

# May We Verify Non-Existing Dispersion Free Ensembles By Application of Quantum Mechanics in Experiments at Perceptive and Cognitive Level?

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## ABSTRACT

Von Neumann in 1932 was the first to outline the possible non-existence of dispersion free ensembles in quantum mechanics, and he used this basic evidence to give a preliminary proof on incompatibility between quantum mechanics and local hidden variables theory. In the present paper, we give a detailed theoretical elaboration on the manner in which such a fundamental subject could be explored at perceptive and cognitive levels in humans. We also discuss a general design of the experiment that we have in progress so to give direct indications to other researchers engaged in such field.

**Key Words:** application of quantum mechanics, psychology, neuroscience, dispersion free ensembles, quantum cognition, quantum contextuality, language analysis

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## Introduction

It is well known that in quantum mechanics Kochen-Specker (KS) theorem (Kochen, Specker, 1967), Peres (1991; 2002), and Mermin (1990) results rule out the non-contextual assignment of values to physical observables. One cannot assume, without falling into contradictions, that an observed entity can enjoy a separate well-defined identity irrespective of any particular context observing it.

Here we will discuss classical contextuality versus quantum intrinsic contextuality. In classical contextuality, the outcome is affected by various aspects of the

environment, but not by the irreducible and nonpredictable specifics of the interaction between the system and the experimental observation. When the situation is analyzed in terms of states, experiments, and outcomes, Kolmogorov's axioms are satisfied, and a classical probability model can be used. Starting with Aerts (Aerts *et al.*, 2000; 2005; 2011) and Khrennikov (Khrennikov *et al.*, 1999; 2004; 2010; Bulskiet *et al.*, 2002; Gribet *et al.*, 2006) the attempt has been made to introduce the basic notion of contextuality and thus quantum mechanics at the level of human cognitive performance. Further studies in this direction, recalling what previously was formulated by us in 1983, were elaborated by us at the theoretical and experimental level (Conte *et al.*, 1983; 2000; 2003; 2004; 2006; 2007; 2008; 2009; 2010; 2011), confirming the central role that could be explained from quantum mechanics in cognitive performance of humans.

Let us explain shortly the basic differences between quantum and classical contextuality. As an example of classical

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contextuality in a cognitive situation, we may consider a task in which we ask to a subject if he likes a shown object. The subject's answer would depend much more on the object than on how the question is posed. Of course, we may conceive other situations in which the outcome is determined through the interaction of the system with the irreducible and nonpredictable properties embedded in the measurement process. This is a case in which we are concerned instead with as intrinsic contextuality. The system and its observation both have an internal relational constitution, such that their interface creates a concrescence of emergent, dynamic patterns, reflecting the process theory of Alfred North Whitehead in 1927. The presence of intrinsic contextuality means that Kolmogorovian axioms are not satisfied, which renders the formal description of the entity non-classical, but quantum mechanical. This is a very important conclusion that of course recalls a result that we have recently obtained.

There are stages of our reality in which we can no longer separate the logic (and thus cognition and hence the conceptual entity) from the features of "matter per se". In quantum mechanics the logic, and thus the cognition and thus the conceptual entity-cognitive performance, assume the same importance as the features of what is being described. We are at levels of reality in which the truths of logical statements about dynamic variables become dynamic variables themselves, so that a profound link is established from in this theory between physics and conceptual entities from the start.

Human beings are adept at drawing context-sensitive and cognitive associations.

