On Complex Fuzzy Graph

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Abstract

In a fuzzy set the range of the fuzzy membership value is limited to [0,1] in a real line, but the complex fuzzy set it will be extended to a unit circle in the complex plane. Complex fuzzy graph is an extension of fuzzy graph. This research explores about the complex fuzzy graph, complex fuzzy subgraph and complex partial fuzzy subgraphs. The study reveals about the degree, order and size of complex fuzzy graph. Some theorems on complex fuzzy graphs are stated and proved in this article.

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1.Introduction

In 1965, the concept of fuzzy set was initiated by Zadeh [8]. In a fuzzy set the membership value of a member lies in a closed interval [0,1]. The complex fuzzy set is a novel expansion of fuzzy set which was introduced by Daniel Ramot et.al in 2002 [2]. The range of member of a complex fuzzy set is not only to [0,1], but it is expanded to a unit circle in a complex plane.

In 1973, Kauffman [5], gave the definition of fuzzy graphs and Rosenfeld et.al [8] gave multiple fuzzy analogues of the concepts of graph theories like cycles, paths etc. The fuzzy complex number concept was investigated by J.J.Buckley [1] in the year 1989. They also investigated the two kinds fuzzy complex numbers based on the forms $z = x + iy$ and $z = re^{i\theta}$. Dong Qiu [3] pointed out some errors in fuzzy complex analysis problems in the year 2008. Another important extension of fuzzy logic theory is complex fuzzy logic which offers the inference systems and basic for control related to complex phenomena which cannot readily adopted in different fuzzy type sets. Naveed Yaqoob et.al [7] associated the objective of graph theory and complex intuitionistic fuzzy sets and discovered the complex intuitionistic fuzzy graphs in the year 2019. They introduced the join, unionand composition of complex intuitionistic fuzzy graph. NagoorGaniA and Basheer Ahamed M (2003) [6] discussed about the size and order of fuzzy graphs. The energy of complex fuzzy graph was discussed by Thirunavukarasu [9]. For crisp graph we can refer the book Graph theory by Harary[4].

In this paper, the order, size, degree, neighbourhood and different complex fuzzy graphs are defined in a complex fuzzy environment. Some theorems based on these concepts are stated and proved.

2.Preliminaries

In this section, the basic concept of complex fuzzy set, complex fuzzy graph and some preliminary definitions are given for further understanding.

Definition 1: Let U be a universe of discourse and $Z \subseteq U$. A complex fuzzy set (CFS) σ_c is defined by $\sigma_c = \{z/r(z)e^{i\theta(z)} : z \in Z\}$ where $r(z)$ is an amplitude and $\theta(z)$ is a phase term of z, $i = \sqrt{-1}$, $0 \leq r(z) \leq 1 \; and \; 0 \leq \theta(z) \leq 2\pi.$

Example 1: An example for complex fuzzy set σ_c is given below.

$$\sigma_c = \{z_1 \,/\, 0.2\, e^{i\pi}, z_2 \,/\, 0.8\, e^{i0.5\pi}, z_3 \,/\, 0.3\, e^{i2\pi}, z_4 \,/\, 0.25\}$$

Definition 2: The complement of a CFS of σ_c is a CFSit is denoted by $\overline{\sigma_c}$ and is defined by $\overline{\sigma_c} = \{z/(1 - r(z))e^{i\theta(z)}: z \in Z\}$.

Example 2: Consider the example 1, the complement of σ_c is given by

$$\overline{\sigma_c} = \{z_1 \,/\, 0.8\, e^{i\pi}, z_2 \,/\, 0.2\, e^{i0.5\pi}, z_3 \,/\, 0.7\, e^{i2\pi}, z_4 \,/\, 0.75\}$$

Remark 1: Complement of a complement CFS is a CFS, (i.e) $\overline{\overline{\sigma_c}}$=$\sigma_c$.

Definition 3: Let $\sigma_c = \{z/r(z)e^{i\theta(z)}: z \in Z\}$ and $\tau_c = \{z/r_1(z)e^{i\theta_1(z)}: z \in Z\}$ be any two CFS. If the complex fuzzy subset τ_c of σ_c, (i.e., $\tau_c \subset \sigma_c$) then $r_1(z) \le r(z)$ and $\theta_1(z) \le \theta(z)$ for all $z \in Z$.

