GEOMETRIC ALGEBRA WITH DIFFERENTIAL EQUATION USING ROTATION VECTOR

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ABSTRACT:

Geometric Algebra (GA) is a powerful mathematical language for expressing physical ideas. It unifies many diverse mathematical formalisms and aids physical intuition. In our various publications and lectures will find many examples of the insights that geometric algebra brings to problems in physics and engineering. The rotation vector differential equation is vital to the attitude algorithm in the modern strapdown inertial navigation system. A substitutive derivation of the rotation vector differential equation in Geometric Algebra (GA) form is provided by utilizing the kinematic equation of rotors. The GA form of the equation inherits the simplicity and efficiency of the vector form. Hence, this model gives better results interms of efficiency and accuracy.

KEYWORDS: **Inertial Navigation, Computational Modeling, Graphical Models, Rotors, Vector**

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I. INTRODUCTION

The rotation vector was introduced by Bortz [1] to represent the rotation of the body frame with respect to a chosen reference frame in the strapdown inertial navigation system. Its time derivative is the sum of the angular velocity vector and the noncommutativity rate vector. The former can be measured by the gyro, while the latter can not be measured. Fortunately, the noncommutativity rate vector is a well behaved function of previous rotations and current angular velocity. Therefore, the rotation vector for the interval can be estimated by making use of the differential formula and the Taylor series of the rotation vector. Geometric Algebra (GA) is a very convenient representational and computational system for geometry.

It integrates many algebraic system, e.g. the algebra of complex numbers, vectors and matrices, into a coherent mathematical language. And this language not only retains the advantages of each special algebra but also possesses powerful new capabilities. GA is found to be eminently suitable to physics, computer vision, biological vision, robotics etc. As Lasenby stated in [2], GA could be considered as "a unified mathematical language for physics and engineering in the 21st century". The spinor theory of rotations and rotation dynamics was introduced by Hestenes [3]. Based on vector algebra, calculations with spinors (or rotors) are more efficient than calculations with matrices. Candy derived a GA form of the rotation vector differential equation, by making use of the kinematic equation of rotors presented by Hestenes [4]. However, the modality of Candy's equation is a little complicated, and the solution of the equation is not provided. We develop a more compact GA form of the rotation vector differential equation. The procedure is analogue to the derivation of the conventional rotation vector differential equation proposed by Savage [5]. The traditional rotation vector algorithms can be easily adapted to resolve the equation derived in this paper.

GA is a powerful and practical framework to represent and solve geometrical problems. Using outer product, vectors can be combined into new elements of computation which represent oriented subspaces of any dimension. And linear transformations on the vector space can be extended to the subspaces constructed by vectors. Operators can be constructed by geometric elements intrinsic to the problem, e.g. the rotation operator is the geometric product of two vectors [6]. Moreover, GA has a special representation of orthogonal transformations (e.g. rotation and translation) which is universal to all geometric elements, in any dimension. Compared to other formalisms for manipulating geometric objects, geometric algebra is noteworthy for supporting vector division and addition of objects of different dimensions.

II. LITERATURE SURVEY

Huazhong Xu, Jinquan He and Changzhe Chen., et.al [7] A vector control strategy of PMSM based on Pan-Boolean algebra self-adapting PID control is proposed in this paper, which is unnecessary to set up the exact mathematical model of the controlled plant. According to the deviation and deviation rate at different time, it is able to eliminate the influences of the parameters of the controlled plant and the disturbance of the external environment by online adjusting the parameters of the PID controller. For the PMSM speed adjustment system, the PI controller is used in current loop and Pan-Boolean algebra self-adjusting PID controller is used in speed loop instead of traditional PI controller. The effectiveness of the proposed control method is verified by simulation based on Matlab and implemented using experimental platform based on DSP chip. The simulation and experimental results show that the performance and robustness of the controller are better than the conventional PI controller in the PMSM control system.

