Internally heated convection in a porous medium saturated couple-stress fluid with the effect of throughflow and variable gravity

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Abstract

Onset of internally heated convection in a porous medium saturated couple-stress fluid with the effect of throughflow and variable gravity is investigated. The governing non-dimension equations are solved using the normal modes, which leads to an eigenvalue problem for the linear stability analysis. We have discussed three cases of gravity field variation: (1). $F(z)=-z$, (2). $F(z)=-z^2$ and (3). $F(z)=-z^3$. The eigenvalue problem is solved by bvp4c in MATLAB R2020a. The critical values of the Rayleigh number are obtained for different prescribed values of the other physical parameters.

KeyWords: linear stability, porous layer, couple-stress fluid

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Introduction

Due to its application in geological and industrial problems, thermal instability in non-Newtonian fluids has been widely studied. Couple-stress fluids were a type of non-Newtonian fluid that differed from Newtonian fluids in that they were used in medical science, lubrication, pure pharmaceutical manufacturing, and other fields. The earliest studies on couple-stress fluids were studied by Stokes **Error! Reference source not found.**. Stokes **Error! Reference source not found.** discussed the behavior of couple stress fluids, and this theory can analyze the rheological behavior of a variety of complex fluids, including bio-fluids **Error! Reference source not found.**, lubrication **Error! Reference source not found.**, and human blood **Error! Reference source not found. Error! Reference source not found.**.

Hsu et al. **Error! Reference source not found.** investigated the effects of couple-stress and surface roughness on the non-Newtonian fluid. The non-linear stability results for couple-stress fluids were obtained by Sunil et al. **Error! Reference source not found.**. Shivakumara et al. **Error! Reference source not found. Error! Reference source not found.** discussed the non-linear instability theory of a couple stress fluid. The problem of thermal instability of a couple-stress fluid in a porous layer with the effect of an internal heat source was studied by Gaikwad and Kouser**Error! Reference source not found.**. Shivakumara and Naveen Kumar **Error! Reference source not found.** investigated the triple diffusive instability in a couple-stress fluid. They used the Fourier series method to study the weakly non-linear theory. The onset of instability of a couple-stress fluid in an anisotropic porous layer with a chemical reaction was investigated by Srivastava and Bera**Error! Reference source not found.**.

The purpose of the current analysis is to investigate the linear theory of internally heated instability in a porous layer saturated couple-stress fluid with the impact of variable gravity and throughflow. The organization of the present analysis is as follows: Section 2 describes the mathematical problem. The linear instability theory is focused on in Section 3 and Section 4 deals with the discussion of results. Conclusions are written in the last section.

Basic Equations

Let us consider a couple-stress fluid saturated porous layer bounded by the planes $z=0$ and $z=d$.

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The inclination angle of the layer is α with respect to the x-axis. z-axis is taken vertically upward. The vertical thermal difference along the walls is ΔT. The governing equations are[?]

$$\nabla . u = 0, \qquad (2.1)$$

$$\frac{\mu}{K} u = -\nabla p + \mu_c \nabla^2 u + \rho_0 \bar{g}(z) \beta (T - T_0) \hat{e}_z, \qquad (2.2)$$

$$\sigma \frac{\partial T}{\partial t} + (u \cdot \nabla) T = \chi \nabla^2 T + q', \qquad (2.3)$$

subject to the boundary conditions

$$u = 0, T = T_0 + \Delta T, \text{ on } z = 0,$$
$$u = 0, T = T_0, \text{ on } z = 1. \qquad (2.4)$$

Here μ, μ_c, K, p, ρ, β, g, σ_1, t, σ, χ, and q' are the dynamic viscosity, couple-stress viscosity, permeability, dynamic pressure, reference density, thermal expansion coefficient, gravity acceleration, electric conductivity, time, heat capacity ratio, thermal diffusivity and internal heat source (where $q' > 0$) respectively. And also, $\bar{g}(z) = g_0 [1 + \delta G(z)]$ is the variable gravity, g_0 is the reference gravity, $G(z)$ is the functional values for the variable gravity field, δ is the gravity variation parameter. The dimensionless quantities are given below

