MIXED VARIATION PROBLEMATIC FOR A GENERALISED DARCY–FORCHHEIMER PERFECT PROMPTED BY HYDRAULIC FRACTURE

Hareesh B
Lecturer in mechanical engineering
Carmel polytechnic college
Alappuzha
Kerala - 688004
Mail id: hareeshpanicker70@gmail.com

Abstract

Hydraulic fracturing leads to a stationary flow in porous media, hence a model that takes inertial phenomena into account is examined. In order to simulate the incompressible fluid in a fluid-driven fracture, a nonlinear Darcy-Forchheimer (DF) equation is used with mixed boundary conditions. With the inclusion of a growth exponent m and inhomogeneous coefficients, the traditional DF equation is generalised. The well-posedness theorem is established for arbitrary $m > 1$ by using a mixed variational preparation of the problem with uncertain fluid velocity and fluid pressure. Optimal fracture shape design is aided by the proposed Lagrange multiplier formalism.

Keywords: Phase-field theory, Hydraulic fracture propagation, Non-Darcy flow, Nonlinear deformations

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1. Introduction

Sustainable project of hydraulic fracturing operations1, which have many different applications including enhanced oil and gas recovery2, geothermal energy carbon dioxide storage, and block cave mining, relies on accurate modelling of the interaction between solid and fluid bulk phases in porous media. Early efforts to characterise fluid-driven fracture propagation were restricted to creating analytical models for impermeable elastic media.3, 4, 5 More sophisticated mathematical models were created to illustrate the interplay between the fluid flow and solid deformation in the fracturing zone and crack-tip as interest in the hydraulic fracturing method grew.6 The fluid density, viscosity, and compressibility, as well as the flow features (such as the leak-off of the pressurised fluid into the porous rock and pore flow velocity), were shown to have a significant influence in the creation and evolution of fractures in laboratory studies.7

Hydraulic fracturing, often referred to as "fracking," is a process used in the oil and gas industry to extract hydrocarbons such as natural gas and oil from deep underground rock formations. Here's an overview of hydraulic fracturing:

Process:

1. ****Well Drilling:**** A well is drilled deep into the Earth's crust to reach the target rock formation. These wells can extend horizontally for long distances.

2. **Injection of Fluids:** A high-pressure fluid, typically a mixture of water, sand, and various chemicals, is injected into the well. This fluid is known as "fracturing fluid."
3. **Fracturing the Rock:** The high-pressure fluid causes the rock to crack or fracture. This creates pathways for hydrocarbons to flow more freely to the well.
4. **Recovery of Hydrocarbons:** As the rock fractures, the hydrocarbons are released and can flow back into the well.
****Key Components:****
- **Fracturing Fluid:** The fluid used in hydraulic fracturing is primarily water, but it also contains proppants (usually sand) to hold open the fractures and various chemicals to improve the efficiency of the process.

- ****Proppants:**** These are solid materials, usually sand or ceramic beads, that are mixed with the fracturing fluid. They help keep the fractures open, allowing gas or oil to flow more easily.
- **Well Casing:** The well is encased in cement and steel to prevent the fracturing fluid from leaking into surrounding groundwater.
****Benefits:****
- **Increased Resource Recovery:** Hydraulic fracturing allows access to oil and gas reserves that were previously inaccessible.

- **Energy Independence:** It has contributed to increased domestic energy production in some countries, reducing dependence on foreign oil and gas.
****Concerns and Risks:****
- **Environmental Concerns:** There are concerns about water contamination, air pollution, and the release of greenhouse gases during the fracking process.
- ****Water Usage:**** Fracking requires large amounts of water, which can strain local water supplies.
- **Induced Seismicity:** Injection of fluids can sometimes trigger small earthquakes.

- **Chemical Use:** The chemicals used in fracking fluid have raised concerns about their impact on the environment and human health.

It's worth noting that hydraulic fracturing is a highly debated and regulated process, and its impacts on the environment and public health are subjects of ongoing research and discussion. Regulations and best practices are in place to mitigate potential risks.
Hydraulic fracture propagation in porous media may be simulated using a number of different numerical approaches. The remeshing methods, and continuum damage modelling have all been used to track crack propagation within the framework of the finite element method.8 For describing displacement jumps and the interaction between fractures, the XFEM offers a solid foundation upon which enrichment functions may be applied to finite elements.9 The correct choice of enrichment functions makes it difficult to couple hydraulic processes to mechanical deformation and cracking, and different governing equations may be needed to model flow inside and outside the fracture,10 which may lead to inconsistency in predicting effective stresses in the solid skeleton. Since poroelastic media are typically ignored in favour of more impermeable ones, hydraulic cracks are typically only considered in discrete-based approaches.8 Nonetheless, they exhibit a high capacity in simulating big deformations because of cracking and capturing the fluid flow inside the discontinuities, and they provide excellent computing efficiency, especially in meshfree techniques.11 Continuum damage theory proposes a useful method for solving complex coupled PDEs for Multiphysics-multidomain processes, since it provides a mathematical context that may be addressed on a unified finite element mesh by pilfering from variational theory. To take use of the strengths of several numerical approaches, hybrid approaches have been created. Examples include using discrete-based approaches of fluid throughout the porous rock and inside the fracture channels.
In the context of linear and hyperelasticity, the phase-field technique has been effectively employed to describe fluid-driven fracture propagation in porous media. and the exchange between these two zones are reliably captured by the phase-field

technique, which models hydro-mechanical processes and fracture propagation across a continuous finite element mesh. When simulating fluid-driven fractures with complicated geometry and the possibility of material heterogeneities, the phase-field technique can take use of the flexibility of the variational approach.20,21 Due to the energy-minimization nature of this method, Neural Networks can now be used to predict as a result of fracture development.

