Mathematical Modelling Of Mucus Transport In Diseased Airways: Effect Of Constriction With Mucus And Serous Fluid Viscosities

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Abstract:

This paper modelled the flow of air, mucus, and serous fluid in a circular tube under a time dependent pressure gradient and mucus transport in a diseased airway (bronchial constriction). It is considered that airways are tubes and flow inside moist air, mucus, and serous fluid flow under the quasi-steady-state laminar condition caused by coughing. The analysis and approximate result show that mucus transport in a constricted airway decreases as the diameter changes and that air flow rate is affected by mucus transport as mucus and serous viscosity increase.

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1. INTRODUCTION

The normal mechanics of mucus transport in the airways are constantly swept from the upper to lower airways to remove mucus. In cases of airway hyperresponsiveness (Asthma), airway inflammation (the production of antigens and cells, enlargement of cells, and fibrosis) can alter these normal mechanics, leading to an increase in mucus production and an altered airway connection with the supporting tissue. Chronic bronchitis, cystic fibrosis, and other lung diseases cause much mucus to be made. This mucus is then expelled through coughing or forced breathing. [1-3].

Several experiments have been conducted to study how mucus transport works in airways during a cough. [4-8]. In particular, Clarke et al. [4] have shown that the resistance to air flow through a liquid lined tube is markedly increased at all flow rates compared to a dry tube. Scherer and Burtz [6], Scherer [7] have studied fluid flow in the context of coughing, using air and liquid blown out of a straight tube by turbulent air jets. They found that the liquid transport efficiency increases as the turbulence level increases and decreases as the viscosity of the liquid increases. Kim et al.[8]found that the elasticity of mucus does not affect its transport in vertical tubes by two phase flow mechanism. Several other investigations of cough machines have been conducted under turbulent flow conditions to simulate mucus transport in the trachea. [9-13].A study by Agarwal et al. [14] has found that mucus transport increases in a simulated cough machine when serous fluid is present. This is because a narrowing of the lumen of the bronchi, restricting airflow to and from the lungs, results. Bronchoconstriction can also be due to an accumulation of thick mucus. Kumar et al. [15] investigated how mucus transport decreases as airway diameter increases and also showed that air flow rate is affected by mucus transport and mucus viscosity in the constricted airway. Chitra and Shabana [16] Two-layered model of the air-mucus interface through constricted human

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mathematical modelling of mucus transport in airways due to cough with quasi - steady state turbulent condition.

It can be noted here that more effort needs to be made to study mucus transport in narrowed airway diameters using a mathematical model.Therefore, this article proposes a three-layer laminar flow model to study mucus transport in the circular airways by considering the simultaneous flow of air and mucus in a tube simulating the transport of mucus in the airways due to coughing under the following assumptions [18]:

- The fluid flow is symmetrical about the central axis.
- The applied pressure gradient is assumed to be a time dependent function representing cough.
- Mucus is assumed to behave as an incompressible Newtonian fluid due to a high shear rate during cough [12].
- Since air is saturated with watery liquid during cough, it is also assumed to behave as an incompressible Newtonian fluid in the lung during cough.
- Air flow is turbulent and is quasi-steady during cough [6, 7].

2. MATHEMATICAL MODEL

We consider the simultaneous and co-axial flow of moist air, mucus, and serous fluid in a tube representing mucus transport in a circular airway. The mucus behaves like a Newtonian fluid and the flow is assumed to be steady, laminar, and axisymmetric.The flow geometry of air, mucus, and serous fluids through the constricted airway has axial symmetry. In diseased airways, the smooth muscles tighten and constrict in the serous layer, which is attached to the wall, and the flow is caused by air motion during coughing. The flow geometry of the air, mucus, and serous is shown in figure 1.