$$x = x^* d, \quad y = y^* d, \quad z = z^* d,$$
$$u = \frac{\chi}{d} u^*, \quad v = \frac{\chi}{d} v^*, \quad w = \frac{\chi}{d} w^*,$$
$$t = \frac{\sigma d^2}{\chi} t^*, \quad T = (T_0 + T\Delta T) T^*, \qquad (2.5)$$

as well as the non dimensional quantities

$$Ra = \frac{g \rho_0 \beta \Delta T K d}{\xi \mu}, \quad C = \frac{\mu_c \kappa}{\mu d^2}, \quad Q = \frac{q' d^2}{\chi \Delta T}. \qquad (2.6)$$

where Ra, C, and Q are the Rayleigh number, Couple-stress parameter, and Internal heat source parameter. The non-dimensional form of the governing Eqs. (2.1) - (2.3) and the corresponding boundary conditions (2.4) is given by

$$\nabla . u = 0, \qquad (2.7)$$

$$u = -\nabla p + C \nabla^2 u + RaT [1 + \delta G(z)] \hat{e}_z, \qquad (2.8)$$

$$\frac{\partial T}{\partial t} + (u . \nabla) T = \nabla^2 T + Q, \qquad (2.9)$$

subject to the boundary conditions

$$u = 0, T = 1 \text{ on } z = 0,$$
$$u = 0, T = 0 \text{ on } z = 1. \qquad (2.10)$$

Basic Flow

The basic stationary flow of Eqs. (2.7) - (2.10) is as follows:

$$u_b = Pe, \qquad (2.11)$$

$$T_b = \frac{Q + Pe}{Pe(1 - e^{Pe})} e^{Pez} - \frac{Q + Pee^{Pe}}{Pe(1 - e^{Pe})} + \frac{Qz}{Pe}. \qquad (2.12)$$

Linear Stability Analysis

The perturbation of basic state for the Eqs. (2.11)-(2.13), as

$$u = u_b + U, T = T_b + \Phi, \; p = P_b + P. \qquad (3.1)$$

By substituting Eq. (3.1) into Eqs. (2.7)-(2.10), one obtains

$$\nabla . U = 0, \qquad (3.2)$$

$$U = -\nabla P + C \nabla^2 u + Ra \Phi [1 + \delta G(z)] \hat{e}_z, \qquad (3.3)$$

$$\frac{\partial \Phi}{\partial t} + U . \nabla T_b + u_b . \nabla \Phi = \nabla^2 \Phi, \qquad (3.4)$$

$$U = 0, \Phi = 0 \text{ on } z = 0,1. \qquad (3.5)$$

Eliminating pressure by taking the third component of curl of curl Eq. (3.3), one obtains

$$\nabla^2 w - C \nabla^4 w - Ra [1 + \delta G(z)] \nabla_h^2 \Phi = 0, \qquad (3.6)$$

$$\frac{\partial \Phi}{\partial t} + w \frac{dT_b}{dz} + Pe \frac{\partial \Phi}{\partial z} = \nabla^2 \Phi, \qquad (3.7)$$

$$z = 0,1 : \; w = 0, \; \Phi = 0. \qquad (3.8)$$

now introduce the normal modes in the form of

$$(w, \Phi) = (W(z), \Phi(z)) e^{i(lx + my - \omega t)}. \qquad (3.9)$$

Where l and m are the wave number along x and y directions, $q = \sqrt{l^2 + m^2}$ is the resulting dimensionless wave number and p is the growth rate of instability. On using Eq. (3.9), Eqs. (3.6) - (3.8) become

$$\left((D^2 - q^2) - C(D^2 - q^2)^2\right) W + Ra [1 + \delta G(z)] q^2 \Phi = 0, \qquad (3.10)$$

$$\left(D^2 - q^2 - PeD\right) \Phi - W \frac{dT_b}{dz} = 0, \qquad (3.11)$$

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$$z = 0,1 : W = \Phi = 0. \quad (3.12)$$

Table 1: Comparison of Ra_c of the present theory with the results of Rionero and StraughanError! Reference source not found. for $Q=0$, $Pe=0$, and $C=0$.

