricted piece results, and the exploratory outcomes was surprising. Likewise clear coupling reaction is a significant variable to consider while computing the strong flexural solidness of a normally covered beams. Pagano [2] demonstrated the adaptability of composite laminated in tube shaped bowing. The utilization of a unidirectional overlay as well as two-and three-layered cross-handle covers exposed to sinusoidal burden is thought of. In the temperature investigation of composite plates, the old-style laminated theory was used. In any event, it's only accurate enough for minor composite laminates. Reddy [3] and Timoshenko [5] developed the laminated plate hypotheses in light of Kirchhoff's idea. Khdeir and Reddy [4] used traditional, first-request, second-request, and third-request hypotheses to investigate symmetric and antisymmetric cross-utilize laminated radiates. Arya *et al.* [6] proposed a crisscross model. The theory meets the shear calm condition at the peak and base of the surface, as well as advancement condition at point of interaction. Murty [7] established a third order beam hypothesis that included cross over shear strain and current (nonlinear) critical temperature. Using constitutive relations, the logical cross over shear temperature scattering over the importance of the beam may be obtained in this idea. The restricted part examination of the symmetric and unsymmetric thick layerwise radiates susceptible to the higher request hypothesis was reported by Maiti and Sinha [8]. The strong immovability grid of a steady layerwise complex beam due to geometrical bending hypothesis be suggested by Li and Hongxing [9]. The broadness bearing tensions are taken into account when resolving a refined laminated beam constitutive situation. By knowing the monitoring differential states of growth, the powerful strength system is immediately characterised from a precise perspective. Swift and Heller [10] concentrated on laminated radiates by tolerating layerwise steady shear strains and a constant cross over dislodging along the depth.

2. The beam under consideration

The beam viable as displayed in Fig. 1.1 possesses in Cartesian direction framework.

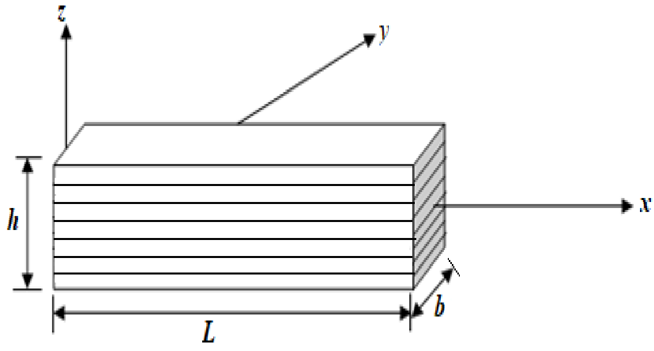

Fig. 1.1.Shape of a covered composite beam

3. The Displacement Field

In light of the before referenced suppositions, the dislodging field of the current composite covered beam hypothesis can be communicated as follows:

$$u(x,z) = u(x) - z\frac{dw}{dx} + \left[h \sinh\left(\frac{z}{h}\right) - \frac{4}{3}\frac{z^3}{h^2}\cosh\left(\frac{1}{2}\right)\right]\phi(x)$$

$$w(x,z) = w(x) \qquad (1.1)$$

Where u is the removal in the x bearing and w is cross over dislodging in the y heading of a point on the bar in mid plane. The strain-relocation relations between strain-removal comparing to the dislodging field are given by

$$\varepsilon_x^0 = \frac{\partial u}{\partial x}, \quad k_x^0 = -\frac{\partial^2 w}{\partial x^2}, \quad k_x^2 = \frac{\partial \phi}{\partial x}$$

$$f\ (z)\ =\ h\sinh\frac{z}{h} - \frac{4}{3}\frac{z^3}{h^2}\cosh\frac{1}{2}$$

$$g\ (z)\ = 1 - f^{'}\ (z)$$

$$g\ (z) = 1 - \left[\cosh\frac{z}{h} - 4\frac{z^2}{h^2}\cosh\frac{1}{2}\right] \tag{1.2}$$

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \\ P_x \\ P_y \\ P_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} & E_{11} & E_{12} & E_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} & E_{12} & E_{22} & E_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} & E_{16} & E_{26} & E_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} & F_{11} & F_{12} & F_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} & F_{12} & F_{22} & F_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} & F_{16} & F_{26} & F_{66} \\ E_{11} & E_{12} & E_{16} & F_{11} & F_{12} & F_{16} & H_{11} & H_{12} & H_{16} \\ E_{12} & E_{22} & E_{26} & F_{12} & F_{22} & F_{26} & H_{12} & H_{22} & H_{26} \\ E_{16} & E_{26} & E_{66} & F_{16} & F_{26} & F_{66} & H_{16} & H_{26} & H_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_{xy}^0 \\ k_x^0 \\ k_y^0 \\ k_{xy}^0 \\ k_x^2 \\ k_y^2 \\ k_{xy}^2 \end{Bmatrix} - \begin{Bmatrix} N^T{}_x \\ N^T{}_y \\ N^T{}_{xy} \\ M^T{}_x \\ M^T{}_y \\ M^T{}_{xy} \\ P^T{}_x \\ P^T{}_y \\ P^T{}_{xy} \end{Bmatrix} \tag{1.3}$$

WhereN, M and P are plane forces, bending and twisting moments and refine bending and twisting moments in x,y and xy plane, $\varepsilon_x^0, \varepsilon_y^0$ and ε_{xy}^0, the mid- plane strains, k_x^0, k_y^0 and k_{xy}^0 the bending and twisting curvatures k_x^2, k_y^2 and k_{xy}^2 the refines bending and twisting curvatures, $A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, H_{ij}\ (i, j = 1, 2, 6)$ are the stiffness coefficient. In the above hypothesis, the constitutive conditions of overlaid composite shaft which represents the Poisson impact are considered as follows. Accept $N_y, N_{xy}, M_y, M_{xy}, P_y$ and P_{xy} identical to nil while $\varepsilon_y^0, \varepsilon_{xy}^0, k_y^0, k_{xy}^0, k_y^2\ k_{xy}^2$ are supposed to be positive. Where N^T=Thermal Resistant and $N^T{}_x$, $M^T{}_x$ and $P^T{}_x$ are force and moment resultant due to thermal loading.