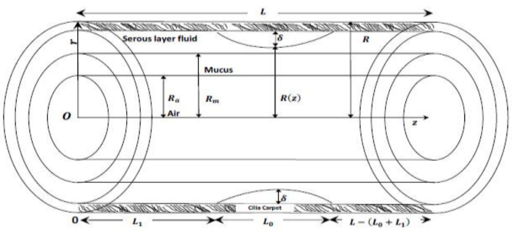

Figure 1: Tube model for mucus transport in the airway

The radius of the circular tube depends upon the geometry of constriction and can be written as follow [19,20]:

$$\frac{R_s(z)}{R_s} = \begin{cases} 1-\frac{\delta}{2R_s}\left\{1+\cos\frac{2\pi}{L_0}\left(z-L_1-\frac{L_0}{2}\right)\right\}, & L_1 \le z \le L_1+L_0 \\ 1, & 0 \le z \le L_1 \text{ and } L_1+L_0 \le z \le L \end{cases} \quad (1)$$

The equation governing the quasi-steady flow of air, mucus and serous layer in a circular tube can be written as follows:

Region I: Quasi steady laminar flow of air($0 \le r \le R_a$):

$$-\frac{\partial p}{\partial z}+\frac{1}{r}\frac{\partial(r\tau_a)}{\partial r}=0 \quad (2)$$

$$\tau_a=\mu_a\left(-\frac{\partial u_a}{\partial r}\right) \quad (3)$$

Region II: Quasi steady laminar flow of mucus($R_a \le r \le R_m$):

$$-\frac{\partial p}{\partial z}+\frac{1}{r}\frac{\partial(r\tau_m)}{\partial r}=0 \quad (4)$$

$$\tau_m=\mu_m\left(\frac{\partial u_m}{\partial r}\right) \quad (5)$$

Region III: Quasi steady laminar flow of serous fluid($R_m \le r \le R_s(z)$):

$$-\frac{\partial p}{\partial z}+\frac{1}{r}\frac{\partial(r\tau_s)}{\partial r}=0 \quad (6)$$

$$\tau_s=\mu_s\left(\frac{\partial u_s}{\partial r}\right) \quad (7)$$

Now, above equations(1)-(7) describe $R_s(z)$ is the radius of circular tube in the constriction region, δ constriction size, and L_0 length of the constriction, L_1 is location of the constriction effect, z is flow direction, r is radial direction fluid flow, R_a is the thickness up to air-mucus interface and u_a, u_m and u_s are the mean velocity components of air, mucus and serous fluid in the z direction respectively, τ_a, τ_m and τ_s are the mean shear stress in the air, mucus and serous fluid layers respectively, μ_a, μ_m and μ_s are viscosities of air, mucus and serous fluid repsectively.

2.1 INITIAL CONDITION

$$u_a=u_m=us=0 \quad and \quad \tau_a=\tau_m=\tau_s=0, \quad \frac{\partial u_m}{\partial r}=0 \quad at \quad t=0 \quad (8)$$

2.2 BOUNDARY CONDITION

$$\frac{\partial u_a}{\partial r}=0 \quad at \quad r=0 \quad (9)$$

$$u_s=0 \quad at \quad r=R_s(z) \quad (10)$$

2.3 MATCHING CONDITION

$$u_a = u_m, \tau_a = \tau_m, \quad at \quad r = R_a \quad (11)$$
$$u_m = u_s, \tau_m = \tau_s, \quad at \quad r = R_m \quad (12)$$

Equations (11) and (12) represents the continuity of the velocity and stress components at the two interfaces. Since during mild coughing or forced expiration the pressure gradient in the lung is time dependent and p is the mean pressure which is constant across two layers, therefore we assume that:

$$-\frac{\partial p}{\partial z} = P = P_0 f(t) \quad (13)$$

where t is time, P_0 is a constant, the magnitude of which depends upon the intensity of prolonged cough and this increases the flow rates increases proportionally. The function $f(t)$ in equation (13) is assumed to be given by:

$$f(t) = \begin{cases} \frac{27}{4T^3} t(T-t)^2, & 0 \le t \le T \\ 0, & t > T \end{cases} \quad (14)$$

where T is the duration of cough and T=0.03 sec.

