IMPACT OF AWARENESS ON THE SPREAD OF HIV INFECTION WITH DELAY EFFECT

Nareshkumar C. Chavda[1]
Assistant Professor, Government Engineering College, Dahod
Email: nareshc.chavda@gmail.com
Ramesh S Damor[2]
L. D. College of Engineering, Ahmedabad
Email : rameshsvnit2010@gmail.com
Ashish A Prajapati[3]
Assistant Professor, Government Engineering College, Dahod
Email: Ashishprajapati14@gmail.com

ABSTRACT

In this paper, a non-linear mathematical model is proposed to study the impact of awareness on the dynamics of HIV. It is assumed that due to awareness susceptible are taking necessary care to avoid the contact with HIV infected while infected population are isolated to prevent contact for the further spread of disease. It is assumed that the susceptible and infected classes are becoming aware at different rates. The non-linear compartmental model is analyzed by using the stability theory of differential equations and numerical simulation.This results in dual impact on the susceptible as well as infected population. Then delay effect is taken into account in order to achieve a better compatibility with reality. Consdiering delay as bifurcation parameter the possibility of periodic solution is considered.

KEYWORDS: Awareness, Basic reproduction number, Stability.
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1. INTRODUCTION

The Mathematical modeling is very useful tool to study and control the spread of infectious diseases in the populations. The classical models governing the spread of infectious diseases depend mainly on the interactions between susceptible and infectives. However, there are other factors, such as vaccination, awareness, migration of population etc., which also affect the spread of infectious diseases. In particular, awareness plays an important role on the dynamics of the diseases.

It is observed that the spread of the infectious or communicable diseases in the population make the people to change their behavior and attitude in such a way that the effect of the disease onto themselves is minimized to prevent themselves and others from contracting the disease (Hays, 2006). These changes in the behavior may be called awareness. The level of awareness not only depends on the behavioral changes imposed by public authorities, but also depends on responses driven by risk and fear of the given disease due to its effects on the life, duration of the sickness due to the disease and most importantly availability of the effective treatment of the disease. Kristiansen et al. (2007) in his work explored the usage of face masks to avoid airborne diseases. Rubin et al. (2009) showed the importance of using better hygiene. Laver et al. (2001) in their study of malaria considered importance of preventive medicine. Ahituv et al. (1996) explored the demand of practicing safer sex using condoms. These actions can change the transmission patterns of the disease. For people to react in some way, they do not necessarily need to have seen the effects of the disease themselves, but they may have heard of it through some sources like media. These, however, usually focus on high-profile diseases like HI V/AIDS, Tuberculosis etc. This kinds of awareness can be developed in the people by hearing about someone having fallen ill by not following some necessary precautions, As the information about the presence of a disease spreads in the population, people adapt their behavior as a result of their awareness of the disease (e.g., Stone burner and Low-Beer,2004).

It is the awareness which make people to take precautions such as vaccination, screening of donated blood to prevent blood borne diseases, adapting to protected sex to prevent sexually transmitted diseases. Therefore, to predict the spread of an infectious disease the effect of the awareness must be considered in the modeling process.

It has been observed in statistical analysis on AIDS awareness programs that public awareness can play an appreciable role in preventing the AIDS epidemic [16]. Some researchers have proposed and analyzed compartmental models with the assumption that the awareness plays a vital role to reduce the contact rate. Liu et al. [3] have considered and analyzed a model with the psychological impact on epidemic outbreaks in his work. Misra et al.[17] have proposed and analyzed a non-linear mathematical model for the effects of awareness programs on the spread of infectious diseases such as flu has been proposed and analyzed.

2. THE MODEL

The total population of interest divided into four mutual disjoint compartments, susceptible class (C_1), aware class) without HIV infection whose members are taking sufficient precautionary actions to o protect themselves from HIV infection, HIV infected class (C_3) whose members are unaware about their own infection or even if they are aware about their infection then also they are not taking sufficient action to stop further spread of HIV infection and the fourth compartment (C_4) consists of those HIV infected individuals who are aware about their HIV infection and interact carefully with others in such way that there is no further spread of HIV infection by their infection. Let S, S_h, I_1 and I_2be the number of individuals in the compartments $C_{1,}$ $C_{2,}$ C_3 and C_4 respectively. Let N be the total population size at time t such that$N = S + S_h + I_1 + I_2$.