### Theoretical Elaboration

Consider now a given quantum system  $S$ . Physically speaking, the Hilbert space of the system  $S$  then contains wave functions belonging to different possible contexts. We may conceive such states as lying in a given energy shell in phase space. Let the considered dimensionality to be  $(m)$ . Each possible state of the system is then represented by a unit vector in the wave function  $\psi(t)$  of the system that lies in the appropriate space  $\Lambda_0(m)$  having dimension. Suppose the orthonormal vectors

$\omega_1, \omega_2, \dots, \omega_m$  are a basis for the projection of  $\psi(t)$  in a co-ordinate axis  $\omega_j$  is

$$c_{t,j}(a) \equiv (\psi(t), \omega_j); \quad j=1,2,\dots,m \quad (1)$$

and it depends directly on the considered context (a). The probability of finding the system at time  $t$  will be given by

$$p(t) = \sum_{j=1}^m |c_{t,j}|^2 \quad (2)$$

and depends on the context (a).

Consider now all the possible contexts. If such all contexts exist at all in principle, there will be many of them, and thus we may average over the different contexts. Consequently one may calculate the average probability of finding the system at a certain time, and it will be given by

$$\langle p(t) \rangle = \sum_{j=1}^m \langle |c_{t,j}|^2 \rangle \quad (3)$$

The spread of the individual values of  $p(t)$  due to the different contexts will be given by the second moment

$$Z = \frac{\langle (p(t) - \langle p(t) \rangle)^2 \rangle}{\langle p(t) \rangle^2} = \frac{\langle p(t)^2 \rangle}{\langle p(t) \rangle^2} - 1 \quad (4)$$

and it may be estimated experimentally.

In detail, consider also the notion of dispersion free ensembles. According to von Neumann

Given the observable  $\mathfrak{R}$ ,

An ensemble is dispersion free if

$$\langle \mathfrak{R}^2 \rangle = \langle \mathfrak{R} \rangle^2 \quad (5)$$

Reasoning in analogy, we conclude that  $Z \rightarrow 0$  in this case.

Let us take now a little step on. In this presentation, being only a brief technical note, we will not enter here in the specialized details of the very specialized field of mental lexicon. It represents a well-advanced field of research and applications that we cannot consider here for the sake of brevity. We will assume that it is well known to the reader and, in any case, we quote here the excellent papers of some authors (Bruza & Cole; 2005), suggesting the reader to read such papers and obviously the whole body of experimental and



theoretical research that has been previously developed and quoted in detail in such papers.

We limit our discussion here to the findings that language is inherently contextual. According to the previous mentioned authors, if we consider that the word “bat” has at least two meanings or *senses* in its standard form. It might refer to a flying mammal that lives in caves, or alternatively it might refer to a sporting implement. This is only a very restricted example. Actually, we may relate the word “bat” to a very large number of contexts. Generally, we can tell the *sense* that another speaker intends through a consideration of the context in which the word appears. These different *senses* of a word can be explored via word association experiments (Bruza & Cole, 2005).

In free association, words are presented to large samples of participants who produce the first associated word to come to mind. The probability or strength of a pre-existing link between words is computed by dividing the production frequency of a response word by its sample size. We can also find out which words are likely to produce the word ‘bat’ (now called a target). For example, the word “ball” may be associated with “bat” as in baseball, but is unlikely to be associated with the word “bat” as in the flying animal. The associational matrix is similar to the quantum matrix or wavefunction. Elements in the association matrix may be associated with multiple senses, as, for example, “bat” may be associated with “swing,” “hit,” “wood,” “boy,” etc. One way of achieving such an association involves a process known as *extra-list cueing*. Here, subjects typically study a list of to-be-recalled target words. For widening on the experiments, we direct the reader (Bruza & Cole, 2005).

### Design of the Experiment and Preliminary Conclusions

Starting with a general model based upon the notion of superposition of states as it is intended in quantum mechanics, the authors in (Bruza & Cole, 2005) introduced a model of the observed behaviour of associative networks and they also developed more sophisticated models of how concepts combine (Bruza & Cole, 2005). These models treat context by representing it as cue words or co-appearing words, and experiments are

currently underway to test the validity of these models.