3 ANALYSIS OF MODEL

3.1 METHOD OF SOLUTION

To see the effect of mucus and serous fluid viscosities we solve the equatins (3.2)-(3.7) using the initial condition (3.8), boundary conditions (3.9), (3.10) and matching conditions (3.11), (3.12).The expressions for the stress and velocity components in each layer can be found as the following:

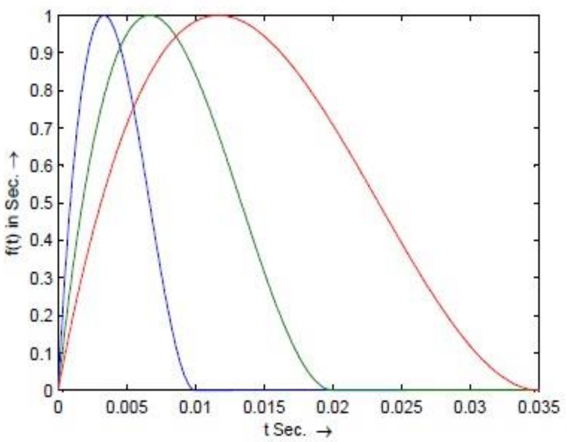

Figure 2: Graphical Representation of $f(t)$for various value of t

$$\tau_a = -\frac{Pr}{2} \quad (15)$$
$$\tau_m = -\frac{Pr}{2} \quad (16)$$
$$\tau_s = -\frac{Pr}{2} \quad (17)$$
$$u_a = -\frac{P}{4\mu_a}(R_a^2 - r^2) + \frac{P}{4\mu_m}(R_m^2 - R_a^2) + \frac{P}{4\mu_s}(R_s(z)^2 - R_m^2) \quad (18)$$
$$u_m = \frac{P}{4\mu_m}(R_m^2 - r^2) + \frac{P}{4\mu_s}(R_s(z)^2 - R_m^2) \quad (19)$$
$$u_s = \frac{P}{4\mu_s}(R_s(z)^2 - r^2) \quad (20)$$

The volumetric flow rates in each layer can be defined as:

$$Q_a = \int_0^{R_a} 2\pi r u_a \, dr; \quad Q_m = \int_{R_a}^{R_m} 2\pi r u_m \, dr; \quad Q_s = \int_{R_m}^{R_s(z)} 2\pi r u_s \, dr \quad (21)$$

Which after using equations (3.18),(3.19) and (3.20) can be written as:

$$\frac{Q_a}{2\pi} = -\frac{PR_a^4}{16\mu_a} + \frac{PR_a^2}{8\mu_m}(R_m^2 - R_a^2) + \frac{PR_a^2}{8\mu_s}(R_s(z)^2 - R_m^2) \quad (22)$$
$$\frac{Q_m}{2\pi} = \frac{P(R_m^2 - R_a^2)^2}{16\mu_m} + \frac{P(R_m^2 - R_a^2)}{8\mu_s}(R_s(z)^2 - R_m^2) \quad (23)$$
$$\frac{Q_s}{2\pi} = \frac{P(R_s(z)^2 - R_m^2)^2}{16\mu_s} \quad (24)$$

Now we will find the pressure drop in each layer. We know, from equation of continuity Q_a, Q_m and $Q_s$are constants. Therefore, the pressure gradient can be obtained from equations (22),(23) and (24) as:

$$-\frac{\partial p}{\partial z} = \frac{Q_a}{2\pi A(R_s(z)^2 - B)} \quad (25)$$
$$-\frac{\partial p}{\partial z} = \frac{Q_m}{2\pi A_1(R_s(z)^2 - B_1)} \quad (26)$$
$$-\frac{\partial p}{\partial z} = \frac{Q_s}{2\pi A_2(R_s(z)^2 - R_a^2)^2} \quad (27)$$

where,

$$A = \frac{R_a^2}{8\mu_s}, \quad B = \left(R_m^2\left\{1 - \frac{\mu_s}{\mu_m}\right\} + \mu_s R_a^2\left\{\frac{1}{\mu_m} + \frac{1}{2\mu_a}\right\}\right);$$
$$A_1 = \frac{(R_m^2 - R_a^2)}{8\mu_s}, B_1 = \left(R_m^2 - \frac{(R_m^2 - R_a^2)\mu_s}{2\mu_m}\right); A_2 = \frac{1}{8\mu_s};$$