$$\begin{Bmatrix} N_x \\ M_x \\ P_x \end{Bmatrix} = \begin{bmatrix} \overline{A}_{11} & \overline{B}_{11} & \overline{E}_{11} \\ \overline{B}_{11} & \overline{D}_{11} & \overline{F}_{11} \\ \overline{E}_{11} & \overline{F}_{11} & \overline{H}_{11} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ k_x^0 \\ k_x^2 \end{Bmatrix} - \begin{Bmatrix} N^T{}_x \\ M^T{}_x \\ M^T{}_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} N_x \\ M_x \\ P_x \end{Bmatrix} = \begin{bmatrix} \overline{A}_{11} & \overline{B}_{11} & \overline{E}_{11} \\ \overline{B}_{11} & \overline{D}_{11} & \overline{F}_{11} \\ \overline{E}_{11} & \overline{F}_{11} & \overline{H}_{11} \end{bmatrix} \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ -\dfrac{\partial^2 w_b}{\partial x^2} \\ -\dfrac{\partial^2 w_s}{\partial x^2} \end{Bmatrix} - \begin{Bmatrix} N^T{}_x \\ M^T{}_x \\ M^T{}_{xy} \end{Bmatrix}$$

The covered solidness coefficients $A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, H_{ij}\ (i, j = 1, 2, 6)$ and the cross over shear firmness F_{55}, which are limit of overlay handle heading, material property and stack progression, are given by the covered solidness coefficients $A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, H_{ij}\ (i, j = 1, 2, 6)$ and the cross over shear firmness F_{55}, which are capability of overlaid utilize direction, material property and stack progression, are given by

$$(A_{ij}, B_{ij}, D_{ij}) = \int_{-h/2}^{h/2} \overline{Q}_{ij}\ (1,\ z,\ z^2)\ dz,$$

$$(E_{ij}, F_{ij}, H_{ij}) = \int_{-h/2}^{h/2} \overline{Q}_{ij}\ f(z)\ (1,\ z,\ f(z))\ dz,$$

$$(O_{ij}, R_{ij}, S_{ij}) = \int_{-h/2}^{h/2} \overline{Q}_{ij}\ \alpha_x (z,\ z^2,\ f(z))\ dz,$$

$$G_{55} = \int_{-h/2}^{h/2} \overline{Q}_{55} \left[g\ (z)\right]^2\ dz, \qquad g\ (z)\ = 1 - f\ '(z) \tag{1.4}$$

The cross over shear force-strain connection for the composite shaft can be express as
All overlays made same orthotropic material, which properties are expected.

$$\frac{E_1}{E_2} = 25,\ E_2 = 1,\ \upsilon_{12} = 0.25,\ G_{12} = G_{13} = 0.5E_2,\ G_{23} = 0.2E_2 \tag{1.5}$$

The power and the second resultants are characterized in the accompanying structure

$$N = \int_{-h/2}^{h/2} \sigma_x^k dz,\ M = \int_{-h/2}^{h/2} \sigma_x^k z\ dz,\ P = \int_{-h/2}^{h/2} \sigma_x^k f(z)\ dz,\ Q = \int_{-h/2}^{h/2} \tau_{zx}^k\ g(z)\ dz \tag{1.6}$$

Where N and Q are power resultant M and P are the minutes resultants. The standard of imaginary work is utilized to acquire the overseeing conditions and limit conditions related with the current hypothesis. The guideline of imaginary work is given as

$$b \int_0^L \int_{-h/2}^{h/2} \left(\sigma_x^k \delta\varepsilon_x + \tau_{zx}^k \delta\gamma_{zx}\right)\ dz\ dx - \int_0^L q\ (\delta w_b + \delta w_s)\ dx = 0$$

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$$b \int_0^L \int_{-h/2}^{h/2} \sigma_x^k \left(\frac{d\partial u_0}{dx} + Z\ \frac{d^2\partial w_b}{dx^2} - f(z) \frac{d^2\partial w_s}{dx^2} \right) dz\ dx + b \int_0^L \int_{-h/2}^{h/2} \tau_{zx}^k\ \mathrm{g}(z) \sigma_x^k\ \frac{d\partial w_s}{dx}\ dz\ dx$$

$$- \int_0^L q\ (\delta w_b + \delta w_s)\ dx = 0$$

$$b \int_0^L \left(\mathrm{N}\ \frac{d\partial u_0}{dx} - \mathrm{M}\ \frac{d^2\partial w_b}{dx^2} - P\ \frac{d^2\partial w_s}{dx^2} + \mathrm{Q} \frac{d\partial w_s}{dx} \right) dx - \int_0^L q\ (\delta w_b + \delta w_s)\ dx = 0 \tag{1.7}$$

where δ is the variational director. Incorporating condition (1.7) by parts and gathering the coefficients of δu, δw, and δϕ, one can get the overseeing conditions and limit states of the pillar related with the current hypothesis utilizing key lemma of analytics of varieties. The variationally steady overseeing conditions of the current hypothesis as far as power and second resultants are as per the following

$$\frac{dN}{dx} = 0\ :\ \delta u,\ \frac{d^2 M}{dx^2} + q = 0\ :\ \delta w,\ \frac{d^2 P}{dx^2} + \frac{dQ}{dx} + q = 0\ :\ \delta\phi \tag{1.8}$$

The accompanying arrangements of limit conditions at the closures x=0 and x=L of the bar are gotten after the coordination of condition (1.8).