The compartmental model is developed according to the schematic diagram shown in figure 1. The following assumptions are made: The susceptible become HIV infected following contact with the HIV infective at the contact rateα.

The susceptible population is getting awareness at the ratem_1 so they take sufficient precaution such that they cannot get the contact with HIV infectives.

HIV infectives are becoming aware at the rate m_2 so that they are taking necessary action to stop further spread of infection by them.It is reasonable to assume that the unaware infected population growth is not instantaneous after contacting susceptible and a discrete time lag for gestation of infected population is required.

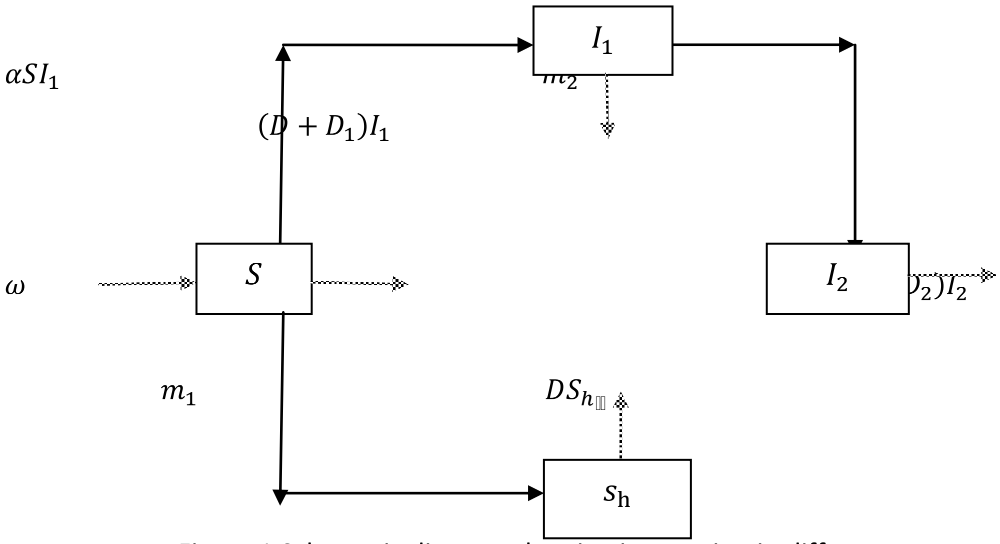

Figure 1 Schematic diagram showing interaction in different compartments.

From the above assumptions, the model equations are

$$\begin{aligned}
\frac{dS}{dT} &= \omega - \alpha S(T-T')I_1(T-T') - m_1 S - DS \\
\frac{dS_h}{dT} &= m_1 S - DS_h \\
\frac{dI_1}{dT} &= \alpha S(T-T')I_1(T-T') - m_2 I_1 - (D+D_1)I_1 \\
\frac{dI_2}{dT} &= m_2 I_1 - (D+D_2)I_2
\end{aligned} \quad (2.1)$$

All parameters are assumed to be nonnegative. The initial conditions associated with system are:

$$S(0)=S_0 \geq 0, I_1(0)=I_{10} \geq 0, I_2(0)=I_{20} \geq 0, S_h(0)=S_{h0} \geq 0$$
$$S(t) \geq 0, I_1(t) \geq 0 \text{ on } [-\tau, 0] \quad (2.2)$$

For the sake of simplicity, the following non-dimensional variables and parameters are introduced in the model system (2.1):

$$t = \mathrm{DT},\ \mathrm{s}=\frac{\mathrm{DS}}{\omega},\ \mathrm{i}_1 = \frac{\mathrm{DI}_1}{\omega},\ \mathrm{i}_2=\frac{\mathrm{DI}_2}{\omega},\ \mathrm{s}_\mathrm{h} = \frac{\mathrm{DS}_\mathrm{h}}{\omega},$$
$$\mathrm{d}_1 = \frac{\mathrm{D}_1}{D},\ \mathrm{A}=\frac{\alpha\omega}{D^2},\ \mu_1 = \frac{m_1}{D},\ \mu_2 = \frac{m_2}{D},\ \tau = \mathrm{DT}'$$