K. Matsumoto and S. G. Sedukhin,et.al [8] It is well-known that the all-pairs shortest paths problem has a similar algorithmic characteristic to the classical matrix-matrix multiply-add (MMA) problem, one of the differences between the two problems is in the underlying algebra: the matrix multiply-add uses linear (+, ×)-algebra whereas the all-pairs shortest paths problem uses (min, +)-algebra. This paper presents an implementation of 64×64 matrix multiply-add kernel in (min, +)-algebra on a short-vector SIMD processor, the so-called Synergistic Processing Element (SPE), of the Cell Broadband Engine (Cell/B.E.). Our implementation for the shortest paths problem adopts an existing fast algorithm of matrix multiply-add with a reduction of the number of required registers. The MMA implementation in (min, +)-algebra achieves the speed of 8.502 Gflop/s, which is about three times as low as that of the (+, ×)-algebra MMA and is very close to the theoretical estimation based on the required number of instructions.

L. Eskor, M. Lepp and E. Tonisson, et.al [9] There can be varying degrees of difference between the commands, syntax, etc., of computer algebra systems. Sometimes, translation from one computer algebra system to another is needed. As the languages of computer algebra systems are similar to programming languages, the translation techniques used in case of programming paper focuses on syntax-directed translation where grammars have a central role. The area of commands is restricted to the commands useful for school mathematics. A prototype is developed for translating the worksheets of Maple, Maxima and WIRIS.

T. Şirin, et.al [10] Symbolic computation techniques for solutions of Partial Differential Equations (PDEs) with

Maxima, an open source Computer Algebra System (CAS) are represented in three new packages. They are prepared in Maxima's user's language. After giving the initial and boundary conditions, packages automatically give the results and plot the graphs. The packages are designed as ready to use and modify instructional tools by users with computer-lab sessions of engineering language seem to be productive. This

courses. Solutions are performed by Finite Difference Method (FDM).

K. S. Achary, P. R. Murarka and M. Reza, et.al [11] To solve complex and large mathematical expression manually using pen and paper is a time taking task which in most cases ends up in an erroneous result. This is a major drawback which may lead to heavy losses to people dealing in numbers. Henceforth we have come up with a vision of Symbolic computation which provides a quick, efficient and user friendly environment to its users. Symbolic Computation is a computer algebra system which has been designed in Java. The Object oriented Programming(OOP) concept and predefined packages of the language have been used to solve expressions consisting of differentiation, integration, series and many more symbols. Moreover these features have also been brought to use to enhance the computation time and provide a better outlook to the application.

B. Erabadda, S. Ranathunga and G. Dias, et.al [12] This paper presents a system that automatically assesses multi-step answers to algebra questions. The system requires teacher involvement only during the question set-up stage. Two types of algebra questions are currently supported: questions with linear equations containing fractions, and questions with quadratic equations. The system evaluates each step of a student's answer and awards full/partial marks according to a marking scheme. The system was evaluated for its performance using a set of student answer scripts from a government school in Sri Lanka and also by undergraduate students. The system accuracy was over 95.4%, and over 97.5%, respectively for the aforementioned data sets.

Jianping Yan, et.al [13] define a logical algebra named MP-algebra and discuss its algebraic properties. We find that MP-algebra not only takes Boole algebra, MV-algebra and R 0 algebra as its special examples but also holds the subdirect representation theorem same as that of on Boole algebra, MV algebra and R 0 algebra. We also explore the basic properties of implication operation of MP-algebra. We prove that an MP-algebra is also a residuated lattice with many good properties. The conclusions we got show that MP-algebra is a well-structure logic algebra when it is taken as the logic truth degree set.

S. A. Matos, C. R. Paiva and A. M. Barbosaet.al [14] It is well-know that conical refraction occurs for electric anisotropic biaxial crystals when the wave vector has the direction of the medium optic axes. In this paper, we show that conical refraction occurs - in an analogous away - for a more general type of biaxial media that have simultaneously electric and magnetic anisotropies. Furthermore, the new coordinate-free approach based on geometric algebra, developed by the authors in previous papers to address anisotropy, is shown to shed new light on this classic topic of optics that is conical refraction.