Integrating equations (25), (26) and (27), we get:

$$-\int_0^z dp = \int_0^z \frac{Q_a}{2\pi A(R_s(z)^2 - B)} dz \quad (28)$$
$$-\int_0^z dp = \int_0^z \frac{Q_m}{2\pi A_1(R_s(z)^2 - B_1)} dz \quad (29)$$
$$-\int_0^z dp = \int_0^z \frac{Q_s}{2\pi A_2(R_s(z)^2 - R_m^2)^2} dz \quad (30)$$

Boundary conditions for pressure are:

$$p = p_{L_1} \quad \text{at} \quad z = L_1$$
$$p = p_{L_1+L_0} \text{ at} z = L_1 + L_0$$

Solving equation(28) with above pressure conditions, we get Q_a:

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$$Q_a = \frac{\Delta P}{L_0} \frac{2\pi A}{\left[a(a^2-b^2)^{-3/2} + Ba(a^2-b^2)^{-7/2}(a^2+\frac{3}{2}b^2)\right]}$$

where $a = \left(R_s - \frac{\delta}{2}\right)$ and $b = \left(\frac{\delta}{2}\right)$

Similarly , solving equation (29) with above pressure conditions, we get Q_m:

$$Q_m = \frac{\Delta P}{L_0} \frac{2\pi A_1}{\left[a(a^2-b^2)^{-3/2} + B_1 a(a^2-b^2)^{-7/2}(a^2+\frac{3}{2}b^2)\right]}$$

Similarly , solving equation (30) with above pressure conditions, we get Q_s:

$$Q_s = \frac{\Delta P}{L_0} \frac{2\pi A_1}{\left[a(a^2-b^2)^{-7/2}(a^2+\frac{3}{2}b^2) + X\right]}$$

where $X = 2L_0 R_a^2 (189a^5(a^2-b^2)^{-11/2}) - 210L_0 a^3 (a^2-b^2)^{-9/2} + 45L_0 a(a^2-b^2)^{-7/2}$

4 RESULTS AND DISCUSSION

To study the effect of various parameters on serous fluid, mucus transport and air flow rate quantitatively apply the model analysis to the airways the expressions for Q_s , Q_m and Q_a have been calculated and plotted in Figures by using the following set of parameters[21].

$T = 0.03\ sec$
$L_0 = 0.40$ cm
$R_s = 41.45 \times 10^{-2} cm$
$R_a = 31.45 \times 10^{-2} cm$
$\mu_m = 1.00 - 10.00\ poise$
$\mu_a = 0.0002\ poise$
$t = 0 - 0.035\ sec$
$\rho_a = 1.00 \times 10^{-3} gm/cm^3$
$R_m = 38.45 \times 10^{-2} cm$
$\mu_s = 0.01 - 0.10\ poise$
$p_0 = 1 \times 10^{-5} gm\ cm^{-2} sec^{-2}$

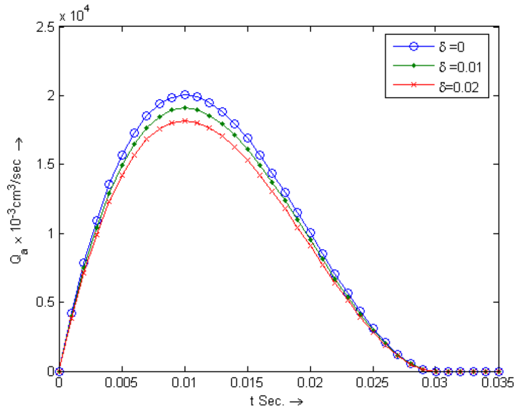

Figure 3: Variation of Q_a with t for different values of δ (for fixed air viscosity = 0.0002 poise and mucus viscosity =1poise.)