Example 3: Consider the example 1, the complex fuzzy subset τ_c of σ_c is

$$\tau_c = \{z_1 \,/\, 0.1\, e^{i0.5\pi}, z_2 \,/\, 0.5\, e^{i0.5\pi}, z_3 \,/\, 0.3\, e^{i\pi}, z_4 \,/\, 0.1\}$$

Definition 4: A complex fuzzy graph (CFG) $G_c = (\sigma_c, \mu_c)$ defined on a graph $G = (V, E)$ is a pair of complex functions $\sigma_c: V \to r(z)e^{i\theta(z)}$, $\mu_c: E \subseteq V \times V \to R(e)e^{i\phi(e)}$ such that μ $c(z_1, z_2) = R(e)\, e^{i\phi(e)}$, where $R(e) \le \min\{r(z_1), r(z_2)\}$ and $\phi(e) \le \min\{\theta(z_1), \theta(z_2)\}$ for all $z_1, z_2 \in V$ and $0 \le r(z_1), r(z_2) \le 1$ and $0 \le \theta(z_1), \theta(z_2) \le 2\pi$.

Example 4:$G_c = (\sigma_c, \mu_c)$ is a complex fuzzy graph, where $\sigma_c = \{z_1 \,/\, 0.2\, e^{i\pi}, z_2 \,/0.5\, e^{i0.5\pi}, z_3 \,/\, 0.7\, e^{i\pi}\}$, $\mu_c = \{(z_1, z_2)/0.1\, e^{i0.5\pi}, (z_2, z_3)/\, 0.5\, e^{i0.2\pi}, (z_1, z_3) \,/\, 0.1\}$.

3. Types of complex fuzzy graph

In this section, the order and size of CFG are defined and some well-known graphs are discussed in complex fuzzy environment.

Definition 5: The order p and size q of a CFG, $G_c = (\sigma_c, \mu_c)$ on $G = (V, E)$ are defined by 169
$p = \sum_{z \in V} r(z).\ e^{i\ \sum_{z \in V} \theta(z)}$; $q = \sum_{e \in E} R(e).\ e^{i\ \sum_{e \in E} \phi(e)}$

Example 5: Consider the example 4, $p = 1.4\, e^{i2.5\pi}$, $q = 0.7\, e^{i0.7\pi}$

Definition 6: The degree of a vertex z_i, in a CFG, $G_c = (\sigma_c, \mu_c)$ on $G = (V, E)$ is defined by $d(z_i) = \sum_{e=(z_i,z_j) \in \mu_c} R(e).\ e^{i.\ \sum_{e=(z_i,z_j) \in \mu_c} \phi(e)}$ such that $\mu_c(z_i, z_j) = R(e)e^{i\phi(e)}$, for all $z_j \in \sigma_c$.

Example 6: Consider the example 4, $d(z_1) = 0.2\, e^{i0.5\pi}, d(z_2) = 0.6\, e^{i0.7\pi}, d(z_3) = 0.6\, e^{i0.2\pi}$.

Note: $e^{i(2n\pi + \theta)} = e^{i\theta}, n \in Z$.

Definition 7: A CFG is said to be complete if for every pair of vertices (z_i, z_j) such that $\mu_c(z_i, z_j) = \min\{r(z_i), r(z_j)\}\, .\, e^{i \min\{\theta(z_i), \theta(z_j)\}}$ for all $z_i, z_j \in V$.

Example 7: Let $G_c = (\sigma_c, \mu_c)$ be a CFG, $\sigma_c = \{z_1 \,/\, 0.5\, e^{i0.7\pi}, z_2 \,/\, 0.8\, e^{i\pi}, z_3 \,/\, 0.6\, e^{i\pi}\}$, $\mu_c = \{(z_1, z_2)/0.5\, e^{i0.7\pi}, (z_2, z_3)/\, 0.6\, e^{i\pi}, (z_1, z_3) \,/\, 0.5e^{i0.7\pi}\}$. Then G_c is a complete complex fuzzy graph.