This leads to dimensionless of equation system of equations

$$\begin{aligned}
\frac{\mathrm{d}s}{\mathrm{d}t} &= 1 - As(t-\tau)i_1(t-\tau) - \mu_1 s - s \\
\frac{\mathrm{d}s_h}{\mathrm{d}t} &= \mu_1 s - s_h \\
\frac{\mathrm{d}i_1}{\mathrm{d}t} &= As(t-\tau)i_1(t-\tau) - \mu_2 i_1 - (1+d_1)i_1 \\
\frac{\mathrm{d}i_2}{\mathrm{d}t} &= \mu_2 i_1 - (1+d_2)i_2
\end{aligned} \tag{2.3}$$

$$\begin{gathered}
s(0) = s_0 \geq 0, i_1(0) = i_{10} \geq 0, i_2(0) = i_{20} \geq 0, s_h(0) = s_{h0} \geq 0 \\
s(t) \geq 0, i_1(t) \geq 0 \text{ on } [-\tau, 0]
\end{gathered} \tag{2.4}$$

3. Boundedness of solution

Lemma. All the solutions of equation (2.3) which initiate in R_+^4 are uniformly bounded.

Proof: Adding the equations of the system (2.3)

$$\frac{\mathrm{d}n}{\mathrm{d}t} = 1 - n - d_1 i_1 - d_2 i_2$$

$$\frac{\mathrm{d}n}{\mathrm{d}t} + n \leq 1$$

Applying theory of differential equation for the above equation

$$0 < n(s, s_h, i_1, i_2) \leq (1 - e^{-t}) + n(s(0), s_h(0), i_1(0), i_2(0))e^{-t}$$

So for $t \to \infty$,

$$0 < n(s, s_h, i_1, i_2) \leq 1$$

Hence, all the solutions of eq. (2.3) that initiate in R_+^4 are confined to the region

$$\Omega = \{(s, s_h, i_1, i_2) \in \mathrm{R}_+^4 : 0 \leq s + s_h + i_1 + i_2 \leq 1\}$$ □

4. Basic Reproduction Number

The basic reproduction number is defined as the number of secondary infections produced by a single infectious individual during his or her entire infectious period.

The basic reproduction number for HIV is obtained as

$$R_0 = \frac{A}{(1+\mu_1)(\delta+\mu_2)}, \quad \text{where, } \delta = 1 + d_1 \tag{4.1}$$

5. Equilibrium points and linear stability of non-delay model

The following equilibrium points of the system (2.3) are obtained:

i) The disease free equilibrium point is $E_0(s^*, s_h^*, 0, 0) = E_0\left(\frac{1}{(1+\mu_1)}, \frac{\mu_1}{(1+\mu_1)}, 0, 0\right)$.

The disease free equilibrium point E_0 always exists.

ii) The endemic equilibrium $E_1(\bar{s}, \bar{s}_h, \bar{i}_1, \bar{i}_2)$ is obtained by solving the four simultaneous equations:

$$1 - Asi_1 - \mu_1 s - s = 0 \tag{5.1a}$$

$$\mu_1 s - s_h = 0 \tag{5.1b}$$

$$As - \mu_2 - \delta = 0 \tag{5.1c}$$

$$\mu_2 i_1 - (1+d_2) i_2 = 0 \tag{5.1d}$$

Solving (5.1c), gives $s = \dfrac{\delta + \mu_2}{A}$. This substitution in equation (5.1b) yields

$$s_h = \frac{\mu_1(\delta + \mu_2)}{A}$$

Substitution of the values of s in equation (5.1a) gives

$$i_1 = \frac{(1+\mu_1)(R_0 - 1)}{A}$$

Finally, the value of i_2 is obtained from the (5.1d) as

$$i_2 = \frac{\mu_2(1+\mu_1)(R_0 - 1)}{A(1+d_2)}$$

Observe that, i_1 and i_2 are positive when $R_0 > 1$.

$$E_1(\bar{s}, \bar{s}_h, \bar{i}_1, \bar{i}_2) = E_1\left(\frac{\delta + \mu_2}{A}, \frac{\mu_1(\delta + \mu_2)}{A}, \frac{(1+\mu_1)(R_0 - 1)}{A}, \frac{\mu_2(1+\mu_1)(R_0 - 1)}{A(1+d_2)}\right).$$