M. T. Pham, T. Yoshikawa, T. Furuhashi and K. Tachibana, et.al [15] Most conventional methods of feature extraction for pattern recognition do not pay sufficient attention to inherent geometric properties of data, even in the case where the data have spatial features. This paper introduces geometric algebra to extract invariant geometric features from spatial data given in a vector space. Geometric algebra is a multidimensional generalization of complex numbers and of quaternions, and it ables to accurately describe oriented spatial objects and relations between them. This paper proposes to combine several geometric features using Gaussian mixture models. It applies the proposed method to the classification of hand-written digits.

III. METHODOLOGY

In this section, the rotation vector differential equation is derived by using GA. The kinematic equation of rotors can be formulated as

$$R=\frac{1}{2}R\Omega \quad \text{--- (1)}$$

where Ω= ω*=ωI is the dual of the angular rate . Define Θ = nΘ to be the rotation vector, we have

$$R= \cos\frac{\theta}{2} +\frac{\theta *}{2}\sin\frac{\theta}{2} \quad \text{---- (2)}$$

Since the pseudoscalar I is invariant with respect to time, it follows that dI/dt=0. Therefore

$$\frac{d}{dt}(\Theta *)=\frac{d}{dt}(\Theta I)=\frac{d\Theta}{dt} I= \Theta * \quad \text{----(3)}$$

Differentiate (2) with respect to time to get

$$\frac{dR}{dt}=-\frac{\theta}{2}\sin\frac{\theta}{2}+\frac{\theta}{\theta}\sin\frac{\theta}{2}-\frac{\theta*\theta}{2\theta^2}\left(2\sin\frac{\theta}{2}-\theta\cos\frac{\theta}{2}\right)$$

--- (4)

Substitution of (2) into the right side of (1) gives

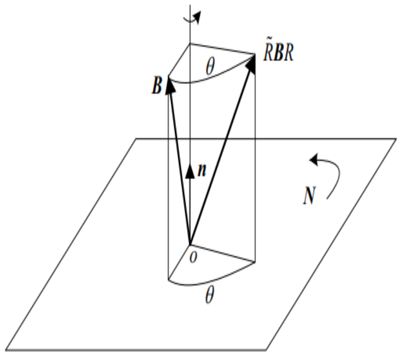

Fig.1: Rotation About An Axis, Or In A Plane

$$\frac{1}{2}R\Omega=\frac{1}{2}\left(\cos\frac{\theta}{2}+\frac{\theta}{2}\sin\frac{\theta}{2}\right)\omega=-\frac{\theta.\omega}{2\theta}\sin\frac{\theta}{2}\omega^*\cos\frac{\theta}{2}-\frac{\theta\wedge\omega}{2\theta}\sin\frac{\theta}{2}$$

--- (5)

Regarding that blades with the same grades should equal to each other in an equation, from (1), (4) and (5) we can get

$$\frac{\theta.\omega}{\theta}=\Theta^* \quad \text{--- (6)}$$

$$\frac{1}{2}\omega*\cos\frac{\theta}{2}-\frac{\theta\wedge\omega}{2\theta}\sin\frac{\theta}{2}=\frac{\theta}{\theta}\sin\frac{\theta*\theta}{2\theta^2}\left(2\sin\frac{\theta}{2}-\theta\cos\frac{\theta}{2}\right)$$

--- (7)

According to the algebraic identity of the inner and outer product

$$\theta.(\theta\wedge\omega)\,\theta 2\,\omega-(\theta.\omega)\,\theta \quad \text{---- (8)}$$

Take (6) into (7) to get

$$\theta\,\theta = \theta\,\omega - \theta.(\theta\wedge\omega)\,/\,\theta \quad \text{--- (9)}$$