Definition 8:$G_{c_1} = (\rho_c, \tau_c)$on $G_1 = (V_1, E_1)$is a partial complex fuzzy subgraph of a CFG $G_c = (\sigma_c, \mu_c)$on $G = (V, E)$if $\rho_c \subseteq \sigma_c$ and $\tau_c \subseteq \mu_c$. Similarly, G_{c_1} is a complex fuzzy subgraph of $G_c$induced by V_1if$V_1 \subseteq V$, $\rho_c = \sigma_c$ $for\ all$ $z \in V_1$ and $\tau_c = \mu_c$ $for\ all$ $z_1, z_2 \in V$.

Definition 9: A complex fuzzy graph $\overline{G_c} = (\overline{\sigma_c}, \overline{\mu_c})$ is said to be a complement of CFG G_c if

i) $\overline{\sigma_c}(z) = \sigma_c(z)$ and

ii) $\overline{\mu_c(z_1, z_2)} = \overline{R(e)}e^{i\overline{\phi(e)}}$, where $\overline{R(e)} = \min\{r(z_1), r(z_2)\} - R(e)$ and $\overline{\phi(e)} = \min\{\theta(z_1), \theta(z_2)\} - \phi(e)$, for all $z_1, z_2 \in V$.

Example 8: Consider the example 4, the complement of CFG is given by $\overline{\sigma_c} = \{z_1 \,/\, 0.2\, e^{i\pi}, z_2 \,/\, 0.5\, e^{i0.5\pi}, z_3 \,/\, 0.7\, e^{i\pi}\}$, and $\overline{\mu_c} = \{(z_1, z_2)/0.1\, , (z_1, z_3)/\, 0.1e^{i\pi}\}$.

Remark 2: For any complement of a complement CFG is also a CFG, symbolically $\overline{\overline{G_c}}$=$G_c$

Definition 10: In a CFG $G_c = (\sigma_c, \mu_c)$ for all $z_i, z_j \in \sigma_c$, the neighbourhood of z_i is defined by $N(z_i) = \{z_j \in \sigma_c/(z_i, z_j) \in \mu_c\}$.

Definition 11: A path P in a CFG $G_c = (\sigma_c, \mu_c)$is a sequence of distinct vertices $z_0, z_1, z_2, \dots, z_n \in V$ (except possibly z_0 and z_n) such that $\mu_c(z_{i-1}, z_i) = R(e_i). e^{i\phi(e_i)}, R(e_i) > 0, \phi(e_i) \ge 0, i = 1,2,3, \dots, n.$ Herethe path's length is n.The successive pairs are said to beedges of the path. The strength of the path in a CFG is defined by $\mu_c(z_{i-1}, z_i) = min\, R(e_i). e^{imin\phi(e_i)}, i = 1,2,3, \dots, n$ and is denoted by S(p).

Definition 12: The strength of connectedness between two vertices $z_i$and z_j which is defined as the maximum amplitude and maximum phase term values of the strength of all paths between z_i and z_j. In symbol we denote it as $\mu_c^\infty(z_i,z_j) = CONN_{G_c}(z_i,z_j)$, $\mu_c^\infty(z_i,z_j)$=$S(e).e^{i\psi(e)}$, $0 \leq S(e) \leq 1, 0 \leq \psi(e) \leq 2\pi$, where $S(e)$ is maximum amplitude value of all paths between z_i and z_j and $\psi(e)$ is maximum phase term value of all paths betweenz_i and z_j. An arc $e = (z_i,z_j)$ is said to be strong if $R(e) \geq S(e)\ and\ \phi(e) \geq \psi(e)$.

Example 9: Consider the CFG $G_c = (\sigma_c, \mu_c)$ where $\sigma_c = \{z_1 / 0.2\ e^{i\pi}, z_2 / 0.5\ e^{i\pi}, z_3 / 0.7\ \}$ $\mu_c = \{e_1 = (z_1, z_2)/0.2\ e^{i0.25\pi}, e_2 = (z_2, z_3)/\ 0.5, e_3 = (z_1, z_3)\ /\ 0.1\}$. The strength of the connectedness of $z_1\ and\ z_3$ is $0.2\ e^{i0.25\pi}$. Here there are two paths $(i)\ P_1: z_1, e_1, z_2, e_2, z_3$the strength is $0.2\ e^{i0.25\pi}(ii)\ P_2: z_1, e_3, z_3$the strength is 0.1, the maximum strength is $0.2\ e^{i0.25\pi}$, $e_1\ and\ e_2$ are strong edges and e_3 is not a strong edge.