	For Case A			For Case B	
δ	Present R	Rionero and Straughan **Error! Reference source not fo**	δ	Present R	Rionero and Straughan **Error! Reference source not fo**
0	39.478	39.478	0	39.478	39.478
1	77.079	77.020	0.2	41.832	41.832
1.5	132.020	132.020	0.4	44.455	44.455
1.8	189.908	189.908	0.6	47.389	47.389
1.9	212.280	212.280	0.8	50.682	50.682

Discussion

The influence of variable gravity field and throughflow on internally heated convective instability in a porous layer saturated couple-stress fluid is studied. The eigenvalue problem for linear theory is examined using the bvp4c routine in MATLAB R2020b. The effect of internal heat source parameter, Q, peclet number, Pe, couple-stress parameter, C, and gravity variation parameter, δ on critical Rayleigh number, Ra_c, and critical wave number, q_c are shown in figs. 1-7. The following three cases of gravity field variance are considered. In tables 2-4, we consider the upward throughflow($Pe>0$) and in tables 5-7, we consider the downward throughflow ($Pe<0$).

$$\begin{cases} (A).G(z) = -z, \\ (B).G(z) = -z^2, \\ (C).G(z) = -z^3. \end{cases}$$

The comparison between present results(for $Q=0$, $Pe=0$, and $C=0$) with the results of Rionero and Straughan**Error! Reference source not found.** has been provided in Table 1. This Table 1 clearly indicates a good agreement of present results with that of Rionero and Straughan**Error! Reference source not found.**.

Table 2 shows the curve of critical Ra versus δ for different values of Pe for three cases. From Table 2, it is observe that with an increase in δ, the Ra_c increases. So δ has a stabilizing effect. The critical q decreases when δ increases.

Table 2: Evaluation of Ra_c and q_c for different values of Pe and δ when $Q=2, C=1.5$.

		case 1		case 2		case 3	
Pe	δ	Ra_c	q_c	Ra_c	q_c	Ra_c	q_c
0.5	0	1071.57137	2.26093	1071.57137	2.26092	1071.57137	2.26089
	0.09	1126.01605	2.25919	1103.69459	2.25914	1092.10766	2.25944
	0.18	1186.26179	2.25733	1137.77572	2.25734	1113.42682	2.25794
	0.27	1253.28280	2.25529	1173.99664	2.25545	1135.57331	2.25638
	0.36	1328.28303	2.25300	1212.56216	2.25344	1158.59493	2.25483
	0.44	1412.76763	2.25054	1253.70362	2.25137	1182.54312	2.25323
	0.53	1508.64272	2.24784	1297.68324	2.24920	1207.47332	2.25157
	0.62	1618.35716	2.24477	1344.79937	2.24695	1233.44533	2.24987
	0.71	1745.10848	2.24141	1395.39278	2.24458	1260.52376	2.24817
	0.80	1893.14897	2.23763	1449.85432	2.24212	1288.77845	2.24641

2	0	1401.24952	2.33414	1401.24952	2.33414	1401.24952	2.33414
	0.09	1478.88269	2.33141	1449.92708	2.33125	1433.84040	2.33161
	0.18	1565.59062	2.32839	1502.07126	2.32810	1467.95396	2.32891
	0.27	1663.05672	2.32496	1558.06181	2.32476	1503.69735	2.32611
	0.36	1773.40632	2.32125	1618.33537	2.32124	1541.18781	2.32323
	0.44	1899.36108	2.31697	1683.39645	2.31755	1580.55385	2.32025
	0.53	2044.46278	2.31214	1753.83087	2.31358	1621.93663	2.31709
	0.62	2213.40511	2.30667	1830.32264	2.30934	1665.49148	2.31394
	0.71	2412.53968	2.30044	1913.67510	2.30474	1711.38972	2.31060
	0.80	2650.67165	2.29308	2004.83763	2.29996	1759.82067	2.30717

Table 3 shows the effect of Q on the system. From Table 3, we observe that with an increase in C the Ra_c increases. So C has stabilizing effect. The critical q decreases when C increases.