Additionally N=0 or $u_0 = 0$

Additionally $\frac{dM}{dx}$=0 or $W_b = 0$

Additionally M=0 or $\frac{dW_b}{dx}$=0

Additionally $\frac{dP}{dx}$+Q=0 or $W_s = 0$

Additionally P=0 or $\frac{dW_s}{dx}=\phi$=0 (1.9)

Using equation (1.4) in equation (1.9) , then we can interms of displacement and variables are

$$\frac{dN}{dx} = 0 \; : For \; N_x = \delta u,$$

$$-\bar{A}_{11}\frac{d^2u}{dx^2} + \bar{B}_{11}\frac{d^3w}{dx^3} - \bar{E}_{11}\frac{d^2\phi}{dx^2} + O_{11}\frac{dT_0}{dx} = 0 \quad (1.10)$$

$$\frac{d^2M}{dx^2} + q = 0 \quad : For \; M_x = \delta w_b,$$

$$-\bar{B}_{11}\frac{d^3u_0}{dx^3} + \bar{D}_{11}\frac{d^4w}{dx^4} - \bar{F}_{11}\frac{d^3\phi}{dx^3} + R_{11}\frac{d^2T_0}{dx^2} = q \quad (1.11)$$

$$\frac{d^2P}{dx^2} + \frac{dQ}{dx} + q = 0$$

$$-\bar{E}_{11}\frac{d^3u_0}{dx^3} + \bar{F}_{11}\left(\frac{d^4w_b}{dx^4}\right) + \bar{H}_{11}\left(\frac{d^4w_s}{dx^4}\right) - G_{55}\frac{d^2w_s}{dx^2} + S_{11}\frac{d^2T_0}{dx^2} = q$$

$$\frac{dp}{dx} = 0 \qquad p_x = d\phi$$

$$-\bar{E}_{11}\frac{d^2u}{dx^2} + \bar{F}_{11}\frac{d^3w_b}{dx^3} - \bar{H}_{11}\frac{d^2\phi}{dx^2} + G_{55}\phi + S_{11}\frac{dT_0}{dx} = 0$$

For symmetrical angle ply $\bar{E}_{11}$ and $\bar{B}_{11}$ is zero.

$$\bar{D}_{11}\frac{d^4w}{dx^4} - \bar{F}_{11}\frac{d^3\phi}{dx^3} + R_{11}\frac{d^2T_0}{dx^2} = q,$$

$$\bar{F}_{11}\frac{d^3w}{dx^3} - \bar{H}_{11}\frac{d^2\phi}{dx^2} + G_{55}\phi + S_{11}\frac{dT_0}{dx} = 0 \quad (1.12)$$

Associated boundary conditions are as follows
Replace u_0=u and w_b=w
Trough boundary *x= 0 and x=L*

$$\bar{B}_{11}\frac{d^2u}{dx^2} - \bar{D}_{11}\frac{d^3w}{dx^3} + \bar{F}_{11}\frac{d^2\phi}{dx^2} = 0 \; or \; \text{w is prescribed,} \quad (1.13)$$

$$\bar{B}_{11}\frac{du}{dx} - \bar{D}_{11}\frac{d^2w}{dx^2} + \bar{F}_{11}\frac{d\phi}{dx} - R_{11}T_0 = 0 \; or \; \frac{dw}{dx} \text{ is prescribed} \quad (1.14)$$

$$\bar{E}_{11}\frac{du}{dx} - \bar{F}_{11}\frac{d^2w}{dx^2} + \bar{H}_{11}\frac{d\phi}{dx} - S_{11}T_0 = 0 \; or \; \phi \text{ is prescribed} \quad (1.15)$$

$$\bar{A}_{11}\frac{du}{dx}-\bar{B}_{11}\frac{d^2w}{dx^2}+\bar{E}_{11}\frac{d\phi}{dx}-O_{11}T_0=0 \text{ or } u \text{ is prescribed} \quad (1.16)$$

4. Navier solution

Following are the limit conditions utilized for essentially upheld overlaid composite beam along the edges x=a and x=l

w=0, M_x=0, N_x=0, P_x=0, Navier's answer methodology is taken on to figure dislodging factors. Coming up next is the arrangement structures for $u_0(x)$, w (x) , and ϕ(x) that fulfills the limit conditions precisely.

$$u_0(x)=\sum_{m=1,3,5}^{\infty} u_m \cos\left(\frac{m\pi x}{l}\right) \quad (1.17)$$

$$w(x)=\sum_{m=1,3,5}^{\infty} w_m \sin\left(\frac{m\pi x}{l}\right) \quad (1.18)$$

$$\varphi(x)=\sum_{m=1,3,5}^{\infty} \varphi_m \sin\left(\frac{m\pi x}{l}\right) \quad (1.19)$$

Where u_m, w_m , and $\phi_m$the obscure coefficients not set in stone. The warm and cross over mechanical burdens are extended in single Fourier sine series as given beneath.

$$T_0(x)=\sum_{m=1}^{\infty} T_{0m} \sin\left(\frac{m\pi x}{l}\right) \quad (1.20)$$

$$q(x)=\sum_{m=1}^{\infty} q_m \sin\left(\frac{m\pi x}{l}\right)$$

where m is the positive number and T_{0m} and q_m are the coefficients of Fourier series developments, separately for warm and cross over mechanical burdens as follows:

5. Numerical examples and discussions

Model 1: A simply supported beam (SSB) with sinusoidally distributed load (SDL),

$q(x)=\sin(\pi x/L)$

The essentially upheld shaft is having its starting point at left help and is basically upheld at *x=0 and x=L.* The beam is exposed to sinusoidally distributed load, $q(x)=\sin(\pi x/L)$

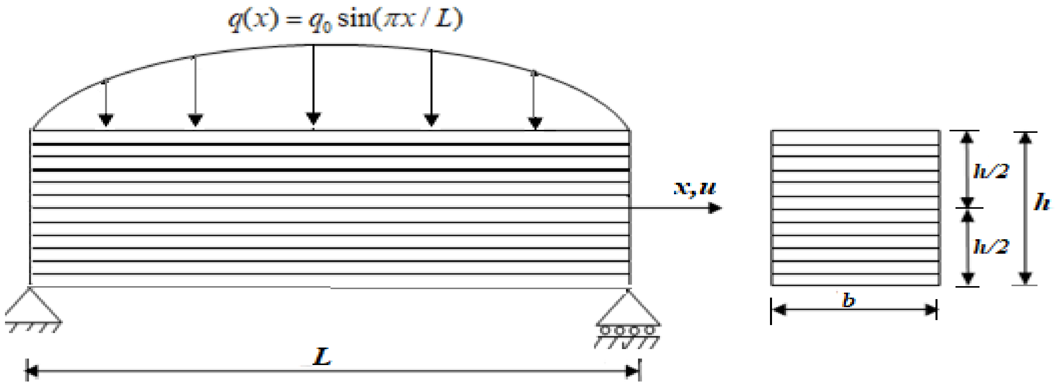

Fig. 1.2. Merely supported single-layer beam with sinusoidally distributed load (SDL) (0^0)

Transverse displacement $\bar{w}$ that is non-dimensional (General)

For 0^0

$$\bar{w}=\left[\frac{1.6701\times10^{5}\alpha_L L^2Th^2\pi\left(2003L^2+2.6950\times10^{4}h^2\pi^2\right)}{\left(4.1728\times10^{7}L^2h^3\pi^4+7.0043\times10^{6}h^5\pi^6\right)}\right]\left[\frac{h}{\alpha_L L^2T}\sin\frac{\pi x}{L}\right]$$

For 90^0

$$\bar{w}=\left[\frac{6680.6599\alpha_L L^2Th^2\pi\left(2003L^2+2695h^2\pi^2\right)}{\left(1.6684\times10^{6}L^2h^3\pi^4+2.0490\times10^{6}h^5\pi^6\right)}\right]\left[\frac{h}{\alpha_L L^2T}\sin\frac{\pi x}{L}\right]$$

For $0^0/90^0/0^0$

$$\bar{w}=\left[\frac{3.4071\times10^{5}6\alpha_L L^2Th^2\pi\left(6.2192\times10^{16}L^2+7.2006\times10^{18}h^2\pi^2\right)}{\left(1.1299\times10^{21}L^2h^3\pi^4+1.7980\times10^{20}h^5\pi^6\right)}\right]\left[\frac{h}{\alpha_L L^2T}\sin\frac{\pi x}{L}\right]$$

For $0^0/90^0/90^0/0^0$

$$\bar{w}=\left[\frac{4342.4624\alpha_L L^2Th^2\pi\left(2456L^2+2.8165\times10^{4}h^2\pi^2\right)}{\left(5.6395\times10^{5}L^2h^3\pi^4+5.3817\times10^{4}h^5\pi^6\right)}\right]\left[\frac{h}{\alpha_L L^2T}\sin\frac{\pi x}{L}\right]$$

For $90^0/0^0/90^0$

$$\bar{w}=\left[\frac{3.3821\times10^{6}\alpha_L L^2Th^2\pi\left(1.2097\times10^{18}L^2+5.2782\times10^{18}h^2\pi^2\right)}{\left(4.4186\times10^{22}L^2h^3\pi^4+6.0284\times10^{21}h^5\pi^6\right)}\right]\left[\frac{h}{\alpha_L L^2T}\sin\frac{\pi x}{L}\right]$$

For $90^0/0^0/0^0/90^0$

$$\bar{w}=\left[\frac{1.7369\times10^{5}\alpha_L L^2Th^2\pi\left(1675L^2+11208h^2\pi^2\right)}{\left(2.7955\times10^{6}L^2h^3\pi^4+1.3773\times10^{5}h^5\pi^6\right)}\right]\left[\frac{h}{\alpha_L L^2T}\sin\frac{\pi x}{L}\right]$$

Non dimensional (General) axial displacement $\bar{u}$

For 0^0

$$\bar{u}=\left[\frac{1}{\alpha_L \mathrm{T}L}\right]\left\{\left[-z\cos\left(\frac{\pi x}{L}\right)\frac{\pi}{L}\right]\left[\frac{\left(1.6701\times10^{5}\alpha_L L^2Th^2\pi\right)\left(2003L^2+2.6950\times10^{4}h^2\pi^2\right)}{\left(4.1728\times10^{7}L^2h^3\pi^4+7.0043\times10^{6}h^5\pi^6\right)}\right]+\left[-\cos\left(\frac{\pi x}{L}\right)\left(h\sinh\frac{z}{h}-\frac{4}{3}\frac{z^3}{h^2}\cosh\frac{1}{2}\right)\left(\frac{\left(5.5616\times10^{9}1\alpha_L LTh^2\pi\right)}{\left(4.1728\times10^{7}L^2h\pi+7.0043\times10^{6}h^3\pi^3\right)}\right)\right]\right\}$$