2. Methodology

To simulate a fluid-driven fracture, the total potential porous combination is minimised using the variational principle. The mathematical steps required to get the relevant governing equations are outlined here. You may have observed that the matrix and vector quantities are denoted in boldface whereas the scalar ones are not.To illustrate the development of cracks in response to external loads, fluid injection, and deformations, we assume a physical porous saturated outside the scope of this research.

Phase-field modelling of media at finite strains

where is the bulk modulus of the solid components (i.e. grains) and is the drained bulk modulus of the matrix. Biot's coefficient can be measured using a number of different laboratory techniques. 43. In this research, the behaviour of the solid skeleton is modelled using an isotropic hyperelastic model, which takes into account geometric nonlinearities. The decision between a compressible and nearly incompressible form of the model is based on the volumetric response of the solid skeleton when subjected to the effective stresses. Since the solid constituent's volumetric strain cannot be ignored in saturated porous rock materials,42 the bulk modulus of the solid cannot be set to infinity. The physical behaviour of most reservoir rocks may be described as being similar to that of a compressible hyperelastic material (Ref. 28), where the range of variation is between 0.5 and 0.8. Although volumetric strain is small in almost incompressible materials (= 1), the solid's volumetric stress must be accounted for in order to prevent strain locking.

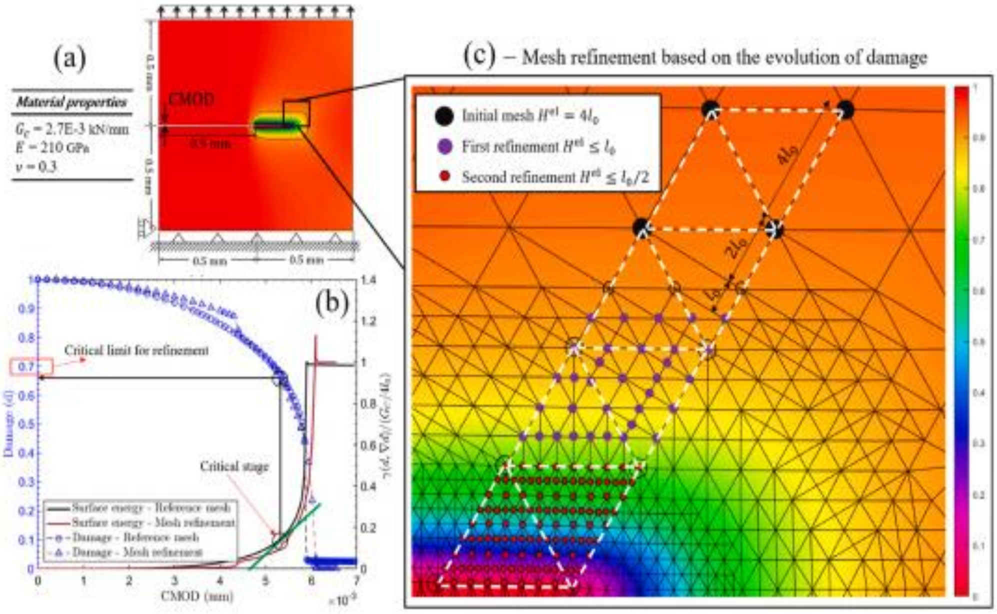

Fig. 1. (a) Is the boundary value problem presented in Ref. 36 for modelling simple fracture propagation from an initial notch; (b) Demonstrates the change in refinement; and (c) Shows a schematic of the node and element size arrangement close to the phase-field crack-tip.

Numerical inaccuracies pressures are generated by stress singularities brought on by fracture propagation, and here is where the second refinement criteria comes into play. The computational mesh changes as the deformations do in a modernised Lagrangian formulation, which can lead to analytical instabilities since the Jacobian () of some components might become negative.54 In order to ensure the reliability of the analysis, the created mesh refinement method must be able to identify the important regions where the mesh quality is inadequate and modify those regions appropriately. In the iterative solution procedure, the local error estimator is used to pinpoint the crucial nodes. Here, we incorporate a second Piola-Kirchhoff strain as independent variables on which the error estimator relies.

conclusion

Corollary 1 is significant because it shows that a mixed laminar-turbulent model may be described by the inhomogeneous Forchheimer coefficient 0 (x) . For large enough x, Darcy's law describes laminar flow, such as when the reservoir is far from the fracture and (x) = 0. Darcy-Forchheimer's law describes the turbulent flow in the complement x f, when (x) > 0 (such as at the fracture). The nonlinear during fracture propagation is taken into account in the suggested formulation, which takes into account the influence of the temporal development of porous media characteristics such as porosity, compressibility, permeability, and mechanical stiffness. Forchheimer's equation is used to account for the impact of gravity and the fracture's nonlinear flow inside the crack. By simulating reference instances, the robustness of the modelling framework may be tested. We discuss how the compressibility of the solid skeleton and the drained bulk modulus, two poroelastic features of porous media, affect the hydro-mechanical and cracking behaviour of porous rocks, as well as the overall energy of the system. Predicting the fracking process's productivity in engineering applications relies heavily on the nonlinearity of fluid flow, which is found to affect zone.

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