Definition 13: A complex fuzzy graph $G_c = (\sigma_c, \mu_c)$ is regular if $d(z_i) = d(z_j)$ for all $z_i,z_j \in \sigma_c$.

Definition 14: The addition of membership values of strong arcs incident at a vertex z is called the strong degree of that vertex. In symbol we write it as$d_s(z)$.

Definition 15: The strong neighbourhood of z_i is defined by $N_s(z_i) = \{z_i \in V / (z_i, z_j)$ is a strong arc$\}$

Definition 16: A CFG $G_c = (\sigma_c, \mu_c)$ is known to be bipartite, the set of all vertices in$\sigma_c$can be split into two non-empty sets $\sigma_{c_1}\ and\ \sigma_{c_2}$ such that $\mu_c(z_i,z_j)$=0 if $z_i,z_j \in \sigma_{c_1}\ and\ z_i\ ,\ z_j \in \sigma_{c_2}$.

Definition 17:A CFG $G_c = (\sigma_c, \mu_c)$ is known to be complete bipartite, the set of all vertices of σ_c can be split into two non-empty sets $\sigma_{c_1}\ and\ \sigma_{c_2}$ such that $\mu_c(z_i, z_j) = R(e)e^{i\phi(e)}$, where $R(e) = \min\{r(z_i), r(z_j)\}, \phi(e) = \min\{\theta(z_i), \theta(z_j)\}$ for $z_i \in \sigma_{c_1}$ and $z_j \in \sigma_{c_2}$. 170

Definition 18: A vertex z_i of a complex fuzzy graph $G_c = (\sigma_c, \mu_c)$ is said to be an isolated vertex if $\mu_c(z_i, z_j) = 0, \forall\ z_j \in V - \{z_i\}$. (ie) $N(z_i) = \emptyset$.

4. Main Results

In this section some theorems related to degree, strong degree, complex fuzzy cycle are stated and proved.

Theorem 4.1: In a CFG G_c the sum of degree of all vertices is equal to two times the sum of membership values of all the edges. Symbolically,

$$\sum_{z_i \in V} d(z_i) = 2 \sum_{(z_i,z_j) \in \mu_c} R(e)\,.\,e^{i2\sum_{(z_i,z_j)\in\mu_c}\phi(e)} \text{and } z_i \neq z_j$$

Proof:

Let $G_c = (\sigma_c, \mu_c)$ be a CFG of graph $G = (V, E)$ is a pair of complex functions $\sigma_c = V \to r(z)e^{i\theta(z)}, \mu_c: E \subseteq V \times V \to R(e)e^{i\phi(e)}$, such that $\mu(z_1, z_2) = R(e).e^{i\phi(e)}$, where $R(e) \leq \min(r(z_1), r(z_2)), \phi(e) \leq \min\{\theta(z_1), \theta(z_2)\}$ and $0 \leq r(z), R(e) \leq 1, 0 \leq \theta(z), \phi(e) \leq 2\pi$. We have

$$\sum_{z \in V} d(z) = \sum_{i=1}^{n} d(z_i) = d(z_1) + d(z_2) + \cdots + d(z_n)$$

degree of each vertex in σ_c is given by

$$d(z_1) = \sum_{(z_1,z_j)\in\mu_c} R(e)\,.\,e^{i\sum_{(z_1,z_j)\in\mu_c}\phi(e)}, \forall z_j \in \mu_c, j = 2,3, \dots, n$$

$$d(z_2) = \sum_{(z_2,z_j)\in\mu_c} R(e)\,.\,e^{i\sum_{(z_2,z_j)\in\mu_c}\phi(e)}, \forall z_j \in \mu_c, j = 1,3, \dots, n$$

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$$d(z_n) = \sum_{(z_n,z_j)\in\mu_c} R(e)\,.\,e^{i\sum_{(z_n,z_j)\in\mu_c}\phi(e)}, \forall z_j \in \mu_c, j = 1,2,3, \dots, n-1$$

Sum of degree of all the vertices is

$= d(z_1) + d(z_2) + \cdots + d(z_n)$

$$= 2 \sum_{(z_1,z_j)\in\mu_c} R(e) \,.\, e^{i2.\sum_{(z_1,z_j)\in\mu_c}\phi(e)}, z_i \neq z_j$$

Hence the theorem.