The endemic $E_1(\bar{s}, \bar{s}_h, \bar{i}_1, \bar{i}_2)$ equilibrium point exists if

$$\mathrm{R}_0 > 1 \tag{5.2}$$

The stability of different equilibrium points can be discussed on the basis of stability matrixat (s, s_h, i_1, i_2) computed as 170

$$J = \begin{bmatrix} -1 - \mu_1 - \mathrm{A} i_1 e^{-\lambda\tau} & 0 & -Ase^{-\lambda\tau} & 0 \\ \mu_1 & -1 & 0 & 0 \\ \mathrm{A} i_1 e^{-\lambda\tau} & 0 & s\mathrm{A}e^{-\lambda\tau} - \mu_2 - \delta & 0 \\ 0 & 0 & \mu_2 & -(1+d_2) \end{bmatrix}$$

The characteristic equation is computed at (s, s_h, i_1, i_2) as

$$\begin{aligned} &(\lambda+1)(\lambda+1+d_2)(\lambda^2 + P_1\lambda + P_0) = 0 \\ &\text{where, } P_1 = 1 + \mu_1 + \mu_2 + \delta + Ae^{-\lambda\tau}(i_1 - s), \\ &\qquad P_0 = (1+\mu_1)(\mu_2 + \delta) + Ae^{-\lambda\tau}\left\{(\mu_2 + \delta) i_1 - (1+\mu_1)s\right\} \end{aligned} \tag{5.3}$$

In absence of delay, $\tau = 0$.

In absence of delay, the stability results are stated in the form of theorems.

Theorem 5.1 The disease-free equilibrium $E_0(\mathrm{s}^*, \mathrm{s}_h^*, 0, 0)$ is locally asymptotically stable when

$$\mathrm{R}_0 < 1 \tag{5.4}$$

Proof: The characteristic equation is computed at $E_0(s^*, s_h^*, 0, 0)$ as

$$(\lambda+1)(\lambda+1+d_2)(\lambda^2+q_0\lambda+q_1)=0,$$

Where, $q_0 = 1+\mu_1+\mu_2+\delta-\dfrac{A}{(1+\mu_1)}$, $q_1=(1+\mu_1)(\mu_2+\delta)-A$

The eigenvalues of the stability matrix J at E_0 are computed as $-1, -(1+d_2), -(1+\mu_1), -\left(\delta+\mu_2-\dfrac{A}{1+\mu_1}\right)$.Clearly, all eigenvalues are negative if $\delta+\mu_2-\dfrac{A}{1+\mu_1}>0$ or $\mathrm{R}_{01}<1$

Hence, the result is proved. □

Remark: It may be noted from (3.4) that the local stability of E_0 ensures the non-existence of equilibrium points E_1. Therefore, when $R_0<1$, one may expect E_0 to be globally stable which is proved by using Lyapunov's second method in form of following theorem.

Theorem 5.2 The locally stable disease-free equilibrium, $\mathrm{E}_0(1,0,0\ ,0\ )$is always globally asymptotically stable

Proof: Consider a function

$$V_1(s,s_h,i_1,i_2)=(s-s^*)^2+\frac{4(1+\mu_1)}{\mu_1^{\ 2}}(s_h-s_h^{\ *})^2+\frac{q}{(1+\mu_1)}i_1+i_2$$

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Where, the positive constant q can be chosen later.

Note: $V_1(s,i_1,i_2,i_3)=0$ only at $E_0(s^*, s_h^*, 0, 0)$ and $V_1>0$ for all points in the region B except E_0.

Computing derivative of $V_1(s,i_1,i_2,i_3)$ using system (2.2) gives

$$\frac{dV_1}{d\tau}=(s-s^*)\{1-(\mu_1+1)s-Asi_1\}+\frac{4(1+\mu_1)}{\mu_1^{\ 2}}(s_h-s_h^{\ *})(\mu_1 s-s_h)$$

$$+\frac{q}{\mu_2+\delta}\{As-(\mu_2+\delta)\}i_1+(\mu_2 i_1-(1+d_2)i_2)$$

$$=-(1+\mu_1)\left\{-(s-s^*)^2-\frac{2(s_h-s_h^{\ *})^2}{\mu_1}\right\}^2-As^2 i_1-(1+d_2)i_2$$

$$+s^*Asi_1+q\left\{\frac{A}{(\mu_2+\delta)(1+\mu_1)}-1\right\}i_1+\{\mu_2 i_1\}$$

$$\frac{dV_1}{d\tau}=-(1+\mu_1)\left\{-(s-s^*)^2-\frac{2(s_h-s_h^{\ *})^2}{\mu_1}\right\}^2-As^2 i_1-(1+d_2)i_2+\phi$$

where,

$$\phi = s^* A s i_1 + \mu_2 i_1 + q\left\{\frac{A}{(\mu_2+\delta)(1+\mu_1)} - 1\right\} i_1$$

$$\leq \left(\frac{A}{(1+\mu_1)} + \mu_2 - q(1-R_0)\right) i_1$$

Since, $R_0 < 1$ there exists a constant q such that $A + \mu_2 - q(1-R_0) < 0$. For such a value, $\frac{dV_1}{d\tau} < 0$ in the domain B.