For 90^0

$$\bar{u}=\left[\frac{1}{\alpha_L \mathrm{T}L}\right]\left\{\left[-z\cos\left(\frac{\pi x}{L}\right)\frac{\pi}{L}\right]\left[\frac{\left(6680.6599\alpha_L L^2Th^2\pi\right)\left(2003L^2+2695h^2\pi^2\right)}{\left(1.6684\times10^{6}L^2h^3\pi^4+2.0490\times10^{4}h^5\pi^6\right)}\right]+\left[-\cos\left(\frac{\pi x}{L}\right)\left(h\sinh\frac{z}{h}-\frac{4}{3}\frac{z^3}{h^2}\cosh\frac{1}{2}\right)\left(\frac{\left(2.2280\times10^{7}\alpha_L LTh^2\pi\right)}{\left(1.6684\times10^{6}L^2h\pi+7.0043\times10^{6}h^3\pi^3\right)}\right)\right]\right\}$$

For $90^0/0^0/90^0$

$$\bar{u}=\left[\frac{1}{\alpha_L \mathrm{T}L}\right]\left\{\left[-z\cos\left(\frac{\pi x}{L}\right)\frac{\pi}{L}\right]\left[\frac{\left(3.3821\times10^{6}\alpha_L L^2Th^2\pi\right)\left(1.2097\times10^{18}L^2+5.2782\times10^{18}h^2\pi^2\right)}{\left(1.1299\times10^{22}L^2h^3\pi^4+6.0284\times10^{21}h^5\pi^6\right)}\right]\right.$$
$$\left.+\left[-\cos\left(\frac{\pi x}{L}\right)\left(h\sinh\frac{z}{h}-\frac{4}{3}\frac{z^3}{h^2}\cosh\frac{1}{2}\right)\left(\frac{\left(1.8495\times10^{25}\alpha_L LTh^2\pi\right)}{\left(4.4186\times10^{22}L^2h\pi+6.0284\times10^{21}h^3\pi^3\right)}\right)\right]\right\}$$

For $0^0/90^0/90^0/0^0$

$$\bar{u}=\left[\frac{1}{\alpha_L \mathrm{T}L}\right]\left\{\left[-z\cos\left(\frac{\pi x}{L}\right)\frac{\pi}{L}\right]\left[\frac{\left(4342.4623\alpha_L L^2Th^2\pi\right)\left(2456L^2+2.8165\times10^{4}h^2\pi^2\right)}{\left(5.6395\times10^{5}L^2h^3\pi^4+5.3817\times10^{4}h^5\pi^6\right)}\right]\right.$$
$$\left.+\left[-\cos\left(\frac{\pi x}{L}\right)\left(h\sinh\frac{z}{h}-\frac{4}{3}\frac{z^3}{h^2}\cosh\frac{1}{2}\right)\left(\frac{\left(1.5552\times10^{8}\alpha_L LTh^2\pi\right)}{\left(5.6395\times10^{5}L^2h\pi+5.3817\times10^{4}h^3\pi^3\right)}\right)\right]\right\}$$

For $90^0/0^0/0^0/90^0$

$$\bar{u}=\left[\frac{1}{\alpha_L \mathrm{T}L}\right]\left\{\left[-z\cos\left(\frac{\pi x}{L}\right)\frac{\pi}{L}\right]\left[\frac{\left(1.7369\times10^{5}\alpha_L L^2Th^2\pi\right)\left(1675L^2+1.1208\times10^{4}h^2\pi^2\right)}{\left(5.6395\times10^{5}L^2h^3\pi^4+5.3817\times10^{4}h^5\pi^6\right)}\right]\right.$$
$$\left.+\left[-\cos\left(\frac{\pi x}{L}\right)\left(h\sinh\frac{z}{h}-\frac{4}{3}\frac{z^3}{h^2}\cosh\frac{1}{2}\right)\left(\frac{\left(2.1170\times10^{9}\alpha_L LTh^2\pi\right)}{\left(2.7955\times10^{6}L^2h\pi+1.3773\times10^{5}h^3\pi^3\right)}\right)\right]\right\}$$

Table 1.1: General Transverse Deflection ($\bar{w}$) at ($x = L$, $z = 0.0$) for Single Layer, Three Layers and Four Layersof Laminated SSB Subjected to SDL $\left[\sin(\pi x/L)\right]$ for length to thickness proportion 2.

AS	Theory	Ply Angle					
		0^0	90^0	$0^0/90^0/0^0$	$90^0/0^0/90^0$	$0^0/90^0/90^0/0^0$	$90^0/0^0/0^0/90^0$
2	Present HYSDT	4.912	0.851	9.951	19.424	11.361	40.972
	TSDT [9]	4.160	0.734	9.045	22.774	10.338	36.152
	HSDT [7]	5.055	0.849	10.562	22.774	11.341	40.972
	FSDT [5]	0.226	0.261	0.462	2.928	0.519	2.668
	ETB [3]	0.202	0.203	0.429	2.899	0.480	2.635

Table 1.2: General Transverse Deflection ($\bar{w}$) at ($x = L, z = 0.0$) for Single Layer, Three Layers and Four Layersof Laminated SSB Subjected to SDL$\left[\sin(\pi x / L)\right]$for length to thickness proportion 4.

AS	Theory	Ply Angle					
		0^0	90^0	$0^0/90^0/0^0$	$90^0/0^0/90^0$	$0^0/90^0/90^0/0^0$	$90^0/0^0/0^0/90^0$
4	Present HYSDT	1.711	0.369	3.468	7.836	3.635	13.102
	TSDT [9]	1.460	0.340	3.022	7.625	3.173	11.591
	HSDT [7]	1.730	0.369	3.533	7.625	3.653	13.102
	FSDT [5]	0.208	0.217	0.437	2.906	0.490	2.643
	ETB [3]	0.202	0.203	0.429	2.899	0.480	2.635

Table 1.3: General Transverse Deflection ($\bar{w}$) at ($x = L, z = 0.0$) for Single Layer, Three Layers and Four Layersof Laminated SSB Subjected to SDL$\left[\sin(\pi x / L)\right]$ for length to thickness proportion 10.