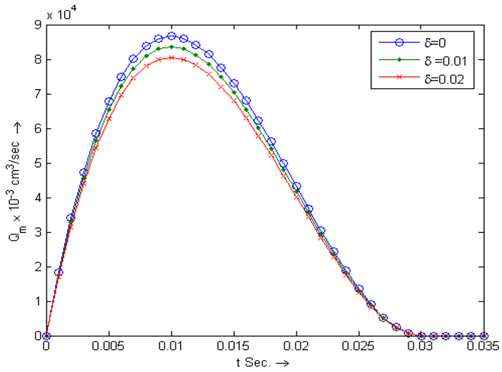

Figure 4: Variation of Q_m with t for different values of δ (for fixed air viscosity = 0.0002 poise and mucus viscosity = 1poise.)

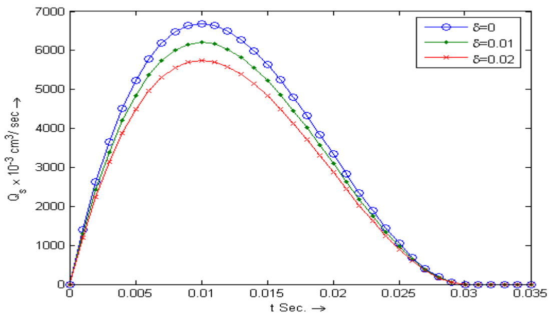

Figure 5: Variation of Q_s with t for different values of δ (for fixed air viscosity = 0.0002 poise and mucus viscosity = 1poise.)

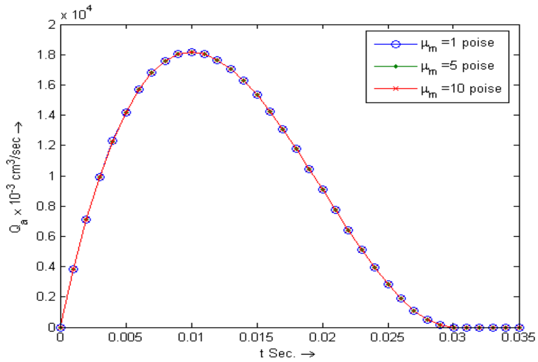

Figure 6: Variation of Q_a with t for different values of μ_m (for fixed air viscosity = 0.0002 poise and fixed value of δ = 0.2 mm)

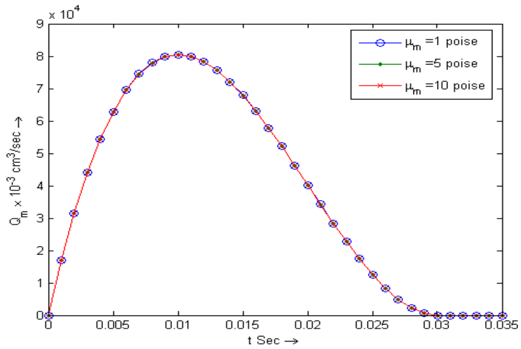

Figure 7: Variation of Q_m with t for different values of μ_m (for fixed air viscosity = 0.0002 poise and fixed value of δ = 0.2 mm)

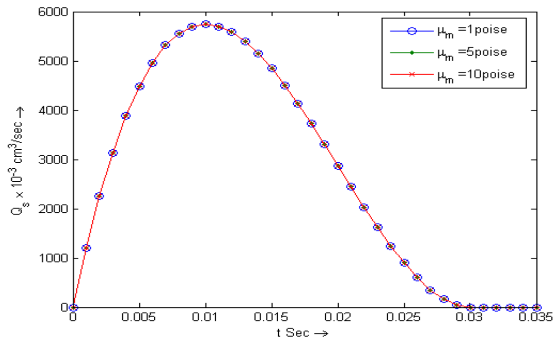

Figure 8: Variation of Q_s with t for different values of μ_m (for fixed air viscosity = 0.0002 poise and fixed value of δ = 0.2 mm)