Table 3: Evaluation of Ra_c and q_c for different values of Pe and C when $Q = 2, \delta = 0.62.$

		case 1		case 2		case 3	
Pe	C	Ra_c	q_c	Ra_c	q_c	Ra_c	q_c
0.5	0.10	170.12574	2.47297	141.41016	2.47541	129.63484	2.47934
	0.53	618.63904	2.28515	514.48472	2.28743	471.99228	2.29024
	0.97	1065.56971	2.25737	886.22975	2.25994	813.11189	2.26275
	1.40	1512.33196	2.24679	1257.83350	2.24872	1154.10027	2.25152
	1.83	1959.04157	2.24084	1629.39312	2.24255	1495.04769	2.24535
	2.27	2405.72795	2.23687	2000.93329	2.23918	1835.97699	2.24198
	2.70	2852.40209	2.23423	2372.46315	2.23637	2176.89677	2.23918
	3.13	3299.06899	2.23224	2743.98695	2.23469	2517.81089	2.23749
	3.57	3745.73116	2.23092	3115.50682	2.23301	2858.72141	2.23581
	4.00	4192.39030	2.22960	3487.02404	2.23188	3199.62944	2.23469
2	0.10	229.35104	2.55459	189.71528	2.55729	172.47253	2.56355
	0.53	843.96765	2.34958	698.50270	2.35220	635.72923	2.35721
	0.97	1455.96506	2.32038	1205.10726	2.32291	1096.96638	2.32745
	1.40	2067.69123	2.30858	1711.48587	2.31132	1557.99484	2.31555
	1.83	2679.33304	2.30236	2217.79426	2.30451	2018.95841	2.30960
	2.27	3290.93777	2.29802	2724.07171	2.30110	2479.89348	2.30563
	2.70	3902.52281	2.29553	3230.33285	2.29838	2940.81348	2.30232
	3.13	4514.09632	2.29367	3736.58430	2.29633	3401.72454	2.30034

	3.57	5125.66241	2.29180	4242.82956	2.29429	3862.62984	2.29902
	4.00	5737.22344	2.29056	4749.07061	2.29361	4323.53133	2.29770

The influence of Q for fixed values of Pe and δ is shown in Table 4. It shows that Ra_c increases with an increase in Q, therefore the system become stable with an increase in Q. The critical q increases when C increases. Finally, we also observe from all these tables that the system become more stable in case A ($G(z) = -z$) and less stable in case C($G(z) = -z^3$).

Table 4: Evaluation of Ra_c and q_c for different values of Pe and Q when $\delta = 0.62, C = 1.5.$

		case 1		case 2		case 3	
Pe	Q	Ra_c	q_c	Ra_c	q_c	Ra_c	q_c
0.5	0.00	1526.35416	2.24730	1275.07908	2.24771	1176.54099	2.24744
	0.67	1556.92535	2.24522	1298.72335	2.24570	1196.15648	2.24679
	1.33	1586.70334	2.24418	1321.62499	2.24570	1214.96091	2.24777
	2.00	1615.42190	2.24522	1343.58062	2.24721	1232.78283	2.25005
	2.67	1642.80190	2.24730	1364.38011	2.24972	1249.44926	2.25363
	3.33	1668.55856	2.25147	1383.81200	2.25475	1264.79034	2.25917
	4.00	1692.40934	2.25773	1401.66970	2.26128	1278.64456	2.26667
	4.67	1714.08364	2.26606	1417.75828	2.26932	1290.86428	2.27579
	5.33	1733.33285	2.27544	1431.90178	2.27987	1301.32129	2.28654
	6.00	1749.94123	2.28795	1443.95037	2.29193	1309.91190	2.29925
2	0.00	1861.95313	2.28314	1548.21314	2.28487	1419.30881	2.28538
	0.67	1969.09851	2.28897	1634.94344	2.29101	1495.59763	2.29151
	1.33	2084.77147	2.29480	1728.35461	2.29714	1577.34850	2.30070
	2.00	2208.84434	2.30646	1828.33035	2.30940	1664.37525	2.31296
	2.67	2340.63478	2.32394	1934.33070	2.32472	1756.13048	2.33136
	3.33	2478.66510	2.34726	2045.21280	2.34925	1851.57621	2.35588
	4.00	2620.43011	2.37932	2159.06832	2.37990	1949.05644	2.38653
	4.67	2762.26115	2.42013	2273.11179	2.41975	2046.23668	2.42638
	5.33	2899.44920	2.46967	2383.76118	2.47186	2140.18053	2.47543
	6.00	3026.74654	2.53088	2486.98402	2.53010	2227.63085	2.53367

Table 5 shows the variation of critical Ra versus δ for $Pe = -0.5, -2$ and for three cases. From Table 5, it is observe that with an increase in δ, the Ra_c increases. So δ has a stabilizing effect. The critical q decreases when δ increases.