AS	Theory	Ply Angle					
		0^0	90^0	$0^0/90^0/0^0$	$90^0/0^0/90^0$	$0^0/90^0/90^0/0^0$	$90^0/0^0/0^0/90^0$
10	Present HYSDT	0.465	0.230	0.965	3.302	1.012	4.356
	TSDT [9]	0.420	0.225	0.869	3.200	0.922	4.099
	HSDT [7]	4.466	0.230	0.960	3.200	1.013	4.356
	FSDT [5]	0.203	0.205	0.430	2.900	0.481	2.636
	ETB [3]	0.202	0.203	0.429	2.899	0.480	2.635

Table 1.4: General Transverse Deflection ($\bar{w}$) at ($x = L, z = 0.0$) for Single Layer, Three Layers and Four Layersof Laminated SSB Subjected to SDL$\left[\sin(\pi x / L)\right]$ for length to thickness proportion 20.

AS	Theory	Ply Angle					
		0^0	90^0	$0^0/90^0/0^0$	$90^0/0^0/90^0$	$0^0/90^0/90^0/0^0$	$90^0/0^0/0^0/90^0$
20	Present HYSDT	0.269	0.210	0.562	2.589	0.613	3.069
	TSDT [9]	0.258	0.210	0.540	2.561	0.590	3.005
	HSDT [7]	0.269	0208	0.563	2.561	0.614	3.069
	FSDT [5]	0.202	0.210	0.429	2.899	0.480	2.635
	ETB [3]	0.202	0.203	0.429	2.899	0.480	2.635

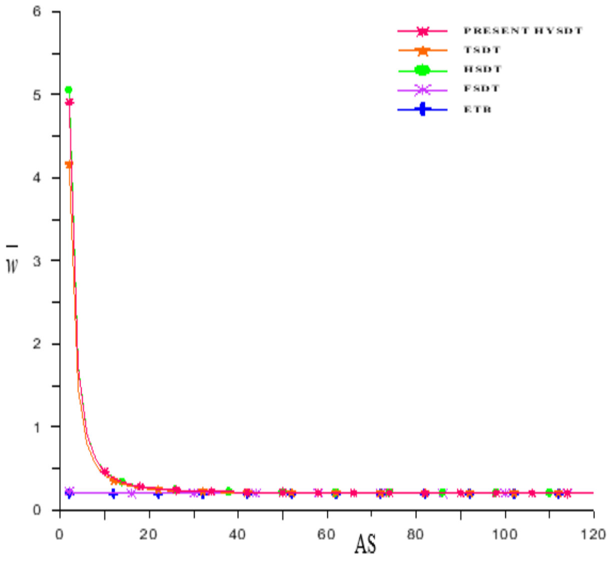

Figure 1.3: Change of extreme transverse displacement ($\bar{w}$) at (*x = L, z = 0.0*) for single layer with ply angle (0^0) of laminated SSBsubjected to SDL$\left[\sin(\pi x/L)\right]$for length to thickness proportion.

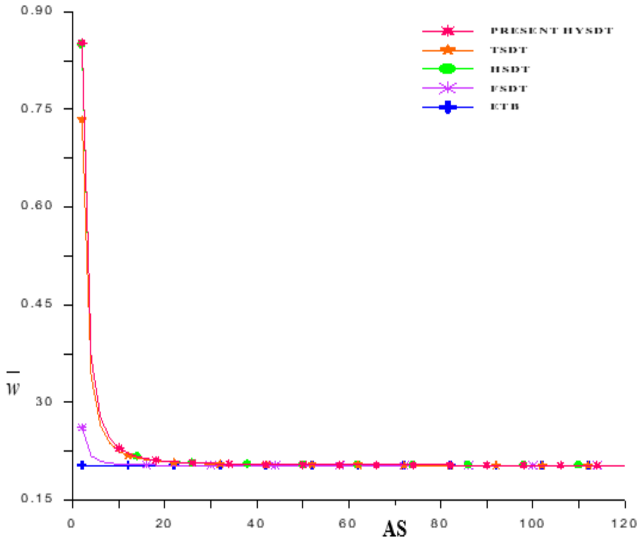

Figure 1.4: Change of extreme transverse displacement ($\bar{w}$) at (*x = L, z = 0.0*) for single layer with ply angle (90^0) of laminated SSB subjected to SDL$\left[\sin(\pi x/L)\right]$for length to thickness proportion.

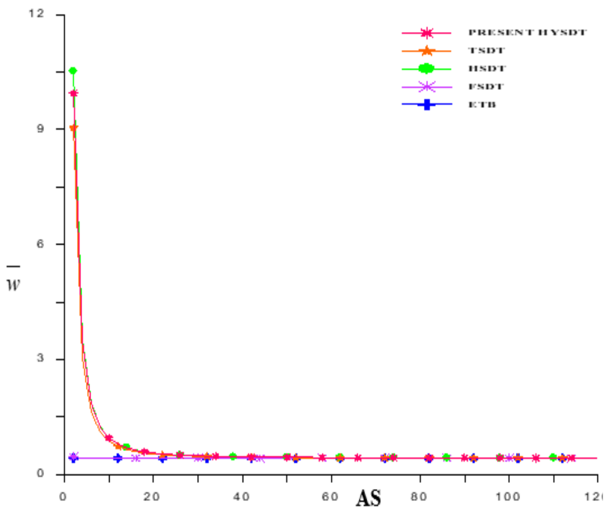

Figure 1.5: Change of extreme transverse displacement ($\bar{w}$) at (*x = L, z = 0.0*) for single layer with ply angle ($0^0/90^0/0^0$) of laminated SSB subjected to SDL$\left[\sin(\pi x/L)\right]$for length to thickness proportion.