Therefore, $\frac{dV_1}{d\tau} < 0$, for $R_0 < 1$

$V_1(s \ , i_1 \ , i_2 \ , i_3 \ )$ is a Lyapunov function.

Hence, the result is proved. □

Clearly, if $R_0 < 1$ then no other equilibrium point exists and only disease free equilibrium point exists and it remains globally stable. This means there will be no disease in the population which is most desirable situation. But suppose if this condition is violated then disease free equilibrium point becomes unstable and equilibrium point E_1 comes into the existence.

When $R_0 > 1$, the endemic equilibrium $E_1(\bar{s}, \bar{s}_h, \bar{i}_1, \bar{i}_2)$ exists.

Theorem 5.3 The endemic equilibrium $E_1(\bar{s}, \bar{s}_h, \bar{i}_1, \bar{i}_2)$, if it exists, *is locally asymptotically stable*

Proof: The characteristic equation of stability matrix at E_1 is obtained as

$$(\lambda+1)(\lambda+\delta)(\lambda^2 + a_0\lambda + a_1) = 0,$$

where, $a_0 = \frac{A}{\delta + \mu_2} > 0; a_1 = A - (1+\mu_1)(\delta+\mu_2) > 0$

It is observed that the quadratic factor gives two eigenvalues having negative real parts.

Hence, the result is proved. □

Theorem 5.4 The locally stable equilibrium point $E_1(\bar{s}, \bar{s}_h, \bar{i}_1, \bar{i}_2)$ is globally asymptotically stable.

Proof: Consider a function

$$V_1(s, s_h, i_1, i_2) = m_1(s-\bar{s})^2 + (s_h - \bar{s}_h)^2 + m_2\left(i_1 - \bar{i}_1 - \bar{i}_1 \log\frac{i_1}{\bar{i}_1}\right)$$

Note: For arbitrarily chosen positive constants m1 and m2, the function $V_1 > 0$ for all points in the region B except at $E_1(\bar{s}, \bar{i}_1, \bar{i}_2, \bar{i}_3)$ and $V_1(E_1) = 0$

Computing derivative of $V_1(s,i_1,i_2,i_3)$ using system (2.2) gives

$$\frac{dV_1}{d\tau}=m_1(s-s^*)\{1-(\mu_1+1)s-Asi_1\}+(s_h-s_h^*)(\mu_1 s-s_h)+m_2\frac{(i_1-\bar{i}_1)}{i_1}\{As-(\mu_2+\delta)\}i_1$$ Clearly,

$$1-(\mu_1+1)\bar{s}-A\bar{s}\bar{i}_1=0, \qquad 1=(\mu_1+1)\bar{s}+A\bar{s}\bar{i}_1$$

$$A\bar{s}-(\mu_2+\delta)=0, \quad \text{means} \quad A\bar{s}=(\mu_2+\delta)$$

$$\mu_1\bar{s}-\bar{s}_h=0 \qquad \mu_1\bar{s}-\bar{s}_h=0$$

$$\frac{dV_1}{d\tau}=m_1(s-\bar{s})\{-(\mu_1+1)(s-\bar{s})-A(i_1-\bar{i}_1)\bar{s}-A(s-\bar{s})\}+(s_h-\bar{s}_h)\{\mu_1(s-\bar{s})-(s_h-\bar{s}_h)\}$$
$$+m_2(i_1-\bar{i}_1)A(s-\bar{s})$$

Simplification gives

$$\frac{dV_1}{d\tau}=-m_1(s-\bar{s})^2\{(1+\mu_1+Ai_1)\}-m_1A(s-\bar{s})(i_1-\bar{i}_1)\bar{s}+\{\mu_1(s_h-\bar{s}_h)(s-\bar{s})-(s_h-\bar{s}_h)^2\}$$
$$+m_2(i_1-\bar{i}_1)A(s-\bar{s})$$