The basic assumption is that the words take different senses depending upon the context in which they occur. As example, when shown out of context, ‘bat’ reminds people of ‘cave’, sporting people and so on. The problem is to estimate the probability of recalling ‘bat’ when some context is present. With reference to the initial elaboration given in (Eq.1-4) let us restrict our “semantic” Hilbert space to  $m=2$ , and identify by  $\omega_1$  the cognitive state of recalling and by  $\omega_2$  the complementary state of not recalling. The recall (or not) of a word can be represented using a superposition state,

$$\psi(t) = c_{1,t}\omega_1 + c_{2,t}\omega_2 \quad (6)$$

with specific notations given in the (Eq.2).

This is the word  $w$ , represented in some context ( $a$ ), as a superposition of recalled, and not recalled. Thus, the word ‘bat’ is a target word, expected to be recalled in an extra-list cueing experiment upon presentation of the cue word ‘cave’ which in this case acts as the context ( $a_1$ ). We have

$$\psi(t) = c_{1,t}(a_1)\omega_1 + c_{2,t}(a_1)\omega_2 \quad (7)$$

The probability of ‘bat’ being recalled in this context is represented by  $|c_{1,t}(a_1)|^2$ , and the probability of not being recalled by  $|c_{2,t}(a_1)|^2$ ,

When given the cue word ‘ball’ we represent ‘bat’ as the new superposition

$$\psi(t) = c_{1,t}(a_2)\omega_1 + c_{2,t}(a_2)\omega_2 \quad (8)$$

where ( $a_2$ ) represents the new context “ball” and the new probabilities result now modified as

$$|c_{1,t}(a_2)|^2, \text{ and } |c_{2,t}(a_2)|^2,$$

respectively, and assuming obviously totally different values respect to the previous case just as they may be retrieved from memory when a subject is presented with the cue ‘ball’ than the cue word ‘cave’.

We would suggest here a general formalism to represent that it provides a very natural representation of contextual effects as they actually occur in language. It is evident that we could continue with the word “bat”, considering each time a different

context  $(a_1, a_2, \dots, a_n)$ , and thus obtaining each time a different representation of the assumed quantum superposition (recalling-not recalling) with different values of the coefficients  $c_{1,t}(a_i)$  and  $c_{2,t}(a_i)$  ( $i=1,2,\dots,n$ ), and each time different values of probabilities. Consequently, we may apply the (Eq.3) in the case  $m=2$ , considering that we may estimate finally the average probability, valued on all the considered contexts. As consequence, selected a given context  $(a_i)$ , we may calculate the spread of the individual value by using the coefficient  $Z$  as indicated in (Eq.3) by the second moment of the considered experimentation.

The simplest assumption is that we have a very large number of alternative possibilities, and that, in order to properly characterize such situation, we must use a continuous distribution. Without loss of generality we may consider

$$c_1 = \cos \alpha \text{ and } c_2 = \sin \alpha ; p_{\text{recalling}} = \cos^2 \alpha ; p_{\text{not-recalling}} = \sin^2 \alpha ; p_{+,-} \text{ being probabilities}$$

where anyone value of  $\alpha$  ( $0 \leq \alpha < 2\pi$ ) now characterizes a different, possible context. The probability of finding an angle in the range  $(\alpha, \alpha + \delta\alpha)$  will be given by

$$f(\alpha) = A \sin^b(2\alpha) d\alpha \quad (9)$$

where  $A$  is a normalizing constant and for a strictly uniform distribution we have ( $b=0$ ), while for a weakly uniform distribution we have possible values ( $b=2$ , or  $b=4$ ) and so on. Under the different theoretical as well as experimental situations, we may also consider more restricted range of possible values for  $\alpha$  as ( $0 \leq \alpha < \pi$ ) or ( $0 \leq \alpha < \pi/2$ ), and so on.

First, consider the very interesting case in which the possible contexts obey a law of strictly uniform distribution.