Dualizing (7) and collecting terms obtains

$$\theta\frac{-\omega\,\theta}{2}\cot\frac{\theta}{2}+(-1)n(n-1)/2\frac{1}{2}\,\theta.\omega\ * + \frac{\theta*\theta}{\theta}\left(1-\frac{\theta}{2}\cot\left(\frac{\theta}{2}\right)\right) \quad \text{-----(10)}$$

where n is the dimension of the space. Employing (9) in (10) and collecting terms gives the final result.

$$\theta=\omega+(-1)^{n(n-1)/2}\frac{1}{2}\theta.\omega^* - \frac{1}{\theta^2}\left(1-\frac{\theta}{2}\cot\left(\frac{\theta}{2}\right)\right)\theta.(\theta^{\omega})$$

---- (11)

This GA form of the rotation vector differential equation is more compact than Candy's, hence, easier to be understood and computed with. It is owing to the flexible switch between the direct and dual representations, which is a powerful way of finding the simplest expressions for geometrical operations. As we can see from the formula is general in all dimensions.

$$\theta = \omega + \frac{1}{2}\theta x\omega + \frac{1}{\theta^2}\left(1 - \frac{\theta}{2}\cot\left(\frac{\theta}{2}\right)\right)\theta x(\theta x\omega)$$

------ (12)

It means that, the GA form of the rotation vector differential formula is the generalization of the conventional vector form of it into all dimensions.

IV. RESULT ANALYSIS

In this performance analysis linear differential equation for geometric algebra is observed in this section.

Table.1: Performance Analysis

Parameters	Geometric Algebra (GA)	Vector Algebra (VA)
Accuracy	99	86
Efficiency	91	84

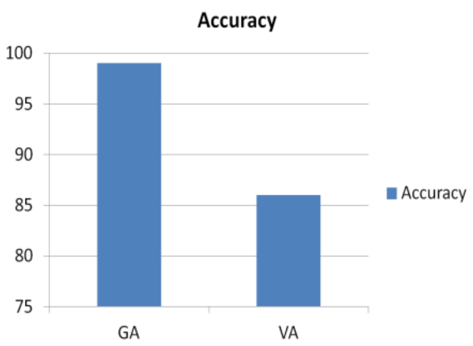

Fig.2: Accuracy Comparison Graph

In Fig.2 accuracy comparison graph is seen between geometric algebra and vector algebra.

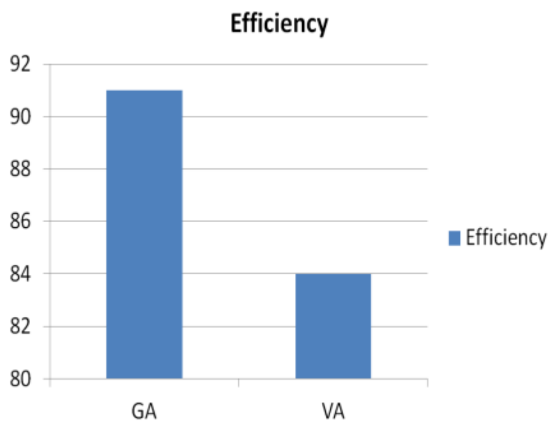

Fig.3 Efficiency Comparison Graph

In Fig.3 efficiency comparison graph is seen between geometric algebra and vector algebra.

V. CONCLUSION

Geometric Algebra (GA) is a powerful mathematical language for expressing physical ideas. It unifies many diverse mathematical formalisms and aids physical intuition. In our various publications and lectures will find many examples of the insights that geometric algebra brings to problems in physics and engineering. The representation of the rotation in GA is very simple and efficient. A compact GA form of the rotation vector differential formula is developed. Like the formula derived by Candy, it is universal in any dimension. Moreover, gyro outputs can be directly fed to estimate rotation vectors.

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