Table 5: Evaluation of Ra_c and q_c for different values of Pe and δ when $Q = 2, C = 1.5$.

		case 1		case 2		case 3	
Pe	δ	Ra_c	q_c	Ra_c	q_c	Ra_c	q_c
-0.5	0	1004.27848	2.24900	1004.27848	2.24912	1004.27848	2.24914
	0.09	1052.63829	2.24780	1031.74129	2.24792	1021.32836	2.24810
	0.18	1105.86410	2.24659	1060.72335	2.24671	1038.95040	2.24731
	0.27	1164.72453	2.24539	1091.35207	2.24571	1057.17302	2.24627
	0.36	1230.15790	2.24379	1123.76923	2.24431	1076.02645	2.24523
	0.44	1303.32140	2.24259	1158.13304	2.24311	1095.54294	2.24445
	0.53	1385.65831	2.24058	1194.62053	2.24190	1115.75689	2.24341
	0.62	1478.99123	2.23898	1233.43034	2.24050	1136.70502	2.24236
	0.71	1585.65400	2.23697	1274.78606	2.23910	1158.42659	2.24132
	0.80	1708.68173	2.23497	1318.94015	2.23770	1180.96356	2.24028
-2	0	1077.91714	2.27721	1077.91714	2.27701	1077.91714	2.27701
	0.09	1125.98375	2.27657	1103.74650	2.27675	1093.28471	2.27675
	0.18	1178.50556	2.27625	1130.81783	2.27623	1109.08056	2.27649
	0.27	1236.12700	2.27593	1159.22086	2.27597	1125.32211	2.27623
	0.36	1299.62189	2.27529	1189.05392	2.27545	1142.02768	2.27571
	0.44	1369.92740	2.27497	1220.42500	2.27519	1159.21660	2.27545
	0.53	1448.18887	2.27465	1253.45297	2.27493	1176.90919	2.27519
	0.62	1535.82032	2.27401	1288.26882	2.27441	1195.12690	2.27493
	0.71	1634.58694	2.27369	1325.01732	2.27415	1213.89232	2.27467
	0.80	1746.71920	2.27369	1363.85872	2.27389	1233.22929	2.27441

Table 6 shows the effect of Q on the system. From Table 6, we observe that with an increase in C the Ra_c increases. So C has stabilizing effect. The critical q decreases when C increases.

Table 6: Evaluation of Ra_c and q_c for different values of Pe and C when $Q = 2, \delta = 0.62$.

		case 1		case 2		case 3	
Pe	C	Ra_c	q_c	Ra_c	q_c	Ra_c	q_c
-0.5	0.10	155.84197	2.46296	130.02552	2.46523	119.80525	2.46874
	0.53	565.63226	2.27874	472.09776	2.27950	435.18745	2.28251
	0.97	974.04057	2.25131	813.00806	2.25296	749.48855	2.25518
	1.40	1382.30084	2.24151	1153.79409	2.24136	1063.67418	2.24492
	1.83	1790.51454	2.23563	1494.54076	2.23638	1377.82321	2.23809
	2.27	2198.70755	2.23171	1835.27056	2.23307	1691.95651	2.23467
	2.70	2606.89001	2.22779	2175.99108	2.22975	2006.08155	2.23126
	3.13	3015.06611	2.22583	2516.70625	2.22809	2320.20132	2.22955