Generally speaking, we know that, given the density function of probability  $f(x)$ , it must be

$$\int_a^b f(x) dx = 1, \langle x \rangle = \int_a^b x f(x) dx, \langle x^2 \rangle = \int_a^b x^2 f(x) dx \quad (10)$$

In the case of strictly uniform distribution ( $b=0$ ), we obtain that for ( $0 \leq \alpha < 2\pi$ )

$$A = 1/2\pi, \langle p_+ \rangle = \langle p_- \rangle = 1/2 ; \langle p_+^2 \rangle = \langle p_-^2 \rangle = 3/8 \quad (11)$$

In this case, under experimentation, we expect to be:

- a) The (Eq.5) is violated (not existing dispersion free ensembles)
- b) The  $Z$ -value, given in (Eq.4) furnishes  $Z = 0.5$
- c) Finally,  $\langle (p_{+,-}(\text{generic } \alpha) - 0.5)^2 \rangle = 1/8$ .

Let us examine now the case of a weakly uniform distribution in ( $0 \leq \alpha < 2\pi$ ), ( $b=2$ ) in the (Eq.9). It results that

$$A = 1/\pi, \langle p_+ \rangle = \langle p_- \rangle = 1/2 ; \langle p_+^2 \rangle = \langle p_-^2 \rangle = 5/16$$

Under experimentation, we expect to be:

- d) The (Eq.5) is violated (not existing dispersion free ensembles)
- e) The  $Z$ -value, given in (Eq.4) furnishes  $Z = 0.25$
- f) Finally,  $\langle (p_{+,-}(\text{generic } \alpha) - 0.5)^2 \rangle = 1/16$ .

For  $b=4$ , we have that

$$A = 4/3\pi, \langle p_+ \rangle = \langle p_- \rangle = 1/2 ; \langle p_+^2 \rangle = \langle p_-^2 \rangle = 7/24$$

Under experimentation we should find that

- g) The (Eq.5) is violated (not existing dispersion free ensembles)
- h) The  $Z$ -value, given in (Eq.4) furnishes  $Z = 0.16$
- i) Finally,  $\langle (p_{+,-}(\text{generic } \alpha) - 0.5)^2 \rangle = 0.04$ .

We do not expect the results to change radically exploring contexts in the range ( $0 \leq \alpha < \pi/2$ ).

Finally, let us evidence further the conceptual foundations and the malleability of the formulation that we have introduced.

Let us examine this time our elaboration for contexts ranging with ( $0 \leq \alpha < \pi/4$ ), ( $b=2$ ). It results that

$$A = 8/\pi, \langle p_+ \rangle = 0.71 ; \langle p_- \rangle = 0.29 ; \langle p_+^2 \rangle = 0.52 ; \langle p_-^2 \rangle = 0.10$$

Under experimentation we should find now that

j) The (Eq.5) is violated (not existing dispersion free ensembles )

k)  $Z_+ = 0.03$ ;  $Z_- = 0.19$

l)  $\langle (p_+(\text{generic } \alpha) - 0.71)^2 \rangle = 0.0151$ ;  
 $\langle (p_-(\text{generic } \alpha) - 0.29)^2 \rangle = 0.0159$

It is clearly seen that the situation is now profoundly modified. Obviously, we could have selected contexts ranging instead with  $(\pi/4 \leq \alpha < \pi/2)$  and, in principle; we could experience a different kind of analytical expression of the density probability function instead of (Eq.9). The conclusion is that, starting with the basic formulation given by the authors in (Bruza & Cole; 2005), we have

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given here a little conceptual extension that may be well built by arranging appropriate experiments.

The most promising evidence is that by this methodology we may also analyse the presence or not of dispersion free ensembles. We remember here a datum that may be of basic interest when exploring quantum cognition. von Neumann in 1932 (1996) was the first to outline the possible non existence of dispersion free ensembles in quantum mechanics, and he used also such basic evidence to give a preliminary proof on incompatibility between quantum mechanics and local hidden variables theory.

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