Theorem 4.2: The sum of strong degrees in a CFG is twice the sum of membership values of strong arcs.

Proof:

Let $G_c = (\sigma_c, \mu_c)$ be a CFG of graph $G = (V, E)$ is a pair of complex functions $\sigma_c = V \to r(z)e^{i\theta(z)}, \mu_c: E \subseteq V \times V \to R(e)e^{i\phi(e)}$. Let W be any subset of V an arc between any two vertices of W is a strong arc. By the definition of degree of a vertex,

$$d_s(z_1) = \sum_{(z_1,z_j)\in\mu_c} R(e) \,.\, e^{i.\sum_{(z_1,z_j)\in\mu_c}\phi(e)}, \forall z_j \in \sigma_c, j = 2,3,\dots,n$$

$$d_s(z_2) = \sum_{(z_2,z_j)\in\mu_c} R(e) \,.\, e^{i.\sum_{(z_2,z_j)\in\mu_c}\phi(e)}, \forall z_j \in \sigma_c, j = 1,3,\dots,n$$

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$$d_s(z_n) = \sum_{(z_n,z_j)\in\mu_c} R(e) \,.\, e^{i.\sum_{(z_n,z_j)\in\mu_c}\phi(e)}, \forall z_j \in \sigma_c, j = 1,2,3,\dots,n-1$$

Sum of degree of all the vertices is

$$= d_s(z_1) + d_s(z_2) + \cdots + d_s(z_n)$$

$$= 2 \sum_{(z_i,z_j)\in\mu_c} R(e) \,.\, e^{i2.\sum_{(z_i,z_j)\in\mu_c}\phi(e)}, \qquad z_i \neq z_j$$

Hence the theorem.

Theorem 4.3: In a complete CFG the degree of atleast a pair of vertices is equal in amplitude value but need not be in phase terms.

Proof:

Let $G_c = (\sigma_c, \mu_c)$ be a complete CFG defined on $G = (V, E)$, $\sigma_c = V \to r(z)e^{i\theta(z)}$, $\mu_c: E \subseteq V \times V \to R(e)e^{i\phi(e)}$ such that $\mu(z_1, z_2) = R(e)e^{i\phi(e)}$ where $R(e) = \min\{r(z_1), r(z_2)\}$, $\phi(e) = \min\{\theta(z_1), \theta(z_2)\}, \forall\, z_1, z_2 \in V$.

Case (i): Suppose $\sigma_c(z)$ are equal, $\forall\, z \in V$. Obviously $\mu(z_1, z_2)$ are all equal $\forall\, z_1, z_2 \in V$. Hence the degree of all the vertices is same.**Case (ii):** Suppose $\sigma_c(z_i)$ are distinct $\forall\, z_i$ such that $\sigma_c(z_1) \neq \sigma_c(z_2) \neq \cdots \neq \sigma_c(z_n)$. Since the graph is complete, from the definition of degree of a vertex, the degree of any two vertices is equal in magnitude value but not in phase terms. Because an edge is the minimum amplitude value of any one of the two vertices which is incident with in it. But the phase term value of that edge is need not be the concern vertex. Hence the theorem.

Remark 2: In a complete CFG the degree of atleast a pair of vertices is equal in amplitude value but not in phase terms.