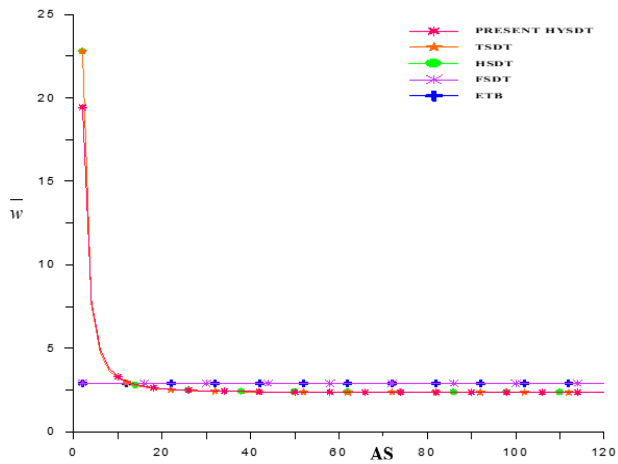

Figure 1.6: Change of extreme transverse displacement ($\bar{w}$) at (*x = L, z = 0.0*) for single layer with ply angle ($90^0/0^0/90^0$) of laminated SSBsubjected to SDL$\left[\sin(\pi x/L)\right]$for length to thickness proportion.

Table 1.5: General Axial Displacement ($\bar{u}$) at *(x = L, z = h/2*) for Single Layer, Three Layers and Four Layersof Laminated SSB Subjected to SDL$\left[\sin(\pi x / L)\right]$ for length to thickness proportion 4.

AS	Theory	Ply Angle					
		0^0	90^0	$0^0/90^0/0^0$	$90^0/0^0/90^0$	$0^0/90^0/90^0/0^0$	$90^0/0^0/0^0/90^0$
4	Present HYSDT	4.661	0.796	9.491	18.853	9.992	32.614
	TSDT [9]	4.777	0.804	9.911	20.657	10.460	33.428
	HSDT [7]	4.713	0.796	9.640	17.913	9.995	32.572
	FSDT [5]	0.318	0.319	0.673	4.553	0.754	4.139
	ETB [3]	0.318	0.319	0.673	4.553	0.754	4.139

Table 1.6: General Axial Displacement ($\bar{u}$) at *(x = L, z = h/2*) for Single Layer, Three Layers and Four Layersof Laminated SSB Subjected to SDL$\left[\sin(\pi x / L)\right]$for length to thickness proportion 10.

AS	Theory	Ply Angle					
		0^0	90^0	$0^0/90^0/0^0$	$90^0/0^0/90^0$	$0^0/90^0/90^0/0^0$	$90^0/0^0/0^0/90^0$
10	Present HYSDT	1.073	0.396	2.199	6.282	2.203	8.814
	TSDT [9]	1.100	0.398	2.261	6.458	2.367	8.974
	HSDT [7]	1.077	0.396	2.207	5.984	2.305	8.810
	FSDT [5]	0.318	0.319	0.673	4.553	0.754	4.139
	ETB [3]	0.318	0.319	0.673	4.533	0.754	4.139

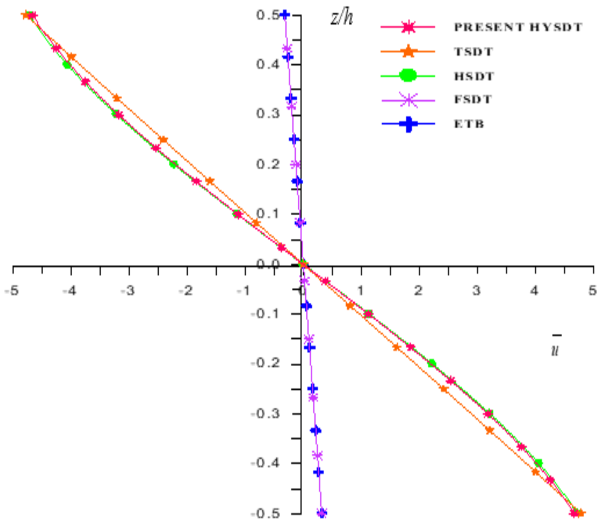

Figure 1.7: Change of extremeaxial displacement ($\bar{u}$) at *(x = L, z = h/2*) for single layerwith ply angle (0^0) of laminated SSB to SDL$\left[\sin(\pi x / L)\right]$for length to thickness proportion 4.

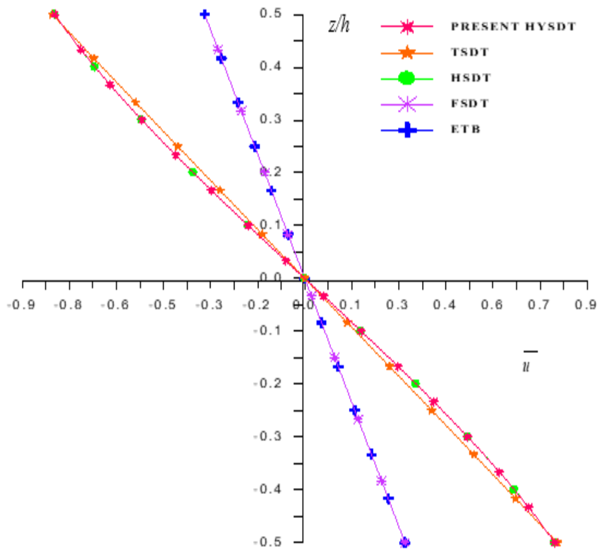

Figure 1.9: Change of extremeaxial displacement ($\bar{u}$) at *(x = L, z = h/2*) for single layerwith ply angle (90^0) of laminated SSB to SDL $\left[\sin(\pi x / L)\right]$for length to thickness proportion 4.