	3.57	3423.23752	2.22583	2857.41809	2.22643	2634.31825	2.22784
	4.00	3831.40634	2.22387	3198.12785	2.22643	2948.43228	2.22784
-2	0.10	161.13648	2.50553	135.27301	2.50548	125.52523	2.50724
	0.53	587.00492	2.31392	492.78884	2.31553	457.29591	2.31573
	0.97	1011.38805	2.28714	849.05952	2.28739	787.91174	2.28784
	1.40	1435.61298	2.27683	1205.19721	2.27683	1118.40407	2.27668
	1.83	1859.78790	2.27065	1561.29307	2.26980	1448.85786	2.27111
	2.27	2283.94073	2.26653	1917.37059	2.26628	1779.29458	2.26739
	2.70	2708.08264	2.26241	2273.43881	2.26452	2109.72229	2.26367
	3.13	3132.21728	2.26241	2629.50105	2.26101	2440.14461	2.26181
	3.57	3556.34685	2.26035	2985.55924	2.26101	2770.56389	2.26181
	4.00	3980.47436	2.25829	3341.61487	2.25925	3100.97990	2.25995

The influence of Q for fixed values of Q and δ is shown in 7. It shows that Ra_c increases with an increase in Q, therefore the system become stable with an increase in Q. The critical q decreases when C increases. Finally, we also observe from all these tables that the system become more stable in case A ($G(z)=-z$) and less stable in case C($G(z)=-z^3$).

Table 7: Evaluation of Ra_c and q_c for different values of Pe and Q when $\delta=0.62, C=1.5$.

		case 1		case 2		case 3	
Pe	Q	Ra_c	q_c	Ra_c	q_c	Ra_c	q_c
-0.5	0.00	1492.15519	2.25302	1251.18519	2.25131	1159.64626	2.25010
	0.67	1488.56517	2.24698	1246.21520	2.24648	1152.94276	2.24648
	1.33	1483.32525	2.24296	1239.93897	2.24286	1145.09033	2.24407
	2.00	1476.50642	2.23894	1232.43006	2.24045	1136.17213	2.24286
	2.67	1468.19565	2.23693	1223.77152	2.23925	1126.27768	2.24166
	3.33	1458.49038	2.23693	1214.05361	2.23925	1115.50085	2.24286
	4.00	1447.49716	2.23693	1203.37148	2.24045	1103.93775	2.24407
	4.67	1435.32889	2.23894	1191.82308	2.24286	1091.68407	2.24769
	5.33	1422.10179	2.24095	1179.50687	2.24528	1078.83455	2.25131
	6.00	1407.93330	2.24497	1166.52062	2.24889	1065.48052	2.25492
-2	0.00	1699.33139	2.30422	1434.38299	2.30141	1338.70388	2.29955
	0.67	1642.25329	2.29137	1383.46218	2.28985	1288.53288	2.28869
	1.33	1586.89574	2.28173	1334.40706	2.28176	1240.48436	2.28146
	2.00	1533.50209	2.27369	1287.37554	2.27482	1194.66493	2.27543
	2.67	1482.22080	2.26727	1242.44673	2.26905	1151.10811	2.27060
	3.33	1433.12681	2.26245	1199.64124	2.26558	1109.79476	2.26698
	4.00	1386.23984	2.25924	1158.93654	2.26211	1070.66895	2.26457
	4.67	1341.53966	2.25763	1120.28085	2.25980	1033.65054	2.26337

	5.33	1298.97592	2.25442	1083.60200	2.25864	998.64437	2.26216
	6.00	1258.47978	2.25442	1048.81514	2.25749	965.54705	2.26216

Conclusions

The onset of magnetoconvection in a porous layer with variable gravity have performed numerically using linear theory. We have analysed four cases of gravity field variation namely, (A) $G(z) = -z$, (B) $G(z) = -z^2$, and (C) $G(z) = -z^3$, and conclusions are listed below:

Couple-stress parameter, C, and variable gravity, δ delay the onset of convection for both upward and downward throughflow.

Internal heat source parameter, Q, has stabilizing effect for upward throughflow and destabilizing effect for downward throughflow on the system.

Convection cell's size increases with increase in C and δ for both upward and downward throughflow.

Convection cell's size decreases with increase in Q upward throughflow ansd increases for downward throughflow.

The system become more stable for linear variation, and less stable for cubic variation.

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