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Choosing the arbitrary constants as $m_1=\frac{\mu_1^2}{4(1+\mu_1)}$ and $m_2=\frac{\mu_1^2(\delta+\mu_2)}{4(1+\mu_1)A}$ gives

$$\frac{dV_1}{d\tau}=-\frac{\mu_1^2}{4}(s-\bar{s})^2+\mu_1(s_h-\bar{s}_h)(s-\bar{s})-(s_h-\bar{s}_h)^2-\frac{\mu_1^2A(s-\bar{s})^2i_1}{4(1+\mu_1)}$$

Or $$\frac{dV_1}{d\tau}=-\left[\frac{\mu_1}{2}(s-\bar{s})-(s_h-\bar{s}_h)\right]^2-\frac{\mu_1^2A(s-\bar{s})^2i_1}{(1+\mu_1)}\leq 0$$

Therefore, $\frac{dV_1}{d\tau}<0$ in the domain B for $R_0>1$

$V_1(s\ ,i_1\ ,i_2\ ,i_3\ )$ is a Lyapunov function.
Therefore, the result is proved. □

6. Equilibrium points and linear stability of the model with delay

Delay can play an important role in the dynamics of population. Small delay can give rise to the possibility of periodic solution. In this section the possibility of periodic solution will be discussed.

For the local stability of the equilibrium points all the eigenvalues of the stability matrix at the equilibrium point must be negative. Now it is very clear from the equation (5.3), two eigenvalues of stability matrix are negative for each equilibrium point so the stability of the equilibrium point will depends on the quadratic factor $(\lambda^2+P_1\lambda+P_0)$ .c So it is necessary to check the roots of

$$(\lambda^2+P_1\lambda+P_0)=0 \qquad (6.1)$$

The equation (6.1) can be expressed as

$$(\lambda^2 + p_1\lambda + p_0) + (n_1\lambda + n_0)e^{-\lambda\tau} = 0$$
$$\text{where, } p_1 = 1 + \mu_1 + \mu_2 + \delta,\ p_0 = (1+\mu_1)(\mu_2+\delta), \quad (6.2)$$
$$n_1 = Ae^{-\lambda\tau}(i_1 - s),\ n_0 = Ae^{-\lambda\tau}\{(\mu_2+\delta)i_1 - (1+\mu_1)s\}$$

Obviously, the $\lambda = i\omega$ is the root of the equation (6.2) if and only if ω satisfies

$$\omega^2 + p_1 i\omega + p_0 = -(n_1\lambda + n_0)(\cos\omega\tau - \sin\omega\tau)$$

Separating real and imaginary parts,

$$\omega^2 + p_0 = -n_0\cos\omega\tau - n_1\omega\sin\omega\tau$$
$$p_1\omega = -n_1\omega\cos\omega\tau - n_0\sin\omega\tau \quad (6.3)$$

Eliminating τ

$$\omega^4 + \left(p_1^2 - n_1^2 - 2p_0\right)\omega^2 + \left(p_0^2 - n_0^2\right) = 0 \quad (6.4)$$

Using $\omega^2 = \eta$

$$\eta^2 + \left(p_1^2 - n_1^2 - 2p_0\right)\eta + \left(p_0^2 - n_0^2\right) = 0 \quad (6.5)$$

Clearly equation (6.5) has all the negative root if

$$p_1^2 - n_1^2 > 2p_0 \text{ and } p_0^2 > n_0^2 \quad (6.6)$$

Now for the disease free equilibrium point

$$p_1 = 1 + \mu_1 + \mu_2 + \delta,\ p_0 = (1+\mu_1)(\mu_2+\delta)$$
$$n_1 = -\frac{Ae^{-\lambda\tau}}{(1+\mu_1)},\ n_0 = -Ae^{-\lambda\tau}$$

It can be easily checked that for the disease free equilibrium point (6.6) are satisfied when $R_0 < 1$ and hence it establishes following result.

Theorem 6.1 The disease-free equilibrium $E_0(s^*, s_h^*, 0, 0)$ is locally asymptotically stable when $R_0 < 1$.