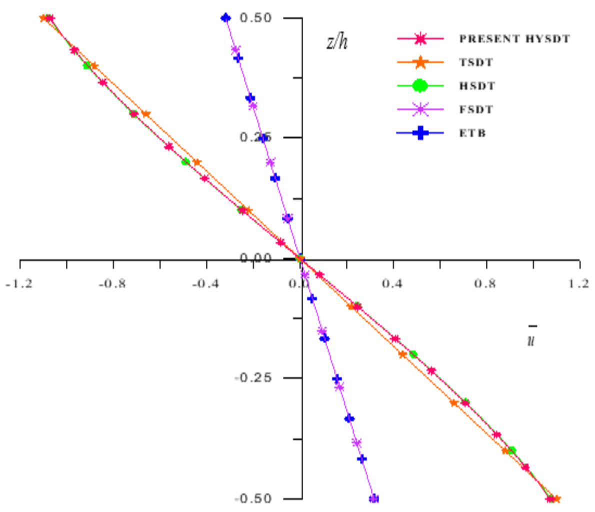

Figure 1.8: Change of extremeaxial displacement ($\bar{u}$) at *(x = L, z = h/2*) for single layerwith ply angle (0^0) of laminated SSB to SDL$\left[\sin(\pi x / L)\right]$for length to thickness proportion 10.

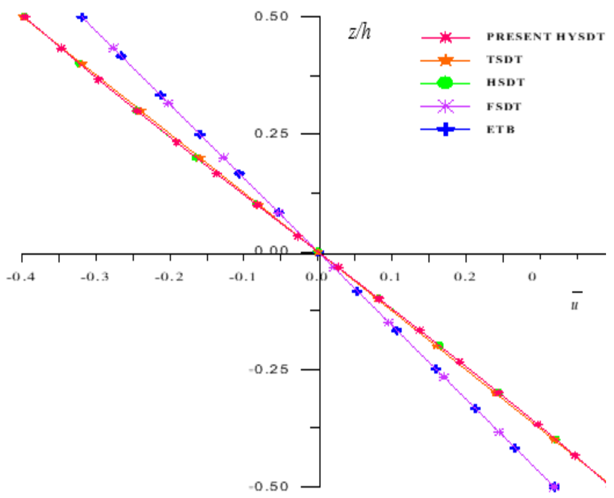

Figure 1.10: Change of extremeaxial displacement ($\bar{u}$) at *(x = L, z = h/2*) for single layerwith ply angle (90^0) of laminated SSB to SDL$\left[\sin(\pi x / L)\right]$for length to thickness proportion 10.

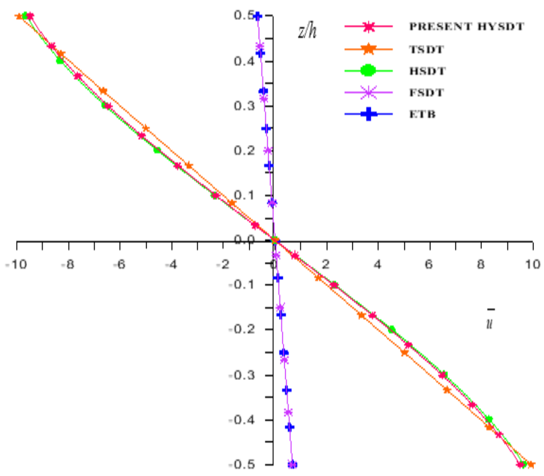

Figure 1.11: Change of extremeaxial displacement ($\bar{u}$) at *(x = L, z = h/2)* for single layerwith ply angle ($0^0/90^0/0^0$) of laminated SSB to SDL$\left[\sin(\pi x / L)\right]$for length to thickness proportion 4.

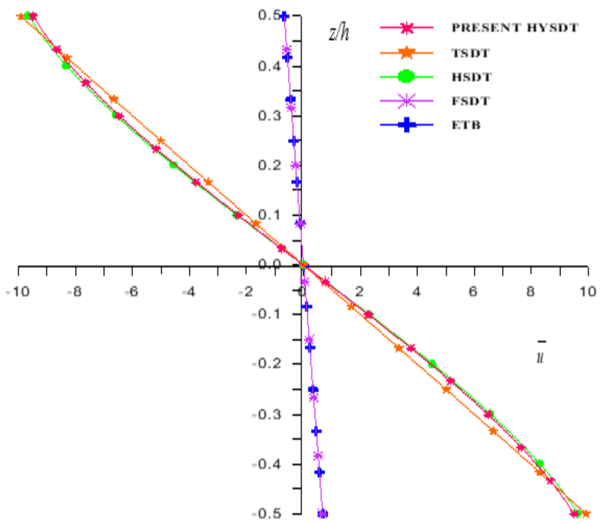

Figure 1.12: Change of extremeaxial displacement ($\bar{u}$) at *(x = L, z = h/2)* for single layerwith ply angle ($0^0/90^0/0^0$) of laminated SSB to SDL$\left[\sin(\pi x / L)\right]$ for length to thickness proportion 10.

6. Conclusions

a) It's a refined shear deformation hypothesis based on elimination.
b) Similar to FSDT, there are an equal number of cryptic components overall.
c) The beams shear deformation is accurately reflected.
d) The hypothesis eliminates the requirement for a trim rectification component.
e) As much as possible, the administering varied conditions and circumstances are an unusual factor.

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