Counter Example: Consider the CFG $G_c = (\sigma_c, \mu_c)$, where $\sigma_c = \{z_1/0.4e^{i\pi/2}, z_2/0.6e^{i\pi/4}, z_3/0.2e^{i3\pi/4}\}$ and $\mu_c = \{(z_1, z_2)/0.4e^{i\pi/4}, (z_2, z_3)/0.2e^{i\pi/4}, (z_1, z_3)/0.2e^{i\pi/4}\}$ and $d(z_1) = 0.6e^{i3\pi/4}, d(z_2) = 0.6e^{i\pi/2}$ degree of z_1 and z_2 is equal in amplitude value but not in phase value. 171

Theorem 4.4: Let $G_c$any CFG it holds following inequality, $|\delta(G_c)| \leq |\Delta(G_c)| \leq |S(G_c)| \leq |O(G_c)|$. In symbolically, $|\delta(G_c)| \leq |\Delta(G_c)| \leq |q| \leq |p|$.

Proof:

Let $G_c = (\sigma_c, \mu_c)$ be a CFG, where $\sigma_c: V \to r(z)e^{i\theta(z)}$ and $\mu_c: E \subseteq V \times V \to R(e)e^{i\phi(e)}$ such that $\mu(z_1, z_2) = R(e)e^{i\phi(e)}$, $R(e) \leq \min\{r(z_1), r(z_2)\}, \phi(e) \leq \min\{\theta(z_1), \theta(z_2)\}$. Since $\delta(G_c)$ and $\Delta(G_c)$ denotes the smallest and largest degree of G_c. Obviously $|\delta(G_c)| \leq |\Delta(G_c)|$ -------(1) we have

$$O(G_c) = \sum_{z\in V} r(z)\, e^{i\sum_{z\in V}\theta(z)}, \forall\, z$$

$$|O(G_c)| = \left|\sum_{z\in V} r(z)\right|$$

$$S(G_c) = \sum_{(z_i,z_j)\in\mu_c} R(e)\,.\,e^{i\sum_{(z_i,z_j)\in\mu_c}\phi(e)}$$

$$|S(G_c)| = \left|\sum_{(z_i,z_j)\in\mu_c} R(e)\right|$$

$|S(G_c)| \geq |\Delta(G_c)|$ --------(2)

Also, in any CFG $\mu_c(z_1, z_2) = R(e)\, e^{i\phi(e)}$, where $R(e) \leq \min\{r(z_1), r(z_2)\}$ and $\phi(e) \leq \min\{\theta(z_1), \theta(z_2)\}$

$|O(G_c)| \geq |S(G_c)|$ ---------(3)

From (1), (2) and (3), we get

$|\delta(G_c)| \leq |\Delta(G_c)| \leq |S(G_c)| \leq |O(G_c)|$.

Definition 4.5: In a complex fuzzy graph the edge which is not strong is called the weak edge.

Theorem 4.6: A complex fuzzy graph remains connected even after deletion of all weak edges.

Proof:

Let G_c be a CFG and $\mu_c^{\infty}(z_1, z_2)$ indicates the strength of connectivity between the vertices z_1 and z_2. Let e_0 represent a weak edge in G_c and $G_c^{'} = G_c - e_0$. To demonstrate $G_c^{'}$ is connected. We will demonstrate this through the tactic of contradiction. Assume that $G_c^{'}$ is not connected. Let $e_0 = (z_1, z_2)$ separates the graph $G_c^{'}$ into more than one component that means there is no path between z_1 and z_2 other than edge e_0 in G_c, $\mu_c(z_1, z_2) = \mu_c^{\infty}(z_1, z_2)$, which is a contradiction.

Definition 4.7: A cycle is called a complex fuzzy cycle C_{σ_c}, if atleast two edges have same minimum membership values in amplitude and phase terms in G_{σ_c}.

Theorem 4.8: All arcs in a complex fuzzy cycle are strong.

Proof:

Let C_{σ_c} is a complex fuzzy cycle. We can establish this theorem using the contradiction approach.Assume that all arcs in a complex fuzzy cycle are not strong. Then there is atleast one weak edge (i.e) the smallest amplitude value and corresponding phase value of $\mu_c(z_i, z_j)$ in G_c. Let us talk about the verification in two cases. **Case (i):** Only one weak edge exists. It has minimum amplitude and phase value than any other edges. But the complex fuzzy cycle must have atleasttwo edges with same amplitude and phase value. This contradicts that not all arcs are strong. **Case (ii):** If there is more than one edge with smallest amplitude and phase value then it will not be considered as a weak edge. Hence the theorem.

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