6.1 Hopf Bifurcation

Now, for the endemic equilibrium point, the coefficients of (6.2) get the following values

$$(\lambda^2 + p_1\lambda + p_0) + (n_1\lambda + n_0)e^{-\lambda\tau} = 0$$
$$p_1 = 1 + \mu_1 + \mu_2 + \delta,\ p_0 = (1+\mu_1)(\mu_2+\delta) \quad (6.7)$$
$$n_1 = -\frac{A - 2(\mu_2+\delta)(1+\mu_1+\mu_2+\delta)}{(\mu_2+\delta)},\ n_0 = -\left(A - 2(1+\mu_1)(\mu_2+\delta)\right)$$

For the endemic equilibrium point, the equation (6.5) has unique positive root because $p_0^2 < n_0^2$. The critical value of delay is computed as

$$p_0^2 > n_0^2 \tau = \frac{1}{\omega}\cos^{-1}\left\{\frac{\omega^2(n_0 - n_1 p_1) - n_0 p_0}{n_0^2 + n_1^2\omega^2}\right\} + \frac{2k\pi}{\omega},\ k = 0,1,2,... \qquad (6.7)$$

Differentiating the equation (6.6) with respect to τ gives $(\lambda^2 + p_1\lambda + p_0) + (n_1\lambda + n_0)e^{-\lambda\tau}$

$$(2\lambda + p_1)\frac{d\lambda}{d\tau} = e^{-\lambda\tau}\left[n_1\frac{d\lambda}{d\tau} - (n_1\lambda + n_0)\left(\frac{d\lambda}{d\tau}\tau + \lambda\right)\right]$$

$$\left(\frac{d\lambda}{d\tau}\right)^{-1} = \frac{2\lambda + p_1 + n_1 e^{-\lambda\tau} - (n_1\lambda + n_0)\tau e^{-\lambda\tau}}{\lambda(n_1\lambda + n_0)e^{-\lambda\tau}}$$

Or

$$\left(\frac{d\lambda}{d\tau}\right)^{-1} = -\frac{\lambda^2 - p_0}{\lambda^2(\lambda^2 + p_1\lambda + p_0)} - \frac{n_0}{\lambda^2(n_1\lambda + n_0)} - \frac{\tau}{\lambda} \qquad (6.8)$$

By Rouche's Theorem [17]

$$\mathrm{sign}\left\{\frac{d(\mathrm{Re}\,\lambda)}{d\tau}\right\}_{\lambda = i\omega} = \mathrm{sign}\left\{\mathrm{Re}\left(\frac{d\lambda}{d\tau}\right)^{-1}\right\}_{\lambda = i\omega} = \mathrm{sign}\left\{n_1^2\omega^6 + 2n_0^2\omega^4 + \omega^2\left(n_0^2 p_1^2 - 2n_0^2 p_0 - n_1^2 p_0^2\right)\right\}$$

It is easily seen that coefficient of ω^2 is positive by substitution of the values

$$p_1 = 1 + \mu_1 + \mu_2 + \delta,\ p_0 = (1 + \mu_1)(\mu_2 + \delta),$$
$$n_1 = -\frac{A - 2(\mu_2 + \delta)(1 + \mu_1 + \mu_2 + \delta)}{(\mu_2 + \delta)},\ n_0 = -\left(A - 2(1 + \mu_1)(\mu_2 + \delta)\right)$$

The above discussion leads to the following result.

Theorem 6.2 For $E_1(\bar{s}, \bar{s}_h, \bar{i}_1, \bar{i}_2)$ the system (11) undergoes Hopf bifurcation if $R_0 > 1$.

7. Numerical Simulation

To support the analytical results, a numerical simulations are carried out using Matlab for the following set of dimensionless parameter values.

$A = 12.245;\ \mu_1 = 0.979311;\ \mu_2 = 0.7;\ d_1 = 0.32;\ d_2 = 0.25;$

3. CONCLUSION

In this paper, a non-linear mathematical model has been proposed and analyzed to study the effects of awareness on the spread of infectious diseases. It has been considered that the growth rate of awareness is proportional to the number of susceptible. It has been assumed further that awareness in the susceptible causes some susceptible to isolate themselves from infective. The awareinfectives isolate themselves from susceptible to limit the spread of infection. the disease-free equilibrium of the model is found to beglobally stable for R_0< 1. The disease-free equilibrium becomes unstable for R_0> 1, which leads to the existence of an endemic equilibrium point. The analysis demonstrates that an endemic equilibrium is locally as well as globally stable under same condition of existence. The awareness reduces the basic reproduction number and hence it is concluded that the awareness helps in reducing the spread of the disease.

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