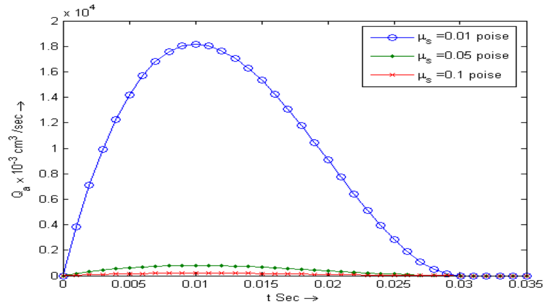

Figure 9: Variation of Q_a with t for different values of μ_s (for fixed air viscosity = 0.0002 poise and fixed value of δ = 0.2 mm)

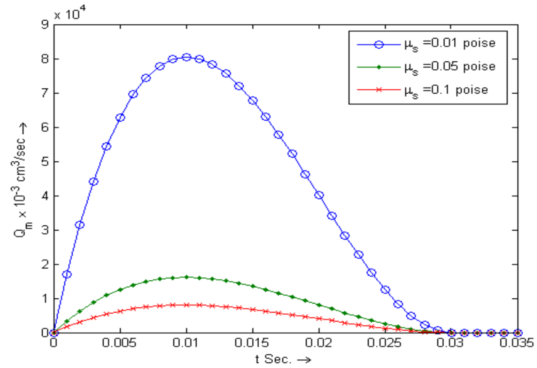

Figure 10: Variation of Q_m with t for different values of μ_s (for fixed air viscosity = 0.0002 poise and fixed value of δ = 0.2 mm)

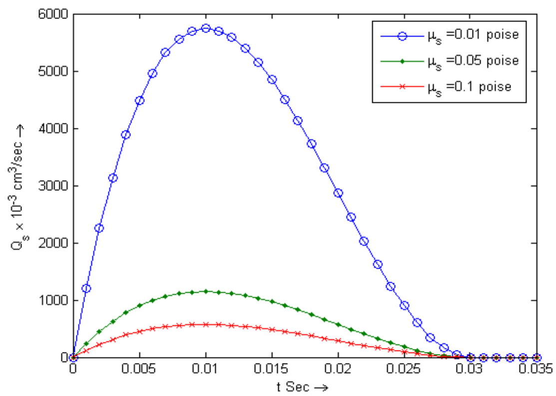

Figure 11: Variation of Q_s with t for different values of μ_s (for fixed air viscosity = 0.0002 poise and fixed value of δ = 0.2 mm)

Figures 3 ,4 and 5 illustrate when the size of constriction is increases then the volumetric flow rates of air ,mucus and serous fluid decreases.

Figures 6,7 and 8 illustrate when the size of constriction if fixed and mucus viscosity increases then their is such no effect on volumetric flow rates of air ,mucus and serous fluid.

Figures 9,10 and 11 illustrate when the size of constriction if fixed and serous fluid viscosity increases then the volumetric flow rates of air ,mucus and serous fluid decreases rapidly.

5 CONCLUSION

We have studied mucus transport in a diseased airway due to prolonged coughing, and the effect of constricted airway diameter on mucus transport and mucus viscosity was observed by representing it as a circular tube. A time-dependent pressure gradient has been observed in prolonged cough. The co-axial flow of air and mucus in a tube is considered to flow under steady laminar conditions. From the analysis of the model, the following results have been obtained:

- Mucus transport decreases in a constricted airway as maximum height of the constriction δ increases but mucus transport same as in normal airway when constriction is very small i.e.δ =0.
- The flow rates of every layer seen a tiny effect of mucus viscosity.
- Serous fluid transport decreases in a constricted airway as maximum height of the constriction δ increases and also decreases the flow of air and mucus.

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